ACI 445R-99
(Reapproved 2015)

Recent Approaches to Shear Design of Structural Concrete

Reported by Joint ACI-ASCE Committee 445

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First printing
June 2000

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Truss model approaches and related theories for the design of reinforced concrete members to resist shear are presented. Realistic models for the design of deep beams, corbels, and other nonstandard structural members are illustrated. The background theories and the complementary nature of a number of different approaches for the shear design of structural concrete are discussed. These relatively new procedures provide a unified, intelligible, and safe design framework for proportioning structural concrete under combined load effects.

Keywords: beams (supports); concrete; design; detailing; failure; models; shear strength; structural concrete; strut and tie.

CONTENTS

ACI 445R-99 became effective November 22, 1999.

CHAPTER 1—INTRODUCTION

1.1—Scope and objectives

Design procedures proposed for regulatory standards should be safe, correct in concept, simple to understand, and should not necessarily add to either design or construction costs. These procedures are most effective if they are based on relatively simple conceptual models rather than on complex empirical equations. This report introduces design engineers to some approaches for the shear design of one-way structural concrete members. Although the approaches explained in the subsequent chapters of this report are relatively new, some of them have reached a sufficiently mature state that they have been implemented in codes of practice. This report builds upon the landmark state-of-the-art report by the ASCE-ACI Committee 426 (1973), *The Shear Strength of Reinforced Concrete Members*, which reviewed the large body of experimental work on shear and gave the background to many of the current American Concrete Institute (ACI) shear design provisions. After reviewing the many different empirical equations for shear design, Committee 426 expressed in 1973 the hope that "the design regulations for shear strength can be integrated, simplified, and given a physical significance so that designers can approach unusual design problems in a rational manner."

The purpose of this report is to answer that challenge and review some of the new design approaches that have evolved since 1973 (CEB 1978, 1982; Walraven 1987; IABSE 1991a,b; Regan 1993). Truss model approaches and related theories are discussed and the common basis for these new approaches are highlighted. These new procedures provide a unified, rational, and safe design framework for structural concrete under combined actions, including the effects of axial load, bending, torsion, and prestressing.

Chapter 1 presents a brief historical background of the development of the shear design provisions and a summary of the current ACI design equations for beams. Chapter 2 discusses a sectional design procedure for structural-concrete one-way members using a compression field approach. Chapter 3 addresses several approaches incorporating the "concrete contribution." It includes brief reviews of European Code EC2, Part 1 and the Comité Euro-International du Béton–Fédération International de la Précontrainte (CEB-FIP) Model Code, both based on strut-and-tie models. The behavior of members without or with low amounts of shear reinforcement is discussed in Chapter 4. An explanation of the concept of shear friction is presented in Chapter 5. Chapter 6 presents a design procedure using strut-and-tie models (STM), which can be used to design regions having a complex flow of stresses and may also be used to design entire members. Chapter 7 contains a summary of the report and suggestions for future work.

1.2—Historical development of shear design provisions

Most codes of practice use sectional methods for design of conventional beams under bending and shear. ACI Building Code 318M-95 assumes that flexure and shear can be handled separately for the worst combination of flexure and shear at a given section. The interaction between flexure and shear is addressed indirectly by detailing rules for flexural reinforcement cutoff points. In addition, specific checks on the level of concrete stresses in the member are introduced to ensure sufficiently ductile behavior and control of diagonal crack widths at service load levels.

In the early 1900s, truss models were used as conceptual tools in the analysis and design of reinforced concrete beams. Ritter (1899) postulated that after a reinforced concrete beam cracks due to diagonal tension stresses, it can be idealized as a parallel chord truss with compression diagonals inclined at 457 with respect to the longitudinal axis of the beam. Mörsch (1920, 1922) later introduced the use of truss models for torsion. These truss models neglected the contribution of the concrete in tension. Withey (1907, 1908) introduced Ritter's truss model into the American literature and pointed out that this approach gave conservative results when compared with test evidence. Talbot (1909) confirmed this finding.

Historically, shear design in the United States has included a concrete contribution V_c to supplement the 45 degree sectional truss model to reflect test results in beams and slabs with little or no shear reinforcement and ensure economy in the practical design of such members. ACI Standard Specification No. 23 (1920) permitted an allowable shear stress of $0.025f_c'$, but not more than 0.41 MPa, for beams without web reinforcement, and with longitudinal reinforcement that did not have mechanical anchorage. If the longitudinal reinforcement was anchored with 180 degree hooks or with plates rigidly connected to the bars, the allowable shear stress was increased to $0.03f_c'$ or a maximum of 0.62 MPa (Fig. 1.1). Web reinforcement was designed by the equation

$$A_v F_v = V' s \sin \alpha / jd \qquad (1\text{-}1)$$

where

A_v = area of shear reinforcement within distance s;
f_v = allowable tensile stress in the shear reinforcement;
jd = flexural lever arm;

V' = total shear minus $0.02f'_c bjd$ (or $0.03f'_c bjd$ with special anchorage);
b = width of the web;
s = spacing of shear steel measured perpendicular to its direction; and
α = angle of inclination of the web reinforcement with respect to the horizontal axis of the beam.

The limiting value for the allowable shear stresses at service loads was $0.06f'_c$ or a maximum of 1.24 MPa, or with anchorage of longitudinal steel $0.12f'_c$ or a maximum of 2.48 MPa. This shear stress was intended to prevent diagonal crushing failures of the web concrete before yielding of the stirrups. These specifications of the code calculated the nominal shear stress as $v = V/bjd$.

This procedure, which formed the basis for future ACI codes, lasted from 1921 to 1951 with each edition providing somewhat less-conservative design procedures. In 1951 the distinction between members with and without mechanical anchorage was omitted and replaced by the requirement that all plain bars must be hooked, and deformed bars must meet ASTM A 305. Therefore, the maximum allowable shear stress on the concrete for beams without web reinforcement (ACI 318-51) was $0.03f'_c$ and the maximum allowable shear stress for beams with web reinforcement was $0.12f'_c$.

ACI 318-51, based on allowable stresses, specified that web reinforcement must be provided for the excess shear if the shear stress at service loads exceeded $0.03f'_c$. Calculation of the area of shear reinforcement continued to be based on a 45 degree truss analogy in which the web reinforcement must be designed to carry the difference between the total shear and the shear assumed to be carried by the concrete.

The August 1955 shear failure of beams in the warehouse at Wilkins Air Force Depot in Shelby, Ohio, brought into question the traditional ACI shear design procedures. These shear failures, in conjunction with intensified research, clearly indicated that shear and diagonal tension was a complex problem involving many variables and resulted in a return to forgotten fundamentals.

Talbot (1909) pointed out the fallacies of such procedures as early as 1909 in talking about the failure of beams without web reinforcement. Based on 106 beam tests, he concluded that

> It will be found that the value of v [shear stress at failure] will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam.… In beams without web reinforcement, web resistance depends upon the quality and strength of the concrete.… The stiffer the beam the larger the vertical stresses which may be developed. Short, deep beams give higher results than long slender ones, and beams with high percentage of reinforcement [give higher results] than beams with a small amount of metal.

Unfortunately, Talbot's findings about the influence of the percentage of longitudinal reinforcement and the length-to-depth ratio were not reflected in the design equations until much later. The research triggered by the 1956 Wilkins

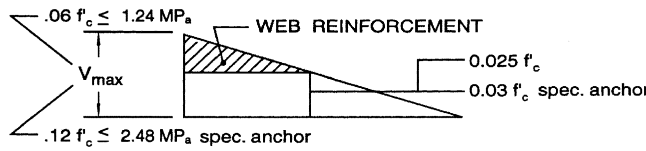

Fig. 1.1—American Specification for shear design (1920-1951) based on ACI Standard No. 23, 1920.

warehouse failures brought these important concepts back to the forefront.

More recently, several design procedures were developed to economize on the design of the stirrup reinforcement. One approach has been to add a concrete contribution term to the shear reinforcement capacity obtained, assuming a 45 degree truss (for example, ACI 318-95). Another procedure has been the use of a truss with a variable angle of inclination of the diagonals. The inclination of the truss diagonals is allowed to differ from 45 degree within certain limits suggested on the basis of the theory of plasticity. This approach is often referred to as the "standard truss model with no concrete contribution" and is explained by the existence of aggregate interlock and dowel forces in the cracks, which allow a lower inclination of the compression diagonals and the further mobilization of the stirrup reinforcement. A combination of the variable-angle truss and a concrete contribution has also been proposed. This procedure has been referred to as the modified truss model approach (CEB 1978; Ramirez and Breen 1991). In this approach, in addition to a variable angle of inclination of the diagonals, the concrete contribution for nonprestressed concrete members diminishes with the level of shear stress. For prestressed concrete members, the concrete contribution is not considered to vary with the level of shear stress and is taken as a function of the level of prestress and the stress in the extreme tension fiber.

As mentioned previously, the truss model does not directly account for the components of the shear failure mechanism, such as aggregate interlock and friction, dowel action of the longitudinal steel, and shear carried across uncracked concrete. For prestressed beams, the larger the amount of prestressing, the lower the angle of inclination at first diagonal cracking. Therefore, depending on the level of compressive stress due to prestress, prestressed concrete beams typically have much lower angles of inclined cracks at failure than nonprestressed beams and require smaller amounts of stirrups.

Traditionally in North American practice, the additional area of longitudinal tension steel for shear has been provided by extending the bars a distance equal to d beyond the flexural cutoff point. Although adequate for a truss model with 45 degree diagonals, this detailing rule is not adequate for trusses with diagonals inclined at lower angles. The additional longitudinal tension force due to shear can be determined from equilibrium conditions of the truss model as $V \cot\theta$, with θ as the angle of inclination of the truss diagonals. Because the shear stresses are assumed uniformly distributed over the depth of the web, the tension acts at the section middepth.

The upper limit of shear strength is established by limiting the stress in the compression diagonals f_d to a fraction of the concrete cylinder strength. The concrete in the cracked web

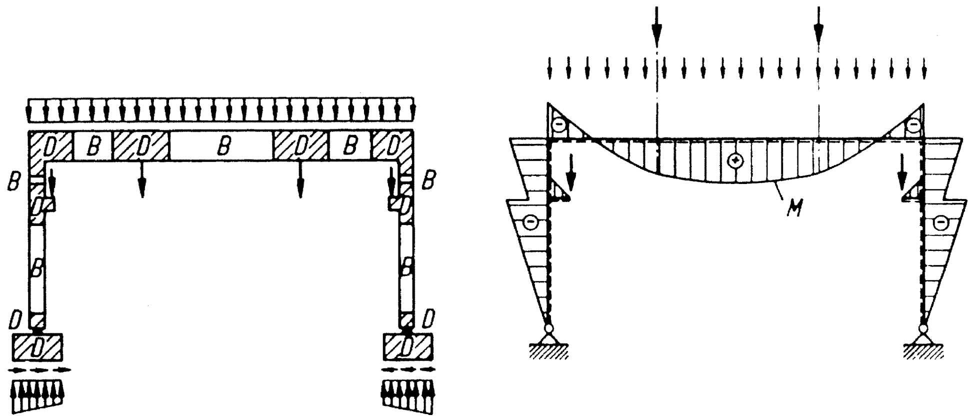

Fig. 1.2—Frame structure containing substantial part of B regions, its statical system, and bending moments (Schlaich et al. 1987).

of a beam is subjected to diagonal compressive stresses that are parallel or nearly parallel to the inclined cracks. The compressive strength of this concrete should be established to prevent web-crushing failures. The strength of this concrete is a function of 1) the presence or absence of cracks and the orientation of these cracks; 2) the tensile strain in the trans-verse direction; and 3) the longitudinal strain in the web. These limits are discussed in Chapters 2, 3, and 6.

The pioneering work from Ritter and Mörsch received new impetus in the period from the 1960s to the 1980s, and there-fore, in more recent design codes, modified truss models are used. Attention was focused on the truss model with diagonals having a variable angle of inclination as a viable model for shear and torsion in reinforced and prestressed concrete beams (Kupfer 1964; Caflisch et al. 1971; Lampert and Thurlimann 1971; Thurlimann et al. 1983). Further development of plasticity theories extended the applicability of the model to nonyielding domains (Nielsen and Braestrup 1975; Muller 1978; Marti 1980). Schlaich et al. (1987) extended the truss model for beams with uniformly inclined diagonals, all parts of the structure in the form of STM. This approach is particularly relevant in regions where the distribution of strains is significantly nonlinear along the depth. Schlaich et al. (1987) introduced the concept of D and B regions, where D stands for disconti-nuity or disturbed, and B stands for beam or Bernoulli. In D regions the distribution of strains is nonlinear, whereas the distribution is linear in B regions. A structural-concrete member can consist entirely of a D region; however, more often D and B regions will exist within the same member or structure [see Fig. 1.2, from Schlaich et al. (1987)]. In this case, D regions extend a distance equal to the member depth away from any discontinuity, such as a change in cross section or the presence of concentrated loads. For typical slender members, the portions of the structure or member between D regions are B regions. The strut-and-tie approach is discussed in detail in Chapter 6.

By analyzing a truss model consisting of linearly elastic members and neglecting the concrete tensile strength, Kupfer (1964) provided a solution for the inclination of the diagonal cracks. Collins and Mitchell (1980) abandoned the assumption of linear elasticity and developed the compression field theory (CFT) for members subjected to torsion and shear. Based on extensive experimental investigation, Vecchio and Collins (1982, 1986) presented the modified compression field theory (MCFT), which included a rationale for determining the tensile stresses in the diagonally cracked concrete. Although the CFT works well with medium to high percentages of transverse reinforcement, the MCFT provides a more realistic assessment for members having a wide range of amounts of transverse reinforcement, including the case of no web reinforcement. This approach is presented in Chapter 2. Parallel to these developments of the truss model with variable strut inclinations and the CFT, the 1980s also saw the further development of shear friction theory (Chapter 5). In addition, a general theory was developed for beams in shear using constitutive laws for friction and by determining the strains and deformations in the web. Because this approach considers the discrete formation of cracks, the crack spacing and crack width should be determined and equilibrium checked along the crack to evaluate the crack-slip mechanism of failure. This method is presented in Chapter 3. The topic of members without transverse reinforcement is dealt with in Chapter 4.

1.3—Overview of current ACI design procedures

The ACI 318M-95 sectional design approach for shear in one-way flexural members is based on a parallel truss model with 45-degree constant inclination diagonals supplemented by an experimentally obtained concrete contribution. The contribution from the shear reinforcement V_s for the case of vertical stirrups (as is most often used in North American practice), can be derived from basic equilibrium consider-ations on a 45-degree truss model with constant stirrup spacing s, and effective depth d. The truss resistance is supplemented with a concrete contribution V_c for both reinforced and prestressed concrete beams. Appendix A presents the more commonly used shear design equations for the concrete contribution in normalweight concrete beams, including effects of axial loading and the contribution from vertical stirrups V_s.