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INTRODUCTION

Crack control is an important issue for primarily two reasons, aesthetics and durability (1,2,3). First, wide cracks detract from a structure visually as well as may unduly alarm the public that there are structural problems. Second, wide cracks may cause durability related problems. Cracks provide a rapid route for oxygen, water, and possible chlorides to reach the reinforcement, which may lead to corrosion and structural deterioration. To combat corrosion, many engineers have been specifying thicker concrete covers. Both research and experience have indicated that thicker covers can increase durability. In designing with thicker covers, however, engineers have found that the common design method for the control of cracking, often referred to as the z-factor method, becomes unworkable.

Research was conducted to investigate the role of concrete cover on cracking and to provide tools for the control of cracking in structures containing thicker covers. This research, presented in Ref. 4, developed a procedure for the calculation of crack widths that was based on the physical phenomenon. Additionally, a design recommendation was presented that ultimately resulted in changes to the ACI building code.

RESEARCH SIGNIFICANCE

For proper application of this new design procedure, it is important to understand the background for its development as well as the limitations imposed by that development. As an example, the building code states that these provisions are not intended for "structures subject to very aggressive exposure or designed to be watertight" (5). This paper explores the limitations and provides tools for the application of this new design approach for specialized structures. In addition, the control of cracking in structures utilizing new reinforcing materials is explored.

BACKGROUND

To understand the limitations of the current design method, it is useful to review the background of its development. As mentioned, research presented in Ref. 4 developed a calculation procedure for the determination of crack widths based on the physical phenomenon. A summary of the physical model is presented here (complete details are available in Ref. 4).

As shown in Figure 1, the crack width at the level of the reinforcement can be calculated as follows:

$$w_c = \varepsilon_s S_c \qquad (1)$$

where:

w_c = crack width

ε_s = reinforcement strain = $\frac{f_s}{E_s}$

S_c = crack spacing

f_s = reinforcement stress

E_s = reinforcement modulus of elasticity

Strain Profile

To determine the crack width at the beam surface, it is necessary to account for the strain gradient. The strain gradient is illustrated in Figure 2, which assumes that plane sections remain plane. The crack width computed above can be multiplied by an amplification factor (β) that accounts for the strain gradient. The factor, β, is computed as follows:

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{h-c}{d-c} \qquad (2)$$

Crack Spacing

Based on the work of Broms (6), it was found that the crack spacing depends primarily on the maximum concrete cover. Specifically, the minimum theoretical crack spacing will be equal to the distance from the point at which the crack spacing is considered to the center of the reinforcing bar located closest to that point. In addition, the maximum spacing is equal to twice this distance. As illustrated in Figure 3, the critical distance for the maximum crack spacing can occur at two locations, and the crack spacing can be calculated as follows:

$$S_c = \Psi_s d^* \tag{3}$$

where:

S_c = crack spacing
d^* = controlling cover distance
Ψ_s = crack spacing factor
1.0 for minimum crack spacing
1.5 for average crack spacing
2.0 for maximum crack spacing

Crack Control

Based on the physical model, the equation for the calculation of maximum crack width is as follows:

$$w_c = 2\frac{f_s}{E_s}\beta\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \tag{4}$$

This equation can be rearranged to solve for the maximum permissible bar spacing, s.

$$s = 2\sqrt{\left(\frac{w_c E_s}{2 f_s \beta}\right)^2 - d_c^2} \tag{5}$$

where:

s = maximum permissible bar spacing, in.
w_c = limiting crack width, in.
E_s = reinforcement modulus of elasticity, ksi
f_s = reinforcing bar stress, ksi
d_c = bottom cover measured from the center of lowest bar, in.

For a given limiting crack width and bar stress, the bar spacing can be plotted versus the concrete cover. The reinforcement stress used in Eq. (5) corresponds with the actual bar stress considered which is typically the service load stress. Alternately, a reinforcement stress of 60 percent of yield may be used to account for service levels. The factor, β, varies as the cover increases. Therefore, based on a review of sections with varying cover, $\beta \approx 1.0 + 0.08 d_c$ was found to provide a reasonable estimate.

Figure 4 is plotted for Grade 60 reinforcement (f_s = 36 ksi, E_s = 29,000 ksi). In this figure, curves are shown for two different limiting crack widths, 0.016 in. and 0.021 in. The crack width of 0.016 in. corresponds to the ACI 318-95 (7) design recommendations for interior exposure conditions while 0.021 in. corresponds to a 1/3 increase in the recommended crack widths. A 1/3 increase in crack widths was considered acceptable due to the large-scatter inherent in crack widths and since Eq. (5) considers the maximum crack width.

DESIGN RECOMMENDATION

Based on consideration of the results from the physical model, a simplified design curve for the maximum spacing of reinforcement was proposed in Ref. 4 as given by Eq. (6). This curve is plotted in Figure 4.

$$s = 12\alpha_s \left[2 - \frac{d_c}{3\alpha_s} \right] \le 12\alpha_s \qquad (6)$$

where:

$$\alpha_s = \frac{36}{f_s}\gamma_c$$

d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in.

s = maximum spacing of reinforcement, in.

α_s = reinforcement factor

γ_c = reinforcement coating factor
- 1.0 for uncoated reinforcement
- 1.5 for epoxy-coated reinforcement, unless test data can justify a higher value.

f_s = Calculated stress in reinforcement at service load, kips, in. Shall be computed as the moment divided by the product of steel area and internal moment arm. It shall be permitted to take f_s as 60 percent of the specified yield strength f_y.

ACI 318-99 DESIGN METHOD

The proposal presented was modified from its original form and resulted in the design equation presented in ACI 318-99 (5) under code section 10.6.4.

$$s = \frac{540}{f_s} - 2.5c_c \leq 12\left(\frac{36}{f_s}\right) \tag{7}$$

where:

s = center-to-center spacing of flexural tension reinforcement nearest to the extreme tension face, in. (where there is only one bar or wire nearest to the extreme tension face, s is the width of the extreme tension face.)

f_s = calculated stress in reinforcement at service loads, ksi It shall be permitted to take f_s as 60 percent of specified yield strength.

c_c = clear cover from the nearest surface in tension to the surface of the flexural tension reinforcement, in.

The ACI 318-99 equation considers the clear cover (c_c) directly rather than using the cover to the center of the bar (d_c). It was felt that the clear cover would be simpler to apply in design. In addition, this modified form of the equation is slightly more conservative than the original proposal. The ACI design curve is also plotted in Figure 4 where c_c is converted to the dimension d_c considering an average bar size of #8 (d_b = 1.0 in.) to provide comparison. It can be seen that this design equation reasonably describes the reinforcement spacing for a range of concrete covers while maintaining crack widths within the ranges previously discussed.

Two primary assumptions, however, were used in the derivation of the original design recommendation and are inherent in the ACI design method. These assumptions may prove to be a limitation for some design applications. First, the crack widths controlled are based on a crack width of approximately 0.016 in. at the beam bottom face. Considering the scatter inherent in cracking (it has been often noted that scatter in crack widths is in the range of 50%), crack widths both below and above this value should be expected in service. Therefore, as indicated in the building code, these provisions are not applicable for structures subject to very aggressive exposure conditions or for structures designed to be watertight.

Second, the maximum spacing was based on a reinforcement modulus of elasticity of 29,000 ksi that corresponds with the use of steel reinforcement. Therefore, the ACI design provisions are not applicable for structures using reinforcement containing a different modulus of elasticity. In fact, all test results used to ascertain the accuracy and applicability of the crack width equation (Eq. (4)) were for steel reinforcement.

SPECIFIED CRACK WIDTH CONTROL

For the design of specialized structures that require tighter control of the crack width, it is important to develop design tools that are applicable. Using the physical model, it is possible to consider any limiting crack width that the designer chooses appropriate. This feature allows for versatility especially for varying structural exposure conditions. In the same manner that the physical model was used to develop the simplified design curves presented by Eq. (6), simplified design curves can be developed for any specified crack width.

The maximum bar spacing can be determined directly using Eq. (5) once the desired limiting crack width is selected. Figure 5 presents the maximum bar spacing versus concrete cover, d_c, for a range of limiting crack widths. The graph was developed for Grade 60 reinforcement stressed at 36 ksi ($0.6f_y$). The range of crack widths presented is based on recommendations for various exposure conditions as provided by ACI Committee 224 (8).

As the limiting crack width is decreased, the spacing of the reinforcement must be decreased. For a structure containing a concrete cover of 1.5 in. and stressed at 36 ksi, it is evident that it is not possible to control the crack width to 0.004 in. With a maximum reinforcement spacing of 3 in., however, it is possible to control to approximately 0.006 in. Again, it must be noted that these crack widths are measured at the beam surface. Smaller crack widths are expected at the reinforcement.

As illustrated, there may be cases where crack width control to a specified crack width may not be possible by reducing only the reinforcement spacing. In these cases, it may also be necessary to reduce the design service level stress in the reinforcement. Figure 6 illustrates the effect of changing the reinforcement stress for a design limiting crack width of 0.006 in. As noted, various reinforcement spacings can be used to achieve the same limiting crack width through control of service load stress.

The designer can directly utilize Eq. (5) to control crack widths to any desired level. Alternately, simplified design curves can be developed as presented in Figure 7. These curves were developed based on the original design recommendation (Eq. (6)) with a factor added in the reinforcement factor, α_s, to account for varying crack control limits. The modified reinforcement factor is presented as follows:

$$\alpha_s = \frac{36}{f_s}\gamma_c\gamma_{w_c} \tag{8}$$

where:

γ_{w_c} = crack width factor = $\dfrac{w_c}{0.016\ in.}$

w_c = desired crack width limit, in.

As noted, the original design recommendation easily permits modification to allow for the control of various crack widths. Since the ACI design equations were based on the same format, they also can be modified to account for various desired crack control levels. The ACI design equation (Eq (7)) can be adjusted to account for varying crack control limits by multiplying f_s by $1/\gamma_{w_c}$.

REINFORCING MATERIALS

As new materials are being considered for use in reinforced concrete design, crack widths will remain of importance. Even though many materials do not have the potential for corrosion, the control of crack widths for aesthetic reasons will continue. Since Eq. (5) is based on the physical phenomenon, it remains applicable for materials with different moduli of elasticity. It must be noted, however, that bond is essential for the development of cracks and a regular crack spacing as computed by Eq. (3). Similar to epoxy coated reinforcement, lower bond strengths of alternative reinforcement can directly affect crack spacing and crack width. Testing of these reinforcement bars is essential to verify adequate development of cracks and the applicability of the crack width calculation.

Assuming adequate bond strength, the modulus of elasticity can be accounted directly in Eq. (5). Alternately, design curves were developed based on the original design recommendation (Eq. (6)) as follows:

$$\alpha_s = \frac{36}{f_s}\gamma_c\gamma_{w_c}\gamma_E \tag{9}$$

where:

γ_E = modulus of elasticity factor = $\dfrac{E}{E_s}$

E = modulus of elasticity of reinforcement, ksi

E_s = modulus of elasticity of steel, 29,000 ksi

This modified form of the reinforcement factor, α_s, contains all multipliers previously presented and is a general form that permits adjustments for epoxy coating, crack width limits, and the reinforcement modulus of elasticity. Similarly, the ACI design equation (Eq. (7)) can also be modified to account for the modulus of elasticity of various reinforcement materials by multiplying f_s by $1/\gamma_E$.

DESIGN EXAMPLES

Design examples are presented to illustrate the use of both the original design proposal as well as the ACI design method. Results from both procedures are provided to allow comparison. The examples also illustrate the incorporation and use of the modification factors presented here. It should be noted that in the typical design case of a structure containing steel reinforcement and not requiring special crack control procedures, all modification factors are 1.0. Therefore the equations simplify back to the basic equations presented by Eq. (6) for the original proposal and Eq. (7) for the ACI method. Therefore, modifications are required to be considered only in special instances.

Example 1

In the first example (Figure 8), it is desired to determine the adequacy of the reinforcement layout for crack control. The beam contains uncoated steel reinforcement and is contained in a structure that does not require special crack control procedures.

Original Design Proposal

As the structure contains uncoated steel and does not require special crack control procedures, no modifications are required (γ_c, γ_{w_c}, and γ_E = 1.0).

$$f_s = 0.6 f_y = 0.6(60\,ksi) = 36\,ksi$$

$$\alpha_s = \frac{36}{f_s}\gamma_c \gamma_{w_c} \gamma_E = \frac{36}{36}(1)(1)(1) = 1$$

$$d_c = 1.5 + 0.375 + \frac{1.128}{2} = 2.44\,in.$$

$$s = 12\alpha_s\left[2 - \frac{d_c}{3\alpha_s}\right] \leq 12\alpha_s$$

$$s = 12(1)\left[2 - \frac{2.44}{3(1)}\right] = 14.24\,in. > 12(1) = 12\,in. \quad \therefore \quad s = 12\,in.$$

Spacing as Designed:

$$s = \left(16 - 2(1.5 + 0.375 + \frac{1.128}{2})\right)/3 = 3.7\,in. < 12\,in. \quad OK$$

ACI Design Method

As the structure contains uncoated steel and does not require specialized crack control procedures, no modifications are required.

$$f_s = 0.6 f_y = 0.6(60\,ksi) = 36\,ksi$$
$$c_c = 1.5 + 0.375 = 1.875\,in.$$

$$s = \frac{540}{f_s} - 2.5c_c \leq 12\left(\frac{36}{f_s}\right)$$
$$s = \frac{540}{36} - 2.5(1.875) = 10.3\,in. \leq 12\left(\frac{36}{36}\right) = 12\,in. \quad \therefore \quad s = 10.3\,in.$$

Spacing as Designed:

$$s = 3.7\,in. < 10.3\,in. \quad OK$$

Both methods indicate that the spacing provided is adequate for crack control. It should be noted that the original design proposal uses the cover to the center of the bar, d_c, while the ACI design method uses the clear cover, c_c. The results provided by the ACI design method are slightly more conservative.

The beam in this example contains two layers of reinforcement. However, as noted in the calculations, only the bottom layer of reinforcement is considered through either the parameter d_c or c_c. Only the bottom layer of reinforcement influences the crack width at the bottom face because this reinforcement is located closest to the surface. Also, in this example, all bars considered are of the same size. The procedure is the same if bar sizes are mixed. For the original design method, the cover to the center of the bar should conservatively consider the largest bar diameter. For the ACI design method, mixed bar sizes do not affect the results as only the clear cover is considered.

Example 2

The second example is provided for the design of a slab (Figure 9). The slab contains uncoated reinforcement spaced at 6 in. on-center. For this structure, it is determined that special crack control procedures are required and the crack width should be limited to approximately 0.006 in. at the slab bottom face.

Original Design Proposal

As the structure contains uncoated steel, γ_c and γ_E = 1.0. However, a modification factor, γ_{w_c}, is required to account for the increased level of crack control.

$$f_s = 0.6 f_y = 0.6(60\,ksi) = 36\,ksi$$

$$\gamma_{w_c} = \frac{w_c}{0.016\,in.} = \frac{0.006}{0.016} = 0.375$$

$$\alpha_s = \frac{36}{f_s}\gamma_c \gamma_{w_c} \gamma_E = \frac{36}{36}(1)(0.375)(1) = 0.375$$

$$d_c = 0.75 + \frac{0.5}{2} = 1.0\,in.$$

$$s = 12\alpha_s\left[2 - \frac{d_c}{3\alpha_s}\right] \le 12\alpha_s$$

$$s = 12(0.375)\left[2 - \frac{1.0}{3(0.375)}\right] = 5.0\,in. > 12(0.375) = 4.5\,in. \quad \therefore \quad s = 4.5\,in.$$

Spacing as Designed:

$s = 6\,in. > 4.5\,in.$ $\quad NG$

The design spacing is too large to provide the desired level of crack control at a service stress of 36 ksi. Therefore, either the spacing needs to be decreased to 4.5 in. or the service load stress should be reduced. A service stress of 27 ksi will be checked.

$$\alpha_s = \frac{36}{f_s}\gamma_c \gamma_{w_c} \gamma_E = \frac{36}{27}(1)(0.375) = 0.5$$

$$s = 12(0.5)\left[2 - \frac{1.0}{3(0.5)}\right] = 8\,in. > 12(0.5) = 6\,in. \quad \therefore s = 6\,in.$$