Design for Crack Control in Reinforced and Prestressed Concrete Beams, Two-Way Slabs and Circular Tanks – A State-of-the-Art

by E. G. Nawy

Synopsis: This paper presents the state-of-the art in the evaluation of the flexural crack width development and crack control of flexural cracks in reinforced and prestressed concrete structures. It is based on extensive research over the past five decades in the United States and overseas in the area of macro-cracking in reinforced and prestressed concrete elements.

Mitigation and control of cracking has become essential in order to maintain the integrity and aesthetics of concrete structures and their long-term durability performance. The trend is stronger than ever towards better utilization of concrete strength, use of higher strength concretes in the range of 12,000-20,000 psi and higher compressive strength, more prestressed concretes and increased uses of limit failure theories - all these trends require closer control of serviceability requirements of cracking and deflection behavior.

The paper discusses and presents common expressions for the mitigation and control of cracking in reinforced concrete beams and thick one-way slabs, prestressed, pretensioned and post-tensioned flanged beams, reinforced concrete two-way action structural floor slabs and plates, and large diameter circular tanks. In addition, recommendations are given for the maximum tolerable flexural crack widths in concrete elements based on the cumulative experience of many investigators over the past five decades. The expressions include the ACI 318-99 crack control provisions in reinforced concrete beams and one-way slabs, and the Concrete Euro Code 1999 for the design of concrete buildings.

Keywords: beams; concrete; concrete strength; crack control; cracking; crack width; environment; equations for reinforced and prestressed beams; Eurocode; flexural crack width; long-term cracking; tanks; tolerable crack widths; two-way action structural slabs

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INTRODUCTION

Presently, the trend is stronger than ever in better utilization of concrete strength, use of higher strength high-performance concretes of 20,000 psi (138 MPa) compressive strength and higher, use of high-strength reinforcement, more prestressed concretes and increased use of limit failure theories - all these trends require closer control of serviceability requirements in cracking and deflection behavior. Hence, knowledge of the cracking behavior of concrete elements, and how to mitigate cracking, become essential.

Concrete cracks early in its loading history. Most cracks are a result of the following actions to which concrete can be subjected:

1. Volumetric change caused by drying shrinkage, creep under sustained load, thermal stresses including elevated temperatures, and chemical incompatibility of concrete components.
2. Direct stress due to applied loads or reactions or internal stress due to continuity, reversible fatigue load, long-term deflection, camber in prestressed systems, or environmental effects including differential movement in structural systems.
3. Flexural stress due to bending.

While the net result of these three actions can be the formation of cracks, the mechanisms of their development cannot be considered identical. Volumetric change generates internal micro-cracking which may develop into full cracking, while direct internal or external stress or applied loads and reactions could either generate internal macro-cracking, such as in the case of fatigue due to reversible load, or flexural micro-cracking leading to fully developed cracking.

This paper will concentrate on the macro-cracking aspect of cracking behavior. Yet it is important to briefly discuss micro-cracking.

MICRO-CRACKING

Micro-cracking may be mainly classified into two principal categories: a) bond cracks at the aggregate-mortar interface, b) paste cracks within the mortar matrix. Interfacial bond cracks are caused by interfacial shear and tensile stresses due to early volumetric change without the presence of external load. Volume change caused by hydration and shrinkage can create tensile and bond stresses of sufficient magnitude as to cause failure at the aggregate-mortar interface.[1-6] As the external load is applied, mortar cracks develop due to increase in compressive stress, propagating continuously through the cement matrix up to failure.

It appears that the damage to cement paste seems to play a significant role in controlling the stress-strain relationship in concrete. The coarse aggregate particles act as stress-raisers that decrease the strength of the cement paste. As a result, micro-cracks develop that can only be detected by large magnification. The importance of additional research lies not only in the evaluation of the micro-cracks, but also in their significance on the development of macro-cracks which generate from those micro-cracked centers of plasticity.

FLEXURAL CRACKING AND CRACKING MITIGATION

External load causes in direct and bending stresses potentially resulting in flexural, bond and diagonal tension cracks. Once the tensile stress in the concrete exceeds its *tensile strength, internal* micro-cracks form. These cracks develop into macro-cracks propagating towards the external fiber zones of the element.

Immediately after the full development of the first crack in a conventionally reinforced concrete element, the stress in the concrete at the cracking zone is reduced to zero and is assumed by the reinforcement[5]. The distribution of ultimate bond stress, longitudinal tensile stress in the concrete and longitudinal tensile stress in the steel can be schematically represented in Fig. 1.

Crack width is a primary function of the bond characteristics and deformation of reinforcement between the two adjacent cracks 1 and 2 in Fig. 1, if the small concrete strain along the crack interval a_c is neglected. Hence, the crack width would be a function of the crack spacing and vice versa up to the level of stabilization of crack spacing (Fig. 2).

The major parameters affecting the development and characteristics of the cracks are: cross-sectional percentage of reinforcement, bond characteristics and size of bar, concrete cover, concrete strength, and the concrete stretched area, namely the concrete area in tension. On this basis, one can propose the following mathematical model:

$$w = \alpha a_c^{\beta} \varepsilon_s^{\gamma} \tag{1}$$

where w = maximum crack width, and

α, ß and γ are nonlinearity constants. Crack spacing a_c is a function of the factors enumerated previously, being inversely proportional to bond strength and active steel ratio (steel percentage in terms of the concrete cross-sectional area in tension). ε_s is the reinforcement strain induced by external load.

The basic mathematical modal in equation (1) with the appropriate experimental values of the constants α, β and γ can be derived for a particular type of structural member. Such a member can be a one-dimensional element such as beam, a two-dimensional structure such as a two-way slab, or a three-dimensional member such as a shell or circular tank wall. Hence, it is expected that different forms or expressions apply for the evaluation of the macro-cracking behavior of different structural elements consistent with their fundamental structural behavior.[2-4]

FLEXURAL CRACKING AND CRACK CONTROL IN REINFORCED CONCRETE BEAMS AND THICK ONE-WAY SLABS

Requirements for crack control in beams and thick one-way slabs (span/thickness ratio about 15) in the ACI 318 Building Code are based on the statistical analysis of maximum crack width data from a number of sources. On the basis of this analysis and the vast amount of data available, the following general conclusions were reached:

1. The steel stress is the most important variable.
2. The thickness of the concrete cover is an important variable, but not the only geometric consideration.
3. The area of concrete surrounding each reinforcing bar is also an important geometric variable.
4. The bar diameter is not a major variable.
5. The size of the bottom crack width is influenced by the amount of strain gradient from the level of the steel to the tension face of the beam.

The simplified expression relating crack width to steel stress is given in Eq. 2 as follows[4] :

$$w_{max} = 0.076\,\beta\ f_s\ \sqrt[3]{d_c A}.10^{-3} \qquad (2)$$

where f_s = reinforcing steel stress, ksi

A = area of concrete symmetric with reinforcing steel divided by number of bars, in.^{2}

d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire closest thereto, in.

β = h_2/h_1 where h_1 = distance from neutral axis to the reinforcing steel, in.

h_2 = distance from neutral axis to extreme concrete tensile surface.

When the strain, $_s$, in the steel reinforcement is used instead of stress, f_s', Eq. (2) becomes

$$w_{max} = 2.2\,\beta\,\varepsilon_s\,\sqrt[3]{d_c A}.10^{-3} \tag{3}$$

and is valid in any system of measurement.

The cracking behavior in thick one-way slabs, namely those with clear cover exceeding 1-1/2 in., is similar to that in shallow beams. For such one-way slabs (Eq. 3) can be adequately applied if β ranging from 1.25 to 1.35 is used.

ACI 224 Report[3] and ACI 340 Report give recommendations for tolerable crack widths under various environmental conditions, as given in Table 1.

ACI 318-99 Code Provisions

The ACI 318-99 mitigates cracking through control of the spacing of the reinforcing bars,[1, 12, 13] as an indirect measure of crack control. The expression that the code requires to be used is:

$$s\ (\text{in.}) = (540/f_s) - 2.5\,c_c \tag{4}$$

but not greater than $12(36/f_s)$, where

f_s = computed stress in the reinforcement at service load = unfactored moment divided by the steel area and the internal arm moment. Alternatively, f_s can be taken as 0.60 f_y.

c_c = clear cover from the nearest surface in tension to the flexural tension reinforcement, inches.

s = center-to-center spacing of flexural tension reinforcement, inches, closest to the tension face of the section.

From these provisions, the maximum spacing for 60,000 psi (414 MPa) reinforcement = 12 [36 / (0.6 x 60)] = 12 in. (305 mm). The maximum allowable spacing of 12 inches is in conformity with the extensive testing performed by the author on over excess 100 two-way action slabs, discussed in subsequent sections. Hence this limitation on the distribution of flexural reinforcement in one-way slabs and wide-web reinforced concrete beams is appropriate. However, in beams of normal web width in usual buildings, these provisions might not be as workable.[14] In the opinion of the author, it is more advisable and perhaps safer to use the applicable crack width equation in designing for the tolerable crack width that the particular environmental condition requires.

The SI expression for the value of reinforcement spacing in Eq. 4 and f_s in MPa units is,

$$s\,(\text{mm}) = (95{,}000/540 f_s) - 2.5\,c_c \qquad (5)$$

but not to exceed 300(252/f_s). For the usual case of beams with grade 420 reinforcement and 50 mm clear cover to the main reinforcement, with f_s = 252 MPa, the maximum bar spacing is 250 mm.

It should be noted that if the concrete structural member is in severe environment, where the ACI bar spacing provisions can result in crack widths of 0.0137 in. or more, satisfying the ACI 318-99 requirements for bar spacing is not adequate for sustaining the long-term structural integrity of the member. This is why, use of the Gergely-Lutz equation in conjunction with Table 1, or the Euro EC2 Code expressions presented in the next section, is the advisable design route to follow for crack control in beams and thick one-way slabs.

Euro EC2 Code Provisions

The Euro Code EC2 requires that cracking should be limited to a level that does not impair the serviceability of the structure or cause its appearance to be unacceptable.[9] It limits the maximum design crack width to 0.3 mm (0.012 in.) under normal environmental conditions and under quasai permanent combination of loads. This ceiling is expected to be satisfactory with respect to appearance and durability. Stricter requirements of tolerable crack width are stipulated for more severe environmental conditions.

The EC2 Code stipulates that the design crack width be evaluated from the following expression:

$$w_k = \beta\, s_{rm}\, \varepsilon_{sm} \qquad (6)$$

where

w_k = design crack width

s_{rm} = average stabilized crack spacing

ε_{sm} = mean strain under relevant combination of loads and allowing for the effect of tension stiffening, shrinkage, etc.

β = coefficient relating the average crack width to the design value

= 1.7 for load-induced cracking and for restraint cracking in sections with minimum dimension in excess of 800 mm (32 in.

The strain, ε_{sm}, in the section is obtained from the following expression:

$$\varepsilon_{sm} = \sigma_s \;/\; E_s \,[\, 1 - \beta_1\beta_2 \,(\, \sigma_{sr} / \sigma_s \,)^2 \,] \tag{7}$$

where

σ_s = stress in the tension reinforcement computed on the basis of a cracked section.

σ_{sr} = stress in the tension reinforcement computed on the basis of a cracked section under loading conditions that cause the first crack.

β_1 = coefficient accounting for bar bond characteristics

= 1.0 for deformed bars and 0.5 for plain bars.

β_2 = coefficient accounting for load duration

= 1.0 for single short-term loading and 0.5 for sustained or cyclic loading

E_s = Modulus of elasticity of the reinforcement

The average stabilized mean crack spacing, s_{rm},, is evaluated from the following expression:

$$s_{rm} = 50 + 0.25\, k_1 k_2\, d_b \;/\; p_t \text{ , mm} \tag{8}$$

where

d_b = bar diameter, mm.

p_t = effective reinforcement ratio = A_s / A_t ; the effective concrete area in tension, A_t , is generally the concrete area surrounding the tension reinforcement, of depth equal to 2.5 times the distance from the tensile face of the concrete section to the centroid of the reinforcement. For slabs where the depth of the tension zone may be small, the height of the effective area should not be taken less than $(c - d_b)/3$, where c = clear cover to the reinforcement.

k_1 = 0.8 for deformed bars and 1.6 for plain bars.

k_2 = 0.5 for bending and 1.0 for pure tension.

In cases of eccentric tension or for local areas, an average value of $k_2 = (\varepsilon_1 + \varepsilon_2) / 2\varepsilon_1$ can be used, where ε_1 is the greater and ε_2 the lesser tensile strain at the section boundaries, determined on the basis of cracked section.

In the absence of rigorous computations as described thus far, choice of minimum area of reinforcement, A_s , for crack control is stipulated such that

$$A_s = k_c k\, f_{ct,eff}\, A_{ct} / \sigma_s \qquad (9)$$

where

A_s = reinforcement area within the tensile zone.

A_{ct} = Effective area of concrete in tension

σ_s = maximum stress permitted in the reinforcement after the formation of the crack. The yield strength may be taken in lieu of σ_s , although lower values may be needed to satisfy crack width limits.

$f_{ct,eff}$ = tensile strength of the concrete effective at the formation of the first crack. A value of 3 N/mm^2 (435 psi) can be used.

k_c = coefficient representing the nature of stress distribution,

= 1.0 for direct tension and 0.4 for bending

k = coefficient accounting for non-uniform stresses due to restraint resulting from intrinsic or extrinsic deformation. It varies between 0.5 and 1.0.

The EC2 Code also stipulates that for cracks dominantly caused by flexure, their width will not usually exceed the standard 0.30 mm (0.012 in.), if the size and spacing of the reinforcing bars are within the range of values in Tables 2 and 3 for bar size and spacing. Evidently, for severe exposure conditions, such as those listed in Table 1 for tolerable crack widths beyond the 0.012 in. crack width level, crack width computations become mandatory.

The Australian Code Provisions on Flexural Crack Control

The Australian Code does not recommend any formula for the calculation of crack widths. Crack control for flexure in reinforced concrete beams is achieved if the center-to-center spacing of bars near the tension face of the beam does not exceed 8 in. (200 mm) and the distance from the side or soffit to the center of the nearest longitudinal bar is not greater than 4 in. (100 mm). In the case of fully prestressed concrete beams, the maximum tensile stress in the concrete due to short-term service loads should not exceed 3 $\sqrt{f_c'}$. To control flexural cracking in partially prestressed concrete beams, the increment in steel stress near the tension face is limited to 29Ksi (200 MPa), as the load increases from its value when the extreme concrete tensile fiber is at zero stress to the short-term service load value; and the center-to-center spacing of reinforcement, including bonded tendons, is limited to 8 in. (200 mm).

Flexural cracking in reinforced concrete slabs is controlled by limiting the center-to-center spacing of bars in each direction to the lesser of 2.5 times the thickness of slab or 20 in (500 mm). In fully prestressed slabs, similar to beams, the maximum tensile stress in the concrete due to short-term service loads is limited to 3 $\sqrt{f_c'}$. For partially prestressed slabs, the incremental steel stress should not exceed 22 ksi (150