VERIFICATION OF THE FINITE ELEMENT MODEL

The last stage of the modeling process is model verification, sometimes referred to as validation. The primary criterion for model verification is rationality, i.e., the degree to which both predictions and effects agree with values obtained from the physical system. Ideally, measured data, either field data or laboratory-measured values, would be available. Even measured data are subject to uncertainty, so other forms of verification can be used. Sensitivity analyses and simulation studies based on measured data are other forms of rationality testing. If such independent resources are not available, then a valid alternative for verification is the comparison of the values obtained from the model with corresponding values from other independently developed models. A reasonable level of agreement between these independent models suggests an acceptable measure of rationality and, therefore, model verification. Two approaches were used to verify the finite element model (Ali [2])). One is the determination of the static and dynamic response of the finite element model of a single pile and the other is the dynamic response of the finite element model of a group of piles. The primary tests of verification were a comparison of the quantitative results of other independently performed analyses.

Single Pile Verification

The finite element model was used to determine the stiffness of a pile embedded in soils with different values of soil shear modulus. A comparison between the values obtained from the finite element solution and the ones obtained from the closed-form solution of Mylonakis and Gazetas [18] was undertaken. The difference between the solutions was within 2% for the different soil shear modulus used. Also, at very low values of shear modulus, the static stiffness of the pile approached the stiffness of a compression member, i.e., the surrounding soils did not contribute to the static stiffness, as expected.

Solutions for the dynamic response of single piles at 5% constant damping ratio were obtained using a threedimensional finite element model, the closed-form plain strain solution presented by Novak [20], and a form of Novak solution modified by Chowdhury and Dasgupta [7] to include the inertial effect of the pile that Novak's solution did not take into account. Figure 3 shows the amplitude response of the pile foundation system comparing the finite element solution with Novak and with Chowdhury and Dasgupta closed-form solutions.



Figure 3—Comparison of Response among FEA, Novak and Chowdhury and Dasgupta at $a_o = 0.30$ [1m = 3.28 ft.]

Pile Group Verification

Petrash et al. [21] determined the dynamic stiffness and damping for a 2 x 2-pile group, spaced at 0.914 m [3.0 ft] center to center using the DYNA5 program. The method used in DYNA5 for calculating the pile dynamic stiffness and damping is based on the plane strain method where elastic waves are assumed to propagate in a horizontal direction, similar to Novak's elastodynamic solution.

To verify the ANSYS model, the soil pile impedance determined by ANSYS was compared with the soil pile dynamic stiffness and damping determined using DYNA. The inputs to the ANSYS model were modified to match the input parameter used by Petrash et al. in the DYNA5 model. Figure 4 shows the ANSYS finite element model of the pile groups.

The ANSYS model was excited using a constant amplitude harmonic excitation force. Figure 5 shows the vertical displacement of the soil pile system at resonance as determined from the ANSYS model.



Figure 4—Modified ANSYS Finite Element Model



Figure 5—Vertical Displacement in ft. for Pile Group at Resonance [1ft = 0.305 m]

Table 2 shows the dynamic stiffness and damping determined from the finite element solution and the ones determined using the DYNA5 solution.

	ANSYS Solution	DYNA5 Solution	% Difference
Vertical Stiffness	1.824 x 10 ⁵ (kN/m) [1.25x10 ⁴ kip/ft.]	2.189 x 10 ⁵ (kN/m) [1.50x10 ⁴ kip/ft.]	20 %
Damping	5.56 x 10 ³ (kN sec/m) [380.1 kip.sec/ft.]	5.808 x 10 ³ (kN sec/m) [398.0 kip.sec/ft.]	4.46 %

Table 2—Comparison of Stiffness and Damping between ANSYS and DYNA5.

The damping values shown in Table 2 are calculated using the Dynamic Magnification Factor (DMF). The calculation of damping was undertaken at resonance where it is the most critical. The DMF for the pile-soil system at each dimensionless frequency parameter (a_0) is calculated as follows:

$$DMF_{max} = \frac{1}{2\zeta\sqrt{1-\xi^2}}$$
(9)

$$\zeta = \frac{c}{c_{\rm cr}} \tag{10}$$

$$c = \zeta(2M_{eff}\omega_n) = \zeta\left(2\frac{K_{pile}}{(2\pi f_n)^2}(2\pi f_n)\right) = \zeta\left(\frac{K_{pile}}{\pi f_n}\right)$$
(11)

The difference in the stiffness between the ANSYS solution and the DYNA5 solution is attributed to the threedimensional effects of the soil pile interaction considered in ANSYS while the DYNA5 solution is based on a twodimension plane strain solution. On the other hand, the damping values between both solutions were relatively close. This is because, in an axially loaded pile, the waves generated along the pile's soil interface propagated mainly in the horizontal direction under essentially plane stain (Dobry, [18]). Hence, damping values show better agreement.

DYNAMIC PARAMETERS DETERMINED

Under the excitation of a vertical harmonic force that acted at the pile cap and assuming a 5% material damping for the soil and no damping for the concrete pile material, three dynamic parameters were obtained: the pile group dynamic stiffness, the pile group damping, and the pile group resonant frequency. The vertical dynamic stiffness of the concrete piles that have compressive strengths of 20.7 MPa [417.709 ksf] was computed for the dimensionless frequency parameter (a_o) that ranged from 0.2 to 2.0, which is consistent with the range used by Novak [20]; however, for most practical applications, a_o will be less than 1.0. The vertical dynamic amplitude response was calculated at the pile cap at various frequencies of excitation. The vertical dynamic displacement amplitude. The pile group damping was calculated using the soil-pile system Dynamic Magnification Factor (DMF), as described above, which is defined as the ratio of the dynamic displacement at resonance to the static displacement for the soil pile system. The resonant frequency of the pile-soil system was the frequency where the maximum vertical dynamic amplitude response occurred.

RESULTS AND DISCUSSION

Static Response

To determine the vertical static stiffness of the pile groups, the finite element models of the group of piles spaced at 2D, 4D, and 6D were subjected to a vertical static load that acted at the center of the pile cap. The vertical deflection of the pile cap center was determined for different soil shear modulus. Figure 6 shows the vertical static stiffness of the pile group as a function of the soil shear modulus, G_{soil} . The vertical stiffness of the soil pile system is determined from the applied load on the pile cap and the displacement of the pile cap.



Figure 6—Vertical Static Stiffness for Pile Groups as a Function of Gsoil

The figure shows that the vertical static stiffness of the pile group increases nonlinearly with the increase in the soil shear modulus. The increase in the soil shear modulus increases the total support of the soil-pile system to the applied load. At a low value of the soil shear modulus, the group of piles acted as end-bearing piles; the surrounding soils had little influence on the stiffness. The pile group spaced at 6D has a higher stiffness than the pile group spaced at 4D and 2D, which is attributed to the largest contribution of the soil between the piles in the group. Forces in the piles along the pile length were also determined and were found to decrease with the pile depth. The decreased portion of the load is being carried by the surrounding soil. The load in the piles in soil with low shear modulus was found to be the same for all the pile locations. Whereas in soils with high shear modulus, the load on the middle pile is less than the load on the edge pile, which is also less than the load on the corner pile. This is because the area of the soil around the middle pile is larger than the area of the soil around the edge pile and the corner pile. Thus the applied load will be shared between the pile and the soil. Since there is more soil around the center pile, the load sharing between the soil and the pile is increased which reduces the load on the pile.

DYNAMIC RESPONSE

Stiffness and Damping of Pile Group

The three finite element models for the pile-soil system were excited with a vertical constant amplitude harmonic excitation force. The dynamic soil properties are defined as a function of the soil dimensionless frequency-dependent parameter (a_o). The maximum amplitude response at different values of a_o of the pile-soil system is measured at the centerline of the pile cap. Figures 7, 8, and 9 show the vertical dynamic stiffness, damping, and the damping ratio of pile groups spaced at 2D, 4D, and 6D, respectively as a function of a_o .



Figure 7—Dynamic stiffness for pile group as a function of a_o .



Figure 8—Damping of pile group as a function of *a*_o



Figure 9—Damping ratio of pile group as a function of *a*_o

The vertical dynamic stiffness and damping of the pile groups are dependent on the pile spacing and a_o . The dynamic stiffness as shown in Figure 7 of the pile groups was decreased by 50%, 33%, and 25% in the piles in a group that are spaced at 2D, 4D, and 6D, respectively, when the dimensionless frequency parameter was increased from 0.20 to 2.0.

Figures 8 and 9 for the damping and the damping ratio respectively show a complicated behavior with the curves having peaks and valleys. This is more pronounced in the case of the spacing of 6D. For the spacing of 2D, the stiffness and damping exhibit a smoother variation with the dimensionless frequency parameter a_0 .

The reason for the oscillatory behavior of the stiffness, damping, and damping ratio as a function a_o is that along the length of each pile at all points on the pile the vertical vibrating piles are sending shear waves into the soil. These waves propagate radially outwards in the horizontal direction with a wave velocity of V_s . These stress waves are generated from each pile in the pile group. These waves emitted from each pile are subject to attenuation with distance, and when encountering a pile in the group, refraction, reflection, and change in phase results. Such wave interaction affects the dynamic response of the pile group. The results of this interaction, as evident in Figures 7, 8, and 9, are a strong oscillatory behavior, i.e., the curves have peaks and valleys. The case of peaks and valleys were also evident in the work of Dobry and Gazetas [8], and they stated that the change in the value of the frequency causes wave interference of the shear waves originating along the pile length and such interference can be constructive where peaks occur or destructive interference where a valley occurs. In the case of damping, the peaks and valleys are more pronounced because the cylindrical stress waves generated from one pile in the group. Thus, the damping of the soil pile system will decrease due to the amplification of the resulting waves. Conversely, the damping of the soil pile system will increase when these stress waves are out of phase due to the de-amplification of the resulting wave, which results in the oscillatory behavior of the damping.

The pile group spaced at 6*D* has higher stiffness and damping than the pile groups spaced at 4*D* and 2*D*. This is attributed to the larger contribution of the soil between each pile in the piles group. With the large soil volume in the case of the pile group spaced at 6*D*, the stiffness increases, as well as the damping. Also for the same frequency, when $a_o = 0.2$, the soil has a large modulus, and when $a_o = 2.0$ the soil has a low modulus. Thus the stiffness at $a_o = 0.2$ is higher than the stiffness at $a_o = 2.0$. As a_o increases, the effect of the soil in the vertical dynamic stiffness of the pile group is minimal and the pile group vertical dynamic stiffness is governed by the structural stiffness of the pile groups (piles structural stiffness and the stiffness of the bearing soil for end-bearing piles). Another difference is that for the case of close spacing, i.e., 2*D*, the response exhibits a smoother variation with a_o compared to the larger

variation in the 4D and the much larger variation in the case of pile group spaced at 6D. With the close spacing, the pile group acts as an isolated embedded foundation, i.e., the soil mass between the piles tends to vibrate in phase with the piles and so the pile groups-soil system responds as a block.

To further show the effect of wave interference especially on the damping, Figures 10 until 15 are presented. Figures 10 until 12 show the vertical displacement fields of the soil-pile systems for the pile groups spaced at 2D, 4D, and 6D at a low frequency range (1.0 Hz), which is quasi-static for soil with $G_{soil} = 8.2 \times 10^5 \text{ kN/m}^2$ [1.713 x 10⁴ ksf], soils such as dense sand and stiff clay. For soils at this low frequency, the displacement fields between the piles show a uniform displacement distribution in the soil continuum, the soil displacement field is well defined around the pile group, and both the soil and the pile move as a block. Figures 13 until 15 show the displacement field of the soil-pile at the resonant frequency for soil with $G_{soil} = 8.2 \times 10^3 \text{ kN/m}^2$ [171.261 ksf]. At the resonant frequency, conversely, the displacement fields between the pile elements are not uniform and show considerable wave interference. Such wave interference is the cause of the oscillatory behavior of the stiffness and especially the damping. As the cylindrical waves travel away from the pile into the soil continuum and depending on the soil type, waves attenuate, refract, and change in phase. When these cylindrical waves meet another cylindrical wave from an adjacent pile, they either become amplified, if both traveling waves have the same frequency and phase, or attenuate when the two traveling waves have different frequencies and phase angles.



Elevation View Plan View **Figure 10**—Vertical displacement in ft. of pile group's response at 1.0 Hz and $G_{soil} = 8.2 \text{ x } 10^5 \text{ kN/m}^2 (17126 \text{ ksf})$ for piles spacing 2D [1ft = 0.305 m]



Elevation View Plan View Figure 11—Vertical displacement in ft. of pile group's response at 1.0 Hz and $G_{soil} = 8.2 \text{ x } 10^5 \text{ kN/m}^2$ (17126 ksf) for piles spacing 4D [1ft = 0.305 m]

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Elevation ViewPlan ViewFigure 12—Vertical displacement in ft. of pile group's response at 1.0 Hz and $G_{soil} = 8.2 \times 10^5 \text{ kN/m}^2$ (17126 ksf)
for piles spacing 6D [1ft = 0.305 m]



Elevation ViewPlan ViewFigure 13—Vertical displacement in ft. of pile group's response at resonance at 10 Hz and $G_{soil} = 8.2 \text{ x} 10^3 \text{ kN/m}^2$
(171.261 ksf) for piles spacing 2D [1ft = 0.305 m]



Elevation View Plan View Figure 14—Vertical displacement in ft. of pile group's response at resonance at 10 Hz and $G_{soil} = 8.2 \text{ x} 10^3 \text{ kN/m}^2$ (171.261 ksf) for piles spacing 4D [1ft. = 0.305m]



Elevation View Plan View **Figure 15**—Vertical displacement in ft. of pile group's response at resonance at 10 Hz and $G_{soil} = 8.2 \text{ x} 10^3 \text{ kN/m}^2$ (171.261 ksf) for piles spacing 6D [1ft. = 0.305m]

PILE INTERACTION AND GROUP EFFICIENCY

The stiffness and damping efficiency factors are determined from equations (2) and (3) for the pile groups spaced at 2D, 4D, and 6D as a function of the dimensionless frequency parameter (a_o) . The results are shown in Figures 16 and 17.



Figure 16—Stiffness Efficiency Factors as a Function of *a*_o



Figure 17—Damping Efficiency Factors as a Function of *a*_o

Both figures show an oscillatory behavior similar to that shown in Figures 7 and 8. The pile group efficiency under dynamic loading differs considerably from that of a pile group under static loads, as the pile stiffness is a function of a_o and a_o is a function of the machine frequency, the pile diameter, and the soil shear modulus. Figure 16 shows that the efficiency factor for the stiffness can be as high as 1.15 for 6D spacing and as low as 0.7 for 2D spacing. The damping could be as high as 3.75 for 6D spacing and as low as 0.4 for 2D spacing, (see Figure 17). In comparison to the static efficiency factors, which are always below unity, the dynamic efficiency factor may exceed unity. Thus, the dynamic group effect can either increase or decrease the response of the supporting structure.

The stiffness and damping efficiency factors were also plotted as a function of the soil shear modulus and at the same machine exciting frequency of 50 Hz, which represent the operating frequency for a wide range of machines, in Figures 18 and 19, respectively.



Figure 18—Pile group stiffness efficiency factors as a function of soil shear modulus (G_{soil})