IN-LB	Inch-Pound Units
SI	International System of Units

Report on Deflection of Nonprestressed Concrete Structures

Reported by ACI Committee 435

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Reported by ACI Committee 435

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This report presents a consolidated treatment of initial and time-dependent deflection of nonprestressed reinforced concrete members such as simple and continuous beams and one-way and two-way slab systems. It presents the current state of practice of deflection prediction as well as analytical methods for computer use in deflection estimation. Topics include material properties, deflection of reinforced concrete one-way flexural members, deflection of two-way slab systems, and reducing deflection of concrete members.

Keywords: camber; cracking; creep; curvature; deflection; modulus of rupture; moments of inertia; serviceability; shrinkage; time-dependent deflection.

CONTENTS

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CHAPTER 1—INTRODUCTION AND SCOPE

1.1—Introduction

Design for serviceability is central to the work of structural engineers and code-writing bodies. It is also essential to users of the designed structures. Increased use of high-strength concrete and higher-strength reinforcing bars, coupled with more detailed computer-aided designs, has resulted in lighter and more material-efficient and, thus, more flexible structural members and systems. This in turn has necessitated better prediction and control of short-term and long-term behavior of concrete structures at service loads.

This report presents a consolidated treatment of initial and time-dependent deflection of nonprestressed reinforced concrete members such as simple and continuous beams and one- and two-way slab systems. It presents current engineering practice in design for control of deformation and deflection of concrete members and includes methods presented in ACI 318 plus selected other approaches suitable for computer-based use in deflection computation. Design examples are given at the end of one- and two-way framing chapters showing how to evaluate deflection and, thus, control it through adequate design for serviceability. The content of the report as well as the step-by-step examples are intended to familiarize practitioners with the current methods for estimating deflections in buildings as well as analytical methods suitable for computer-based application. The examples apply ACI 318 requirements and a recommended alternative approach with a lower cracking moment (to account for shrinkage restraint). Methods for predicting initial and time-dependent deflections of prestressed concrete are not addressed in this document, although prestressing can be an effective tool for controlling both short- and long-term deflections.

1.2—Scope

The principal causes of deflections taken into account in this report are those due to elastic deformation, flexural cracking, creep, shrinkage, and temperature effects. This document is composed of two introductory chapters and four main chapters that provide information on calculating and controlling deflections of members constructed using reinforced concrete. The organization of the report is:

a) **Chapter 1—Introduction and Scope** provides background information on the document.

b) **Chapter 2—Notation and Definitions** provides a listing of the notation used throughout the document.

c) **Chapter 3—Material Properties** discusses material properties that affect deflections.

d) **Chapter 4—Deflection of Reinforced Concrete One-Way Flexural Members** discusses behavior of uncracked and cracked members, and time-dependent effects. It also includes the relevant code procedures and expressions for deflection computation in reinforced concrete beams. Numerical examples are included to illustrate the standard calculation methods for simply supported and continuous concrete beams.

e) **Chapter 5—Deflection of Two-Way Slab Systems** covers the deflection behavior of reinforced two-way-action slabs and plates. This chapter gives an overview of classical and other methods of deflection estimation, such as the crossing beam analogy and the finite element method for immediate deflection computation. It also discusses approaches for determining the minimum thickness requirements for two-way slabs and plates and gives a detailed computational example for evaluating the long-term deflection of a two-way reinforced concrete slab. The chapter emphasizes the uncertainties inherent in estimating deflections of two-way slab systems.

f) **Chapter 6—Reducing Deflection of Concrete Members** gives practical and remedial guidelines for improving and controlling the deflection of reinforced concrete members, hence enhancing their overall long-term serviceability.

It should be emphasized that the magnitude of actual deflection in concrete structural members, particularly in buildings, which are the emphasis and the intent of this report, can only be estimated with limited accuracy. This is because of the large variability in the properties of the constituent materials of these members, the quality control exercised in their construction, and the construction methods used. Therefore, for practical considerations, the computed deflection values in the illustrative examples at the end of each chapter should be interpreted with this in mind.

In summary, this single document gives design engineers the key tools for estimating, and thereby controlling through design, the expected deflection in nonprestressed reinforced concrete building structures. The material presented and the design examples will help to enhance serviceability when used judiciously by the engineer. Designers, constructors, and codifying bodies can draw on the material presented in this document to achieve serviceable deflection of constructed facilities.

CHAPTER 2—NOTATION AND DEFINITIONS

2.1—Notation

A_s = area of nonprestressed tension steel, in.^{2} (mm^2)
A_s' = area of nonprestressed steel in compression zone, in.^{2} (mm^2)
b = width of the section, in. (mm)
b_w = web width, in. (mm)
C_t = creep coefficient of concrete at time t, days
C_u = ultimate creep coefficient of concrete
c = depth of centroidal axis, in. (mm)
d = distance from the extreme compression fiber to centroid of tension reinforcement, in. (mm)
d' = distance from the extreme compression fiber to centroid of compression reinforcement, in. (mm)
d_b = bar diameter, in. (mm)
E = modulus of elasticity, psi (MPa)
E_c = modulus of elasticity of concrete, psi (MPa)
f_c = stress in concrete, psi (MPa)
f_c' = specified compressive strength of concrete, psi (MPa)
f_{cr} = stress to cause cracking in concrete, psi (MPa)
f_r = modulus of rupture of concrete, psi (MPa)
f_{res} = stress from restraint to shrinkage, psi (MPa)
f_s = stress in nonprestressed steel, psi (MPa)
f_y = specified yield strength of nonprestressed reinforcing steel, psi (MPa)
h = overall thickness of a member, in. (mm)
h_f = flange thickness, in. (mm)
I = moment of inertia, in.^{4} (mm^4)
I_{cr} = moment of inertia of the cracked section transformed to concrete, in.^{4} (mm^4)
I_e = effective moment of inertia for computation of deflection, in.^{4} (mm^4)
I_g = moment of inertia for gross concrete section about centroidal axis, neglecting reinforcement, in.^{4} (mm^4)
k = depth of compression zone divided by d
ℓ = span length, in. (mm)
ℓ_n = distance from the inside of the support to the inside of support, clear span, in. (mm)
M = bending moment, lb-in. (N-mm)
M_1 = moment at End 1 of a continuous member, lb-in. (N-mm)
M_2 = moment at End 2 of a continuous member, lb-in. (N-mm)
M_a = maximum service load moment (unfactored) at stage deflection is computed, lb-in. (N-mm)
M_{cr} = cracking moment, lb-in. (N-mm)
M_D = moment due to dead load, lb-in. (N-mm)
M_L = moment due to live load, lb-in. (N-mm)
M_m = midspan moment, lb-in. (N-mm)
N = axial member load, lb (N)
n = modular ratio E_s/E_c
T = temperature, °F (°C)
t = time, s
w = uniformly distributed load (load per unit length), lb/in. (N/mm)
w_c = unit weight of normalweight concrete or equilibrium density of lightweight concrete, lb/ft^3 (kg/m^3)
y_t = distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, in. (mm)
Δ = elastic deflection of a beam or slab, in. (mm)
Δ_{cr} = additional deflection due to creep, in. (mm)
Δ_{inc} = incremental deflection that occurs after attachment on nonstructural elements (includes long-term deflection Δ_{LT} from sustained loads and immediate deflection from the remaining part of live load that is not sustained, in. (mm)
Δ_L = initial (immediate) deflection due to live load, in. (mm)
Δ_{LT} = deflection from long-term effects, in. (mm)
Δ_{sh} = additional deflection due to shrinkage, in. (mm)
Δ_{sus} = initial (immediate) deflection due to sustained load, in. (mm)
ε_{cf} = strain due to stress in the concrete
ε_o = free strain, such as unrestrained shrinkage
ε_{sf} = strain due to stress in nonprestressed steel
ε_{sh} = shrinkage strain of concrete
$(\varepsilon_{sh})_t$ = shrinkage strain of concrete at time t, days
$(\varepsilon_{sh})_u$ = ultimate shrinkage strain of concrete
ε_t = total strain
ζ = distribution coefficient
κ = cross section curvature, in.$^{-1}$ (mm^{-1})
κ_{sh} = shrinkage curvature, in.$^{-1}$ (mm^{-1})
λ_c = creep multiplier for long-term deflection
λ_{sh} = shrinkage warping multiplier for long-term deflection
λ_t = total multiplier for long-term deflection
λ_Δ = time-dependent multiplier for long-term deflection
ν = Poisson's ratio
ρ = nonprestressed tension reinforcement ratio (A_s/bd)
ρ' = reinforcement ratio for nonprestressed compression steel (A_s'/bd)
ξ = time-dependent multiplier for deflection

2.2—Definitions

Please refer to the latest version of ACI Concrete Terminology for a comprehensive list of definitions.

CHAPTER 3—MATERIAL PROPERTIES

3.1—Objective

Deflections in reinforced concrete structures are affected significantly by numerous material properties, including concrete and reinforcement moduli of elasticity, concrete modulus of rupture, creep, and shrinkage. The purposes of this chapter are to: 1) briefly address how each of these material properties affects deflection; and 2) provide brief guidance on the most common expressions recommended by various ACI committees for estimation of these parameters during the design process. This chapter is not intended to provide a comprehensive review of all the material property expressions that have been suggested by researchers, committees, and design codes.

3.2—Material properties affecting deflection

The primary material properties that affect the deflection of reinforced concrete structures are identified in the following.

3.2.1 *Concrete modulus of elasticity*—The elastic modulus of concrete (E_c) has a significant impact on deflections in reinforced concrete structures, as it is a direct measure of material stiffness. Based on simple mechanics principles developed for elastic theory, deflection is inversely related to the elastic modulus of the concrete. However, the concrete elastic modulus also affects the sectional moment of inertia at the cracked section (I_{cr}) and, thus, the relationship between concrete elastic modulus and deflection is far more complex.

In indeterminate structures, the member forces and moments themselves (and thus resulting deflections) will be affected by the modulus of elasticity if it varies across the different components of the structure.

3.2.2 *Concrete modulus of rupture*—The concrete modulus of rupture (f_r) affects deflection primarily in that it establishes the stress threshold at which a member cracks. Upon reaching the modulus of rupture, the member (or a portion of the member) transitions from an uncracked to a cracked state and there is a significant reduction in stiffness accompanied by an increase in deflection. While most reinforced concrete members are designed to be cracked at service loads, the extent of cracking along the length of the member will have an impact on the member deflection.

For indeterminate structures, the extent of cracking has a significant effect on member stiffness, which in turn affects internal member forces and moments used in evaluating deflections.

3.2.3 *Time-dependent concrete properties*—Creep and shrinkage of concrete have a pronounced effect on the deflection behavior of reinforced concrete members. Creep of concrete will cause additional deformation over time, resulting in increased deflection of flexural members under sustained load. In many cases, the additional deflection will exceed the instantaneous deflection. The consideration of creep is not as simple as using a multiplier based solely on the creep coefficient of the concrete material because creep also causes a redistribution of internal stresses over time. This redistribution of internal stresses can be considered to cause a variation in effective sectional properties (at any point in the member) with time because the neutral axis and moment of inertia will change as the compressive strain in the concrete increases with age due to creep.

Similarly, shrinkage of concrete will directly lead to time-dependent deflections whenever warping is present due to differential shortening within the depth of the member. Differential shortening may occur due to an unsymmetric arrangement of internal reinforcement or external environmental effects. Restraint to shrinkage, either due to external restraint or internal restraint of embedded reinforcement, can also have a significant effect on deflection by inducing tensile stresses in the concrete that effectively reduce the external load required to cause cracking and thereby increase deflection. The most common approach for calculating deflection is to use a lower cracking moment computed with a reduced modulus of rupture to account for the tensile stresses that develop from restraint to shrinkage. For indeterminate structures, the member forces and moments themselves (and thus resulting deflections) may be significantly affected by creep and shrinkage. Creep effects can lead to a redistribution of forces among the members in a structure. Shrinkage restraint can also develop among the members in a structure, leading to cracking and a subsequent loss in stiffness.

3.2.4 *Reinforcement modulus of elasticity*—The effect of reinforcement modulus of elasticity on deflections can be significant but is limited primarily to the effect on sectional properties and the restraint of concrete shrinkage. The use of a lower modulus reinforcement will lead to smaller sectional moments of inertia for a constant reinforcement ratio, particularly for the cracked section, which in turn will lead to larger deflections. On the other hand, a lower modulus reinforcement will offer less internal restraint to concrete shrinkage and thus will not have as significant an effect in reducing the effective cracking moment.

3.3—Concrete material properties

Expressions recommended by ACI committees for estimation of concrete material properties during the design process are presented in this section. In general, these properties vary widely among different concretes or even among different batches of concrete produced using a single set of mixture proportions. Therefore, these expressions should only be expected to provide rough estimates for these concrete material properties, with a relatively low degree of accuracy. When a higher level of accuracy in deflection calculations is desired, concrete material properties should be measured using appropriate ASTM standards on test specimens produced using field trial batches.

3.3.1 *Concrete modulus of elasticity*—The modulus of elasticity is strongly influenced by the constitutive materials and proportions used and is especially affected by the properties and quantity of the coarse aggregate. An increase in the modulus of elasticity is commonly associated with an increase in compressive strength because the slope of the ascending branch of the stress-strain diagram becomes steeper for higher-strength concretes, but at a lower rate than the compressive strength. The value of the secant modulus of elasticity at $0.45f_c'$ for normal-strength concretes at 28 days is usually approximately 4×10^6 psi (28,000 MPa), whereas for higher-strength concretes with compressive strengths in excess of 8000 psi (56 MPa), values as high as 8×10^6 psi (56,000 MPa) have been reported (Myers and Yang 2004).

ACI 318 recommends Eq. (3.3.1) for computing the modulus of elasticity of concretes with densities in the range of 90 to 150 lb/ft^3 (1445 to 2325 kg/m^3) based on the secant modulus at the $0.45f_c'$ intercept.

$$\begin{aligned} E_c &= 33w_c^{1.5}\sqrt{f_c'} \quad \text{(psi)} \\ E_c &= 0.043w_c^{1.5}\sqrt{f_c'} \quad \text{(MPa)} \end{aligned} \tag{3.3.1}$$