New Prediction Models for Creep and Shrinkage of Concrete

by H.S. Muller

Synopsis: The Comité Euro-International du Béton (CEB) has prepared a new model code for the design and analysis of concrete structures (CEB-FIP Model Code 1990) which includes new prediction models for creep and shrinkage of concrete. These models have been derived and optimized on the basis of a computerized data bank. For the prediction of shrinkage a diffusion theory type model has been chosen. The prediction of creep is based on a simple product type approach. Though the new creep model resembles some of the features of the model presented by ACI 209, various basic improvements could be achieved. The coefficients of variation for shrinkage and creep have been found to be approximately 33% and 20%, respectively. The developed prediction models, both for creep and shrinkage represent a reasonable compromise of accuracy and simplicity. They meet the requirements for presentation in a code. In this paper both models are presented and some comparisons with test data are shown.

Keywords: Coefficient of variation; creep properties; models; modulus of elasticity; shrinkage; stresses; temperature

H. S. Müller is Director and Professor at the Federal Institute for Materials Research and Testing (BAM) in Berlin, Germany. He holds a PhD in civil engineering from the Technical University of Karlsruhe, Germany. He is an active member of the Comité Euro-International du Béton (CEB) and of ACI Committee 209.

GENERAL

The analytical relations given for creep and shrinkage in chapter 2.1 of CEB-FIP Model Code 1990 (CEB MC 90; CEB = Euro-International Committee for Concrete; FIP = International Federation for Prestressing) [1] are based primarily on the work of the former CEB General Task Group 9 (GTG 9) "Evaluation of the time dependent behavior of concrete" [2]. The prediction methods have been developed on the basis of a data bank on laboratory test results on structural concrete at normal ambient temperatures. Only such parameters which are known to the designer in practically all cases have been taken into account in the prediction methods. To avoid any confusion, all equations have been given in a non-dimensional presentation.

The models have been developed primarily to predict the mean cross-section behavior of concrete, i.e. the mean properties of a given cross-section are estimated considering the average relative humidity and member size. Local stress and moisture states as well as local cracking cannot be taken into account with such a model. As a crude approximation, however, such models may also be used as a basis for a FE point by point analysis if the relations for very thin sections are properly calibrated and other relations given in chapter 2.1 of CEB MC 90 [1] are taken into account such as the data on fracture properties and the data on moisture movement.

CREEP

Background and Range of Validity

In modeling the creep behavior of concrete as a linear code-type approach two types of formulations have been used in the past: The summation and the product models. They differ in their mathematical approach. Some aspects will be briefly summarized in the following; details may be found in [2], [3], [4].

The product model (also called: aging creep model) is characterized by the feature that creep is calculated from the product of a function describing the effect of age at loading and a function describing the effect of duration of loading. The prediction methods given in various codes and recommendations, e.g.

in [5], [6] and [7], and also the BP Model [8] belong to the group of product models.

The specific feature of the summation model (also called: rate of flow model or improved Dischinger) is the separation of creep into delayed elasticity and flow. The formulation of the flow term reveals an additional characteristic of the summation model, i.e. the effect of age at loading and the time development are expressed by one unique time function. This type of creep prediction model underlies the approaches given in the German Standard DIN 4227 [9] (DIN = German Institute for Standardization) and in principle also the method given in CEB-FIP Model Code 1978 [10].

The differences between the two types of linear creep models become particularly evident when the models are used to predict the effect of variable stresses or strains in combination with the principle of superposition. However, even for the prediction of creep under constant stress, some systematic differences may be observed. Fig. 1 summarizes the basic differences of the models concerning their accuracy of prediction for various load histories. It should be emphasized that this comparison considers the models itself in view of their mathematical approach and evaluates their aptness to predict various effects. Nevertheless, the conclusions which may be drawn from this comparison are also valid for prediction methods as given in recommendations, which are developed on the basis of these models.

It is obvious that neither approach gives correct answers in all cases. This is also true for improved product and summation models as they have been presented e.g. in [8], [4] and [2]. The major reason for the weaknesses of the models is that creep is in fact a nonlinear phenomenon which can only be modeled correctly by means of a nonlinear approach. In addition, creep is strongly interrelated with shrinkage and elastic strain. Where linear prediction models are under consideration, a summation model as presented e.g. in [2] and extended in [11] seems to be the best compromise. However, there are loading cases where such an approach may still be erroneous. Thus, the controversy about the "best" linear creep law will continue.

In CEB MC 90 a product type formulation has been chosen because it can be presented and dealt with in a simpler manner than a summation formulation [2]. In addition, the designer may adopt practical approaches, i.e. approximate constitutive equations [1] to calculate the effects of variable stresses or strains, e.g. the age-adjusted-effective-modulus method.

Unless special provisions are given the model is valid for ordinary structural concrete having a compressive strength at the age of 28 days, f_{cm}, ranging from 20 N/mm^2 $\leq f_{cm} \leq$ 90 N/mm^2 subjected to a compressive stress $\sigma_c \leq 0.4\ f_c(t_0)$ at an age at loading t_0 and exposed to mean relative humidities in the range of 40 % to 100 % at mean temperatures from 5 °C to 30 °C. Some extensions of the model have been developed to allow an estimate of the effects of temperature and high stresses upon creep [1]. Details on the applicability of the relations presented are given in [2].

Basic Equations

The total load dependent strain at time t, $\varepsilon_{c\sigma}(t,t_o)$, of a concrete member uniaxially loaded at time t_o with a constant stress $\sigma_c(t_o)$ is subdivided as follows:

$$\varepsilon_{c\sigma}(t,t_o) = \varepsilon_{ci}(t_o) + \varepsilon_{cc}(t,t_o) \tag{1}$$

where $\varepsilon_{ci}(t_o)$ is the initial elastic strain at loading and $\varepsilon_{cc}(t,t_o)$ represents the creep strain at time $t > t_o$. Both strain components may also be expressed by means of the tangent moduli of elasticity $E_c(t_o)$ and E_c, and the creep coefficient $\phi(t,t_o)$, respectively:

$$\varepsilon_{ci}(t_o) = \frac{\sigma_c(t_o)}{E_c(t_o)} \tag{2}$$

$$\varepsilon_{cc}(t,t_o) = \frac{\sigma_c(t_o)}{E_c} \cdot \phi(t,t_o) \tag{3}$$

In eqs. 2 and 3, $E_c(t_o)$ represents the tangent modulus of elasticity at a concrete age t_o and $E_c = E_c(t_o = 28 \text{ days})$. With help of eqs. 2 and 3, eq. 1 may be written as:

$$\varepsilon_{c\sigma}(t,t_o) = \sigma_c(t_o) \cdot \left[\frac{1}{E_c(t_o)} + \frac{\phi(t,t_o)}{E_c}\right] \tag{4}$$

$$= \sigma_c(t_o) \cdot J(t,t_o) \tag{5}$$

where $J(t,t_o)$ is the creep function or the creep compliance, representing the total stress dependent strain per unit stress.

Prediction of the Tangent Modulus of Elasticity

Values of the tangent modulus of elasticity for normal weight concrete, E_c, can be estimated from eq. 6:

$$E_c = E_{co} \cdot (f_{cm}/f_{cmo})^{1/3} \tag{6}$$

where f_{cm} is the mean compressive strength of concrete cylinders (in [N/mm^2]), 150 mm in diameter and 300 mm in height stored in water at 20 ± 2 °C, and tested at the age of 28 days in accordance with ISO 1920, ISO 2736/2 and ISO 4012 (ISO = International Organization for Standardization); E_{co} = 21500 N/mm^2 and f_{cmo} = 10 N/mm^2.

Eq. 6 is valid for concretes made of quartzitic aggregates. For concrete made of basalt, dense limestone, limestone or sandstone the modulus of elasticity according to eq. 6 may be calculated by multiplying E_c with a coefficient α_E = 1.2; 1.2; 0.9 or 0.7, respectively.

The modulus of elasticity at an age $t \neq 28$ days, $E_c(t)$, may be estimated from eq. 7:

$$E_c(t) = E_c \cdot \exp\left[\frac{s}{2} \cdot \left(1 - (28/(t/t_1))^{0.5}\right)\right] \tag{7}$$

where E_c = modulus of elasticity from eq. 6;
s = coefficient which depends on the type of cement; s = 0.2; 0.25; 0.38 for concretes made with rapid hardening high strength cement (RS), normal (ordinary) or rapid hardening cement (N,R) and slowly hardening cement (SL), respectively;
t = age of concrete in [days], taking into account curing temperature T according to eq. 17;
t_1 = 1 day.

The effect of elevated and reduced temperatures on the modulus of elasticity of concrete at an age of 28 days may be estimated from eq. 8:

$$E_c(T) = E_c \cdot (1.06 - 0.003\, T/T_0) \tag{8}$$

where $E_c(T)$ = modulus of elasticity at the temperature T;
E_c = modulus of elasticity at T = 20 °C from eq. 6;
T = temperature of concrete in [°C];
T_0 = 1 °C.

Eq. 8 is primarily valid if no moisture exchange takes place. It may also be used for other concrete ages than 28 days.

Prediction of the Creep Coefficient

The creep coefficient at time t, $\phi(t,t_0)$, when concrete is loaded at time $t_0 \leq t$, may be estimated from the following general relation:

$$\phi(t,t_0) = \phi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) \cdot \beta_c(t-t_0) \tag{9}$$

where

$$\phi_{RH} = 1 + \frac{1 - \frac{RH}{RH_0}}{0.46 \cdot (h/h_0)^{1/3}} \tag{10}$$

$$\beta(f_{cm}) = \frac{5.3}{(f_{cm}/f_{cmo})^{0.5}} \tag{11}$$

$$\beta(t_o) = \frac{1}{0.1 + (t_o/t_1)^{0.2}} \tag{12}$$

$$\beta_c(t-t_o) = \left[\frac{(t-t_o)/t_1}{\beta_H + (t-t_o)/t_1}\right]^{0.3} \tag{13}$$

with $$\beta_H = 150 \cdot \left[1 + \left(1.2 \cdot \frac{RH}{RH_o}\right)^{18}\right] \cdot \frac{h}{h_o} + 250 < 1500 \tag{14}$$

where RH = relative humidity of the ambient environment in [%];
h = $2A_c/u$; A_c = cross-section of the structural member in [mm^2]; u = perimeter of the structural member in contact with the atmosphere in [mm];
f_{cm} = mean compressive strength of concrete in [N/mm^2] at the age of 28 days;
t = age of concrete in [days] at the moment considered;
t_o = age of concrete at loading in [days];

and RH_o = 100 %, h_o = 100 mm, f_{cmo} = 10 N/mm^2, t_1 = 1 day.

Effect of Type of Cement

The effect of type of cement on the creep coefficient of concrete may be taken into account by modifying the age at loading t_o according to eq. 15:

$$t_o = t_{o,T} \cdot \left[\frac{9}{2 + (t_{o,T}/t_{1,T})^{1.2}} + 1\right]^{\alpha} > 0.5 \text{ days} \tag{15}$$

where $t_{o,T}$ = age of concrete at loading in [days] according to eq. 17;
$t_{1,T}$ = 1 day;
α = coefficient which depends on type of cement;

$$\alpha = \begin{cases} -1 & \text{for slowly hardening cement, SL} \\ 0 & \text{for normal or rapid hardening cement, N or R,} \\ 1 & \text{for rapid hardening high strength cement, RS.} \end{cases}$$

The value for t_o according to eq. 15 has to be used in eq. 12; the duration of loading $t-t_o$ to be used in eq. 13 is the actual time under load in [days].

Effect of Elevated Temperatures

The creep coefficient at elevated temperatures may be roughly estimated from eq. 16:

$$\phi_T(t,t_0) = \phi_{T,st}(t,t_0) + \Delta\phi_{T,trans} \tag{16}$$

where $\phi_{T,st}$ = steady state creep coefficient, which may be calculated using eq. 9 ($\phi_{T,st}(t,t_0) = \phi(t,t_0)$) and eqs. 10 to 15, considering the modifications given in eqs. 17 to 20;

$\Delta\phi_{T,trans}$ = transient creep coefficient which may be estimated from eq. 21.

Effect of Elevated Temperatures - Steady State Creep

The effect of an elevated temperature T to which concrete is exposed prior to or during loading - the load being applied after temperature rise - may be taken into account employing eqs. 17 to 20:

$$t_T = \sum_{i=1}^{n} \Delta t_i \cdot \exp\left[13.65 - \frac{4000}{273 + T(\Delta t_i)/T_0}\right] \tag{17}$$

where t_T = modified age of concrete at loading in [days], which has to be used in eqs. 7 and 15;

$T(\Delta t_i)$ = temperature in [°C] during the time period Δt_i;

Δt_i = number of days prior to loading, where the temperature T prevails;

T_0 = 1 °C.

$$\beta_{H,T} = \beta_H \cdot \exp\left[\frac{1500}{273 + T/T_0} - 5.12\right] \tag{18}$$

where $\beta_{H,T}$ = temperature dependent coefficient replacing β_H in eq. 13;

β_H = coefficient according to eq. 14.

$$\phi_{RH,T} = \phi_T + [\phi_{RH} - 1] \cdot \phi_T^{1.2} \tag{19}$$

with $$\phi_T = \exp\left[0.015 \cdot (T/T_0 - 20)\right] \tag{20}$$

where $\phi_{RH,T}$ = temperature dependent coefficient which replaces ϕ_{RH} in eq. 9;

ϕ_{RH} = coefficient according to eq. 10.

In eqs. 18 and 20, T is a constant temperature while concrete is under load and T_0 = 1 °C.

Effect of Elevated Temperatures - Transient Creep

Transient temperature conditions, i.e. the increase of temperature while the structural member is under load, leads to an additional creep $\Delta\phi_{T,trans}$, which may be calculated from eq. 21:

$$\Delta\phi_{T,trans} = 0.0004 \cdot (T/T_0 - 20)^2 \tag{21}$$

where T is the temperature in [°C] to which the structural member, being under load, is heated and T_0 = 1 °C.

Effect of High Constant Stresses

For stresses in the range of $0.4\ f_c(t_0) < \sigma_c \leq 0.6\ f_c(t_0)$, where $f_c(t_0)$ is the mean compressive strength of concrete at the age t_0, the increased creep due to stress level dependent nonlinearity may be taken into account using eq. 22:

$$\phi_{\bar{\sigma}}(t,t_0) = \begin{cases} \phi(t,t_0) \cdot \exp\left[\alpha_\sigma(\bar{\sigma}-0.4)\right] & \text{for} \quad 0.4 < \bar{\sigma} \leq 0.6 \\ \phi(t,t_0) & \phantom{\text{for}} \quad \bar{\sigma} \leq 0.4 \end{cases} \tag{22}$$

where $\phi(t,t_0)$ = creep coefficient according to eq. 9;
$\bar{\sigma}$ = stress-strength ratio $\sigma_c/f_c(t_0)$;
α_σ = 1.5.

For mass concrete and for creep at very high relative humidities the coefficient α_σ may be as low as α_σ = 0.5.

Comparison with Test Data and Applicability of the Model

Eq. 6 is a rather crude empirical relation between strength and stiffness of hardened concrete. As a consequence, a considerable scatter band is observed if test results on the modulus of elasticity are plotted versus the compressive strength of the concrete (Fig. 2). However, a clear tendency may be observed which is predicted reasonably well by eq. 6. The cubic root of the compressive strength correlates better with E_c than the square root, which underlies corresponding models in some other codes.

Fig. 3 illustrates the development of the modulus of elasticity with time as observed in more than hundred different tests taken from the literature (solid lines) and as evaluated from eq. 7 (dashed lines).

Apparently, the effect of aging upon the modulus of elasticity is reasonably well predicted by the CEB model. It is worth noting that the time function for the development of modulus of elasticity differs from that for the development of strength as given in [1].

The coefficient of variation for the prediction of the creep function, i.e. the sum of elastic and creep strains per unit stress, has been found to be V = 20,4 %. More refined models will result in lower coefficients of variation. However, the room for improvements is limited [2]. Fig. 4 illustrates the prediction accuracy for the effects of age at loading and type of cement on creep of concrete; $\varepsilon_{cc,365}(t_0)$ represents the creep strain after a duration of loading of 365 days when the age at loading is t_0. Here, the test results for concretes made of rapid hardening high strength (RS) and slowly hardening (SL) cements taken from various authors and represented by symbols have been converted to normal or rapid hardening (N,R) cements using eq. 15. Two examples of the prediction for the creep function are illustrated in Figs. 5 and 6. Further comparisons are shown in [2].

It has to be pointed out, that the simple formulae to predict the effects of elevated temperatures and high stresses upon creep allow only a very rough estimate of the mean concrete behavior. The restrictions of applicability given in [1] and in the basic document [2] have to be followed. Nevertheless, in many cases of practical significance, above all for creep of ordinary structural concrete at high ambient humidity (or thick structural members), satisfactory results may be obtained for the prediction of temperature effects. This is underlined by the example of Fig. 7, where relative creep strains as predicted by the model and experimental results of [11] are compared.

As far as the creep prediction at normal temperatures and stress levels is concerned the CEB MC 90 model resembles some of the features of the model given in the CEB-FIP Model Code 1970 [6] and of the model proposed by ACI Committee 209 [5]. However, various basic improvements could be achieved. The time development of creep which is described by a hyperbolic function (eq. 13) includes the member size as a parameter taking pattern from diffusion theory. In addition, the effects of the ambient environment and member size on creep of concrete are interrelated (eq. 10). Nevertheless, the chosen product model inevitably includes the typical constitutive weaknesses of these models as indicated in Fig. 1. Further informations and restrictions of validity of the model have been discussed in [2].

SHRINKAGE

The prediction model for shrinkage given in the CEB-FIP Model Code 1990 predicts the mean time dependent strain of a non-loaded, plain structural concrete member which is exposed to a dry or moist environment after curing. The model is valid for ordinary normal weight structural concrete, moist cured at normal temperatures not longer than 14 days and exposed to mean relative humidities in the range of 40 to 100 percent at mean temperatures ranging from 5 °C to 30 °C. Some extensions of the model have been developed to allow an estimate of the effect of temperature upon shrinkage [1]. Details, in particular on the applicability of the relations presented, may be found in [2].

Prediction Formulae

The strain due to shrinkage or swelling at normal temperatures may be calculated from eq. 23:

$$\varepsilon_{cs}(t,t_s) = \varepsilon_{cso} \cdot \beta_s(t-t_s) \qquad (23)$$

where ε_{cso} = notional shrinkage coefficient according to eq. 24;

β_s = coefficient to describe the development of shrinkage with time according to eq. 28;

t = age of concrete in [days];

t_s = age of concrete in [days] at the beginning of shrinkage or swelling.

The notional shrinkage coefficient may be obtained from eq. 24:

$$\varepsilon_{cso} = \varepsilon_s(f_{cm}) \cdot \beta_{RH} \qquad (24)$$

with

$$\varepsilon_s(f_{cm}) = [160 + 10 \cdot \beta_{sc} \cdot (9 - f_{cm}/f_{cmo})] \cdot 10^{-6} \qquad (25)$$

where β_{sc} = coefficient which depends on type of cement;

$$\beta_{sc} = \begin{cases} 4 \text{ for slowly hardening cement, SL} \\ 5 \text{ for normal or rapid hardening cement, N,R} \\ 8 \text{ for rapid hardening high strength cement, RS,} \end{cases}$$

and

$$\beta_{RH} = \begin{cases} -1.55 \cdot \beta_{sRH} & \text{for } 40\ \% \leq RH < 99\ \% \\ +0.25 & \phantom{\text{for }} RH \geq 99\ \% \end{cases} \qquad (26)$$

where

$$\beta_{sRH} = 1 - \left(\frac{RH}{RH_o}\right)^3 \qquad (27)$$

In eqs. 25 and 27, f_{cm} is the mean compressive strength of concrete in [N/mm^2], and RH is the mean relative humidity of the ambient atmosphere in [%], respectively; f_{cmo} = 10 N/mm^2 and RH_o = 100 %.

The development of shrinkage with time is given by:

$$\beta_s(t-t_s) = \left[\frac{(t-t_s)/t_1}{\beta_{sH} + (t-t_s)/t_1}\right]^{0.5} \qquad (28)$$

with

$$\beta_{sH} = 350 \cdot (h/h_o)^2 \qquad (29)$$