deflection as the secant between the maximum load and one-half of its value. A plot of the average modulus of elasticity versus testing age is illustrated in Fig. 6.

Flexural Strength (Modulus of Rupture)

The flexural strength was computed from the maximum load at cracking by applying the flexural formula. A plot of the average flexural strength at the different testing ages is illustrated in Fig. 7.

DISCUSSION OF TEST RESULTS

Flexural Properties at Early Age

A summary of all results is shown in Fig. 8 in order to observe the behavior of the material as age increases during the 11-hour period. Each data point represents the average of at least five tests, and although the individual properties would very likely change with change in the material, the trend of values is established. In some of the figures reversal of trends are shown at five-hour age, but since this is the only data point between three and eight-hour ages, it is recognized that the actual age of reversal could possibly be before or after the five-hour age.

Figure 8 shows that the curvature at first cracking decreases rapidly as time increases from 3 to 5 hours, reaching the minimum value at 5 hour test age. The curvature then begins to increase, at a somewhat slower rate, with time until tests terminated at the 11 hour age. The reason why the curvature behaves in this fashion can be explained by the study of the development of the flexural strength and modulus of elasticity with age.

The modulus of elasticity grows more rapidly than the flexural strength between 3 and 5 hours ages and the reverse occurs between 5 and 11 hours (Fig. 8). This is further illustrated in the figure by the plot of the ratio of flexural strength to modulus of elasticity versus age. Comparing the plot of the curvature at first cracking versus age with that of the ratio of strength to modulus versus age, it is seen that both are very similar in shape. Flexural theory, in the equation for curvature, indicates that the radius of curvature decreases (curvature increases) with the increase of the ratio of concrete strength to modulus of elasticity. From this, it might be reasoned that a similar relationship would hold between curvature, or the ratio of flexural strength to modulus of elasticity, and age. It is seen in Figure 8 that this is indeed the case, and the data supports the reasoning, even though the material is not perfectly elastic at these early ages.

The only investigation in the literature in which the bending curvature of concrete was measured at early ages was conducted by Hilsdorf and Lott(5). They showed that a concrete surface curvature of approximately 50 x 10^{-5} inch^{-1} was required to develop cracking in 6-inch thick reinforced concrete slabs when deflected upwards. Their tests were conducted between 2 and 4 1/2 hours after mixing. In comparing the cracking curvatures of the present tests with those of Hilsdorf and Lott, the thicknesses of the test beams must be taken into account. The beams reported here were 2-inches thick, whereas the slabs of Hilsdorf and Lott were 6-inches thick. In an elastic model, the beam curvature is inversely proportional with the distance from the neutral axis to the top fibers of the beam, which is a function of the beam thickness. Thus the thickness of the beam or slab has a very important bearing on the curvature. In order to have a fair comparison between the curvature determined by Hilsdorf and Lott and those obtained in this study, the former curvature is modified by multiplying it by a factor of 6/2, the ratio of beam thicknesses. This modification is based on the assumption that the same strain occurs at cracking irrespective of beam thickness. Hilsdorf and Lott's modified curvature at cracking is almost identical to the average curvature from the test beams, and the ages of the two sets of tests correspond reasonably well. Hilsdorf and Lott's slabs cracked at a curvature of 150 x 10^{-5} inch^{-1} versus 148.9 x 10^{-5} inch^{-1} for the test beams of the present study. The former test specimens were made of reinforced concrete whereas the latter were of non-reinforced mortar. There is not enough data available to know if the differences in the materials, which includes different water to cement ratios, had any effect on the curvatures. The agreement in curvature is very interesting and possibly of considerable significance, despite the differences in materials noted.

Hilsdorf and Lott determined that the slabs deflected 4 1/2 hours after mixing had wider cracks than those deflected 2 hours after mixing. Assuming an elastic beam, the depth of the crack will vary directly with the width of the crack, and hence the depth of the uncracked portion is greater at 2 hours than that at 4 1/2 hours. Assuming a constant curvature 1/ = f/Ec (where f, is the flexural strength; E, is the modulus of elasticity; and c is one half the uncracked depth), it can be said that the ratio f/E at the younger age is greater than that at the later age. This signifies that the modulus of elasticity has increased faster than the flexural strength between these time periods, which agrees with the findings of the present study.

Data on the ultimate tensile strain of concrete at early ages has been reported by Byfors(6) and Kasai(7), and came primarily from direct tension tests. The data generally agrees reasonably well with the behavior of the mortar specimens tested in the present study, and the general trend of the relations between ultimate tensile strain and age, is very similar to the curvature-age relationship of Fig. 5.

Kasai(7) designed a tensile testing apparatus for studying the tensile properties of early-age concrete. He tested different mix proportions of concrete, and presented relations between tensile strain and age for the different mixes used. He found that the tensile strain decreases rapidly, with the elapse of time, at 2 to 3 hours showing the minimum value at 8 to 15 hours, and thereafter increases slightly with the elapse of time. He also found that the age at which the tensile strain is at the minimum varies with the kind of cement. It was concluded that the mix composition has a major effect on the properties of the material at early ages. Thus it is felt the curvatures at first cracking determined in the present study are expected to apply only for the specific mix and curing conditions used.

Actual Versus Elastic Curvature

In an elastic system the curvature is related to the flexural stress, f, and the modulus of elasticity in flexure, E, by the relation

$$\text{Curvature} = \frac{f}{E\,c}$$

where c is the distance from the neutral axis to the outer fibre of the beam. For a rectangular section, c is one-half the depth of the section.

The above equation was used to estimate the curvature at cracking by using the average values obtained from the tests, for the flexural strength and modulus of elasticity at a particular test age. Since the test specimens were 2 inches thick, a value of unity was used for c. The curvatures at first cracking, at the different test ages, computed by using the elastic relation, were compared to the actual average curvatures determined from the deflected geometry of the test specimens. The results are plotted as a function of age in Figure 9.

Surprisingly, for the ages of 5, 7, 9, and 11 hours the differences are small. The maximum difference of about 13 percent occurs at the 7 hour age. At the 3 hour age a large difference of about 62 percent occurs. Initial set occurs at about 3 hours age, and at that age the products of hydration are poorly formed and there is little interlock. The situation improves with time as hydration products increase to fill water spaces, and interlock between these products becomes better developed. Sufficient data is not available to draw any final conclusions. However, it can be said that at the very early ages, from time of casting up to 3 hours, the load-deflection curves and consequently the stress-strain curves for the mortar specimens, were very non-linear and thus the elastic formula did not yield good results. At the ages 5, 7, 9, and 11 hours the load-deflection relations of the different tests, as shown in Figure 4, had more the form of a straight line agreeing closely with curvatures obtained from the elastic formula.

Additional research is needed to investigate the extent of the applicability of the elastic formula at early ages. If this formula can be applied within reason, it can be extremely useful. The flexural strength and the modulus of elasticity can be estimated from simple tests and curvatures at early ages can, therefore, be computed without resorting to complicated experiments. The elastic formula, if applied, should be used with caution. The accuracy of the formula is considerably dependent on the modulus of elasticity whose magnitude depends greatly on the way it is defined.

SUMMARY

A summary of some of the significant results obtained in this study may be itemized as follows:

1. The average curvature at first cracking at the 3 hour test age, as determined from seven tests, was 149×10^{-5} in^{-1}. It decreased to a minimum of 27×10^{-5} in^{-1}, the average of six tests, at the 5 hour test age which corresponds approximately to the setting age of the material. Thereafter, the curvature at first cracking increased with the elapse of time up to 11 hours, the oldest material of the study.

2. The modulus of elasticity increased rapidly between the 3 and 5 hour test ages. Thereafter it increased at a much slower rate.

3. The flexural strength increased slowly between the 3 and 5 hour test ages. Thereafter its rate of growth was much faster.

4. The ratio of flexural strength to modulus of elasticity behaved in the same manner as the curvature in item 1, i.e., it decreased to a minimum at the 5 hour test age and thereafter increased with the elapse of time up to 11 hours.

5. The calculated curvatures of the beams when the mortar first cracked using the elastic formula agreed very closely with the actual curvatures at the test age of 5, 7, 9, and 11 hours. The maximum differences in values was only about 13 percent. However, at the 3 hour test age, the elastic formula yielded results which were about 62 percent in error when compared to the actual curvature.

CONCLUSIONS

The mortar deformation behavior in flexure at early age is dependent chiefly on the relative growth of the modulus of elasticity and flexural strength. At very early ages, approximately less than 2 hours from mixing, the mortar is almost in a fluid state and has an extremely large deformation capacity in flexure. Very large curvatures can be tolerated before cracking occurs. After 2 to 3 hours, at approximately initial set, the material begins to stiffen. The modulus of elasticity grows more rapidly than the flexural strength, causing a decrease in the curvature capacity (or strain). This continues up till about 5 hours, as determined in the present study. However, the literature indicates that the age at which the deformation capacity is minimal is somewhat dependent on the mix composition and type of cement(7). After the critical age at which the deformation capacity is at a minimum, strength grows at a moderately faster rate than the modulus. Thus curvature shows an increase with elapse of time. This increase is faster at the early ages and decreases as the mortar gets older. In the present study, ages only up till 11 hours were investigated. Results show that the growth in curvature capacity with age was rapid between 5 and 9 hours, at approximately the time of final set, and decreased between 9 and 11 hours. Other investigators showed that ultimate strain capacity, from the time it assumes a minimum, starts increasing at a slow rate and continues to an age of at least 28 days.

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Table 1. Summary--All Test Results

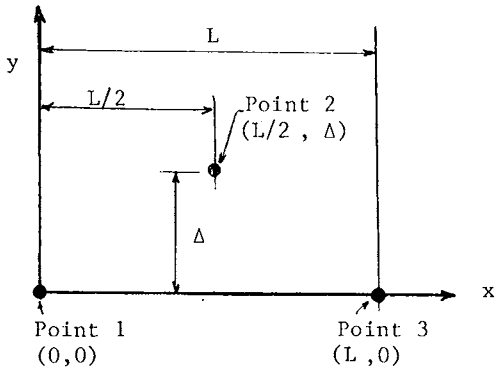
Average Age (hours)	No. of Tests	Average P_{max} (lbs)	Average Δ_{max} (10^{-3} in)	Average Slope (lb/in)	Average Curvature ($10^{-5} in^{-1}$)	Average Mod. of Elasticity (10^{6} psi)	Average Flexural Strength (psi)
3.01	7	3.46 (1.2)*	60.9 (9.6)	44.6 (37.2)	148.9 (14.4)	.00160 (.00087)	3.86 (1.27)
5.03	7	21.1 (5.8)	11.0 (3.1)	2575 (683.7)	26.7 (7.3)	.0787 (.02)	23.7 (6.5)
7.03	6	76.6 (10.3)	35.8 (10.3)	2790 (955)	87.4 (254)	.0852 (.029)	85.5 (11.8)
9.03	5	142.1 (20.1)	61.2 (13.2)	3510 (679)	149.5 (32.4)	.1067 (.021)	158.0 (23.1)
11.02	5	177.9 (34.5)	68.9 (18.3)	4045 (1221)	168.0 (44.6)	.1234 (.038)	199.0 (38.9)

*Numbers in parenthesis are the standard deviation.

APPENDIX A

METHOD OF COMPUTING CURVATURE

Points 1, 2, and 3 represent points on the test beam. Points 1 and 3 are support points of zero deflection. The deflection of point 3 was measured.



The equation of a second degree parabola is:

$$y = A + Bx + Cx^2$$

where A, B, and C are constants to be determined from the available boundary conditions. The available boundary conditions are:

At	$x = 0$	$y = 0$
At	$x = L$	$y = 0$
At	$x = L/2$	$y = \Delta$

Applying the boundary conditions, the parabolic relation becomes:

$$y = \left(\frac{4x}{L} - \frac{4\,x^2}{L^2}\right) \cdot \Delta$$

therefore, curvature at midspan of the beam $= \frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{-8\Delta}{L^2}$

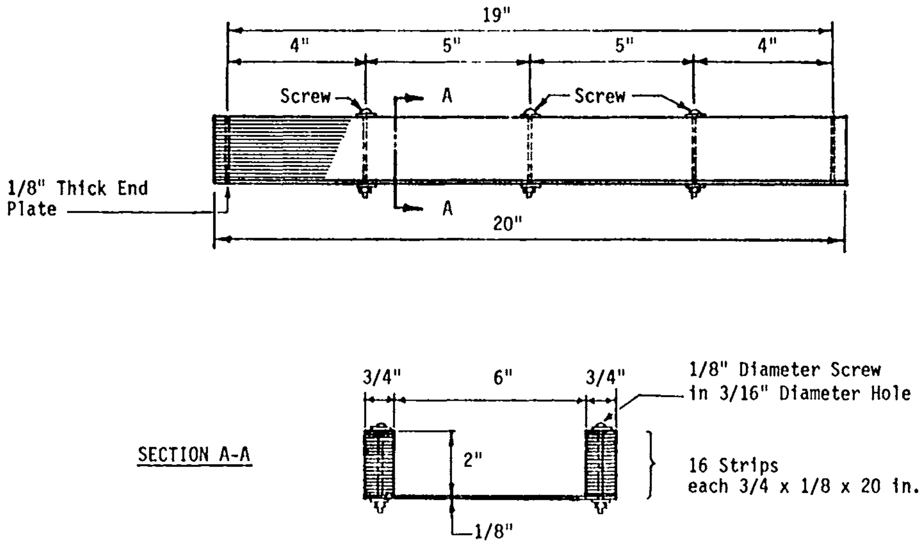


Fig. 1--Details of plexiglass test mold

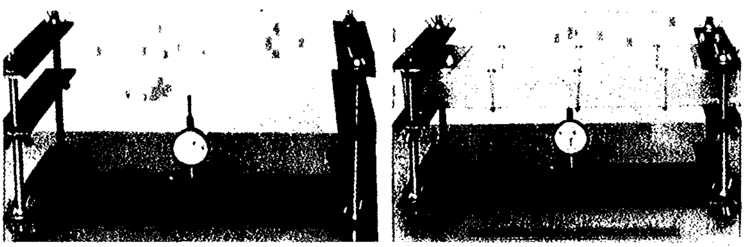


Fig.2--Testing frame before and after inserting test specimen

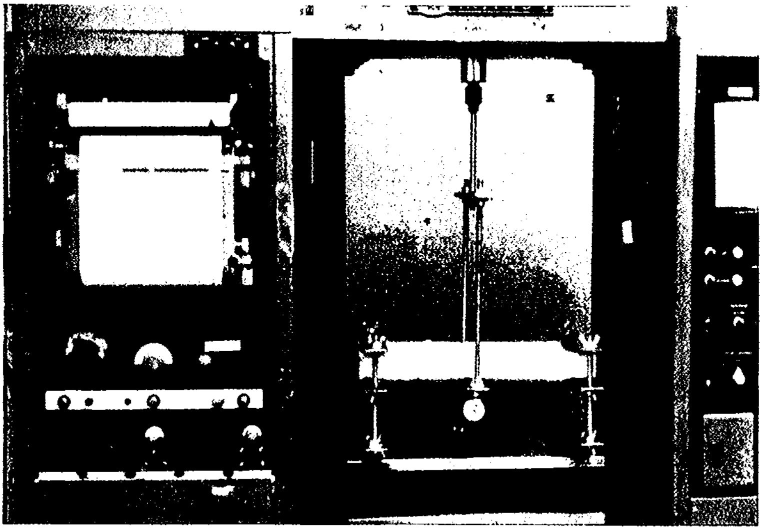


Fig. 3--Test arrangement

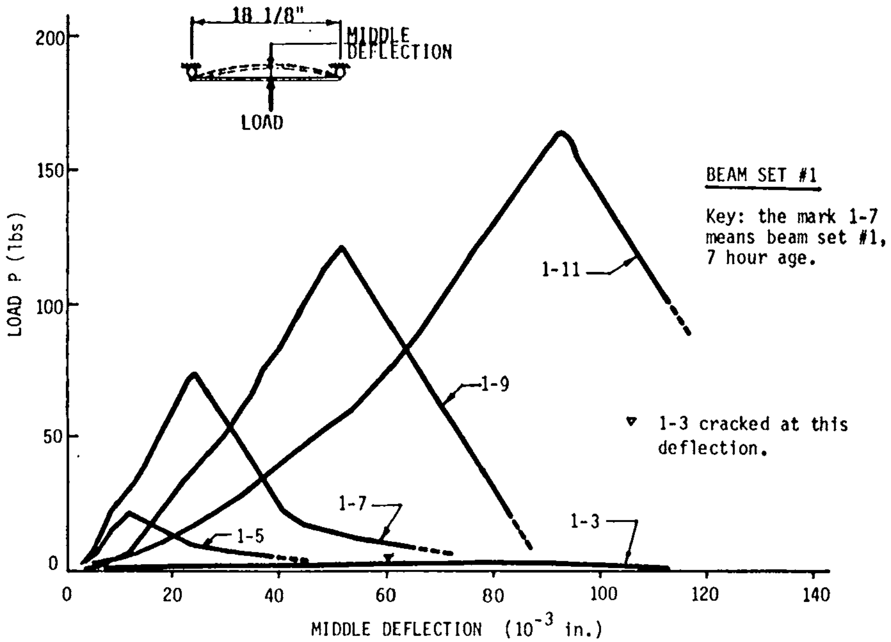


Fig. 4--Load-deflection curves, beam set #1 — mortar beam only