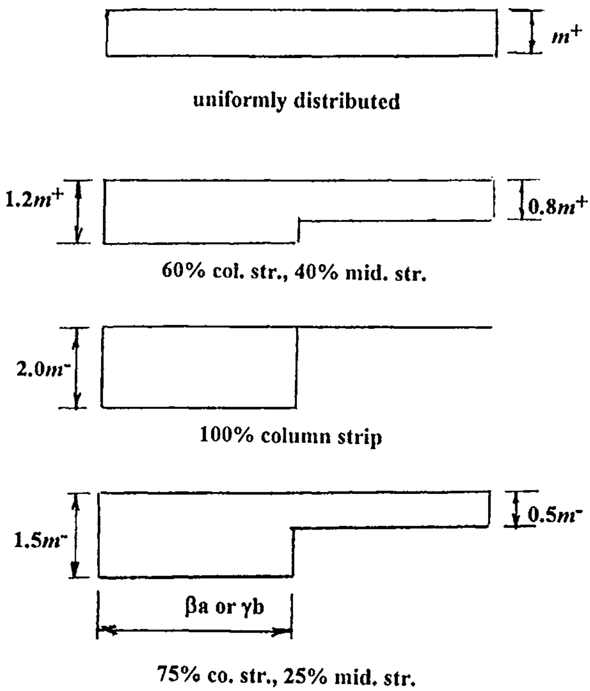

Fig. 7—Possible distributions of edge moments for corner supported elements ($\beta = \gamma = 0.5$).

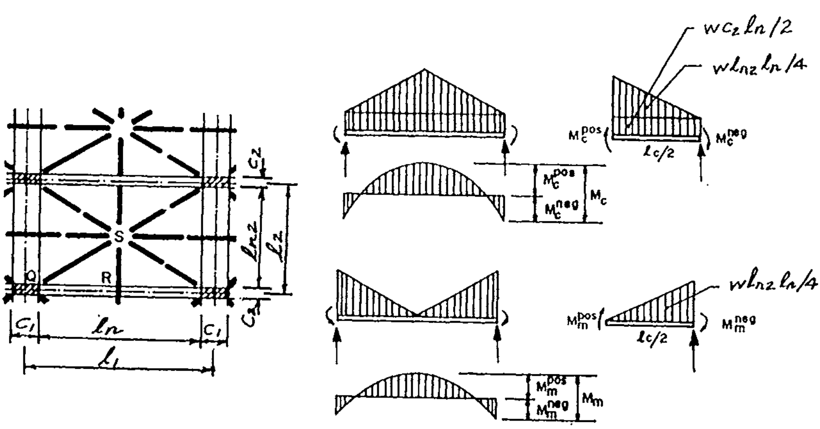

Fig. 8—Segment equilibrium method for rectangular panels.

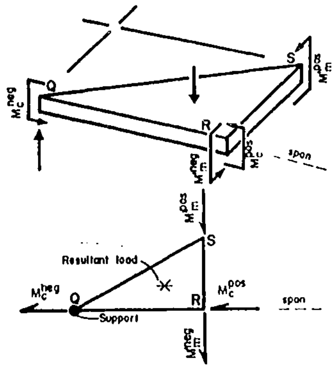

Fig. 9—Arrow definition used in segment equilibrium method.

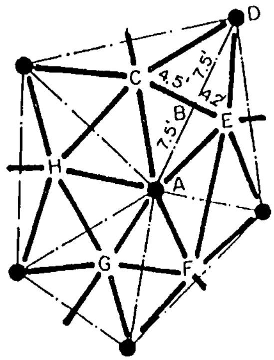

Fig. 10—Slab with random column spacing.

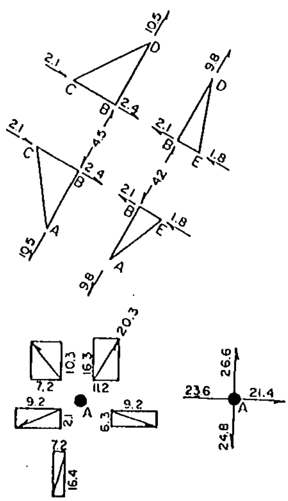

Fig. 11—Negative slab moments at Column A.

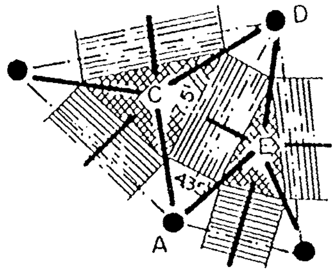

Fig. 12—Bottom reinforcement for slab with random column spacing.

SP 183-6

Strip Method for Flexural Design of Two-Way Slabs

by S. Alexander

Hillerborg's strip method of design (1, 2) is a powerful and versatile technique for designing two-way reinforced concrete slabs and plates. The method is based on the lower bound theorem of plasticity, meaning that a design based on the strip method is always safe. The purpose of this paper is to provide an overview of the strip method, including design examples.

The strip method is usually divided into two parts. The simple strip method is used to design edge supported slabs. Many designers will recognize this as an application of the strong-band concept. The advanced strip method is used to design slabs with column supports or reentrant edge supports.

Keywords: columns; design; plates; reinforced concrete slabs

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INTRODUCTION

Hillerborg's strip method of design (1, 2) is one of the most powerful and versatile techniques for designing two-way reinforced concrete slabs and plates. Like yield-line analysis, the strip method is based on the theory of plasticity. Unlike yield-line analysis, the strip method satisfies, for the most part, the lower bound theorem of plasticity. This means that, so far as flexural strength is concerned, the designer is always on the safe side using the strip method.

With the strip method of designing a two-way slab, one first assumes a pattern of load distribution and then determines average design moments that are consistent with that assumed load distribution. The load distribution scheme is made up of two types of load distribution elements: edge supported elements and corner supported elements.

The treatment of edge supported elements is usually referred to as the simple strip method. Many designers will recognize aspects of the simple strip method as a strong band approach to designing two-way slabs. While it is possible to design any slab using simple strips, certain support conditions such as columns or reentrant walls are awkward. To overcome this difficulty, Hillerborg developed a corner supported element, commonly referred to as the advanced strip method.

The purpose of this paper is to review, with design examples, the simple and advanced strip methods. The material contained here is available from a number of other sources and interested readers are urged to consult these. The most complete presentations of the strip method are in Hillerborg's books (1, 2). The first of these provides detailed theoretical development of both the simple and the advanced strip methods. The second focuses on the application of these methods in design. A journal article by Hillerborg (3) provides a concise presentation of the advanced strip method. Many textbooks on reinforced concrete design present the simple strip method and a few include material on the advanced strip method. Notable among these is Nilson (4), which devotes several chapters to slab design and one entire chapter to the strip method.

EDGE SUPPORTED ELEMENTS: THE SIMPLE STRIP METHOD

Not counting membrane behaviour, two-way slabs carry load by means of three internal mechanisms. Relative to a rectangular coordinate system, these are bending in the x and y directions and torsion. It follows that the total load, q, at any point on the slab is made up of three components; that carried by flexure in the x and y directions (q_x and q_y respectively) and that carried by torsion (q_{xy}).

Hillerborg recognized that, because of the tremendous ductility of two-way slabs, virtually any combination of q_x, q_y, and q_{xy} may be used as the basis for the design of a slab as long as; (i) the slab is designed at all points to resist the moments resulting from the assumed load distribution, (ii) the shears and moments resulting from the assumed load distribution do not violate any boundary conditions (for example, the shear and moment at a free edge must be zero) and (iii) the assumed load distribution satisfies equilibrium. That is:

$$q_x + q_y + q_{xy} = q \qquad [1]$$

The simple strip method refers to one class of load distributions that is convenient for orthogonally reinforced, edge supported slabs. Choosing the x and y axes to be parallel to the reinforcement, the torsional moment relative to these axes is set to zero throughout the slab. As a result, the magnitude of q_{xy} is also zero and the total load, q, is divided between the two bending components. Equation [1] is replaced with

$$q_x + q_y = q \qquad [2]$$

The designer divides the total load at every point on the slab into two parts; that which spans in the x direction and that which spans in the y direction. Any strip of slab in either the x or y direction is treated as a one-way beam and designed to carry the load assigned to it.

To illustrate the simple strip method, consider a simply supported square slab of side length a = 5m subjected to a uniform load of q = 12 kN/m², illustrated in Figures 1 through 4. One possible load distribution, shown in Figure 1, has, at every point on the slab, one-half of the load spanning in the x direction and one-half spanning in the y direction (i.e. $q_x = q_y = q/2 = 6$ kN/m²). Following this load distribution, the slab would be designed across its full width, in both directions, for a mid span moment of $qa^2/16 = 18.75$kN·m/m.

There are two problems with this design. First, an average design moment of $qa^2/16$ is too large for a square slab and requires too much reinforcement. Second, reinforcing for a uniform design moment across the full width of the slab places too much steel at the edge of the slab parallel to the support where curvatures are small, and too little in the middle of the slab where curvatures are large.

An alternate load distribution is shown in Figure 2. In this case, the slab is partitioned along the diagonals with each support carrying a triangular segment of slab. At any point on the slab, all the applied load is carried in either the x or y direction. The mid span moment for a thin strip of slab located a distance $s \le a/2$ from the x-axis is $qs^2/2$. The design moment across the width of the slab varies parabolically, from zero at the edge to a maximum value of $qa^2/8$ in the middle.

The average value of the design moment across the width of the slab is $qa^2/24$ = 12.5 kN·m/m, which is exactly the same result as one would obtain using a simple yield-line analysis with yield-lines following the load dispersion lines in Figure 2. The coincidence of upper and lower bound indicates that the solution is, in fact, exact. The solution is, however, impractical since it requires continual variations in the spacing of the reinforcement. Note that with a uniform distribution of reinforcement, the governing yield-line pattern differs from the simple load dispersion lines shown in Figure 2 in the vicinity of the simply supported corners. Accounting for this corner effect increases the average required reinforcement by approximately 10 per cent.

Figure 3 illustrates a load distribution that leads to a banded reinforcement layout. In the central region and four corner regions, the load is distributed equally in the x and y directions. For regions along the edges of the slab all load is carried to the nearest support. This resulting design moment in both directions for the middle half of the slab is $5qa^2/64$ = 23.4 kN·m/m. The design moment for the outside quarters of the slab is $qa^2/64$ = 4.7 kN·m/m. The average design moment over the full width of the slab is $3qa^2/64$ = 14.1 kN·m/m, which is only 13% greater than the solution shown in Figure 2.

The solutions shown in Figures 2 and 3 do not account for minimum reinforcement requirements. One of the strengths of the strip method of design is that it allows more effective use of minimum reinforcement. To satisfy deflection requirements, the example slab would have a thickness of about 150 mm (5.91in.). Minimum reinforcement for the slab would provide a factored moment resistance of about 12.3 kN·m/m. While the average design moment for the load distribution shown in Figure 3 is 14.1 kN·m/m, significant portions of the slab would be governed not by the calculated design moment but by minimum reinforcement requirements. With these regions reinforced for 12.3 kN·m/m in lieu of 4.7 kN·m/m, the average moment resistance provided is 17.85 kN·m/m.

One way to make better use of minimum reinforcement is to increase the width of the edge strips, as shown in Figure 4. The maximum width of edge strip that can be supported by minimum reinforcement is conservatively estimated as:

$$\sqrt{\frac{2 \times 12.3\ \text{kN·m/m}}{6\ \text{kN/m}^2}} = 2.02\ \text{m} \approx 2.0\ \text{m}$$

In other words, minimum reinforcement will support all load within the four 2m square corner regions. This leaves a 1m wide central strip in each direction with a mid span design moment of 30.75 kN·m/m. With this scheme, the average moment resistance provided is 16.0 kN·m/m.

The preceding examples illustrate some important characteristics of the strip method. First, the economy of the method depends upon the choice of load distribution patterns. As a rule, carrying all load to its closest support will result in the lowest average design moments and, therefore, the most economical design. This means that load distributions are best determined using the areas of slab that are tributary to each support. As a guide, it is useful to visualize the yield-line pattern for the slab, with the yield-lines defining ideal load distribution lines. For economy, these optimal load distributions may have to be adjusted to account for minimum reinforcement. Second, unlike a yield-line analysis which provides only average reinforcement requirements, the strip method gives a clear indication of where reinforcement should be placed for greatest benefit. Bar cutoffs are easily determined since complete moment diagrams can be drawn for all strips.

Designing with the Simple Strip Method

The following examples provide a sampling of the sort of design problems using the simple strip method. Each example concerns a slab that is 150 mm (5.91in.) thick and supports a uniform factored load, including self-weight, of 12 kN/m² (251 p.s.f.) Minimum reinforcement of grade 400 (58 k.s.i.) provides a flexural capacity of approximately 12.3 kN·m/m (2.77 ft·kips/ft.).

Rectangular slab with mixed supports — The rectangular slab of Figure 5 has two fixed and two simple supports, and spans 4.6 m (15.1 ft.) by 8 m (26.2 ft.). A simplified yield-line pattern for this slab is shown in Figure 5(a).

Figure 5(b) shows a load distribution scheme obtained by "squaring off" the yield line pattern of Figure 5(a). The slab is divided into eight elements of three different types: two central elements that span exclusively in the short direction, two end elements that span in the long direction, and four corner elements that carry half their load in the long direction and half in the short direction.

To produce average design moments that are close to minimum, Hillerborg suggests that the width of the central elements should be equal to the long span less one-half of the short span and the width of the end elements should be equal to one-half of the short span. With mixed support conditions, as in this example, the ratio of dimension a to the short span should range between 0.35 and 0.39. This corresponds to ratios of support to span moment between 1.45 and 2.45. The same proportions can be used for the end regions, making dimensions c and d one-half of dimensions a and b respectively. Following these rules, the design moments for the end elements spanning in the long direction will always be one-quarter the design moments for the central elements spanning in the short direction. The design moments for the corner elements will be one-eighth the design moments for the central elements. In this example, a is chosen to be 1.7m making b equal to 2.9m. Dimensions c and d are 0.85m and 1.45m, respectively. The resulting moment diagrams for strips A-A, B-B, C-C, and D-D are shown.

As was the case for the square slab of Figure 1, consideration of minimum reinforcement requirements leads to a somewhat different load distribution scheme. Figure 6 illustrates a simple load distribution that makes more efficient use of minimum reinforcement. As before, there is a central region that spans entirely in the short direction. The remaining design moments are set to 12.3 kN·m/m, that provided by minimum reinforcement. All that left to do is to calculate the width of the various design strips.

In the end regions, a fraction of the total load, q_l, spans in the short direction while the remainder spans in the long direction. The load q_l that can be supported by minimum top and bottom reinforcement is:

$$\left(\frac{\sqrt{4 \times 12.3\ \mathrm{kN{\cdot}m/m}} + \sqrt{2 \times 12.3\ \mathrm{kN{\cdot}m/m}}}{4.6\ \mathrm{m}}\right)^2 = q_l = 6.78\ \mathrm{kN/m^2}$$

The minimum reinforcement in the long direction must support a load of 12 - 6.78 = 5.22 kN/m² in the end regions. The maximum width of these end zones is:

$$e = \sqrt{\frac{2 \times 12.3\ \mathrm{kN{\cdot}m/m}}{5.22\ \mathrm{kN/m^2}}} = 2.17\ \mathrm{m}\ \text{ and}$$

$$f = \sqrt{\frac{4 \times 12.3\ \mathrm{kN{\cdot}m/m}}{5.22\ \mathrm{kN/m^2}}} = 3.07\ \mathrm{m}$$

For the load distribution of Figure 6, minimum reinforcement provides sufficient resistance to support all of the slab except for a central band spanning in the short direction. While the design moments for this central band are the same as those calculated for the load distribution in Figure 5, the width of the central band is reduced from 5.7m to 2.86m.

<u>Slab with a Free Edge</u> — The slab in Figure 7 spans 5m by 3m. It is unsupported on one long side and is simply supported on the other three. One design strategy would be to span all the load in the long direction, resulting in a uniform design moment of 37.5 kN·m/m. While this approach has the virtue of

simplicity, it fails to make any use of reinforcement that will be provided in the short direction. A more economical design involves the use of a strong band along the free edge of the slab.

Strong bands are an effective way to handle unsupported edges or large openings in slabs. A strong band is a strip of slab that acts like a beam, providing an internal support for other parts of the slab. To do this, the strong band must be designed to carry the applied load plus the internal support load.

Figure 7 shows a load distribution using a 1m wide strong band along the free edge. Outside of the strong band, the slab will have minimum reinforcement in the long direction. The maximum load that can carried in the long direction by minimum reinforcement is:

$$\frac{12.3\ \text{kN}\cdot\text{m/m} \times 8}{(5\ \text{m})^2} = 3.94\ \text{kN/m}^2$$

Carrying 3.94 kN/m² in the long direction leaves 8.06 kN/m² to be carried in the short direction. From the loading diagram shown for section C-C, in Figure 7(b), the internal support load q_1 is calculated by summing moments about the simple support.

$$q_1 = \frac{8.06\ \text{kN/m}^2 \times 2\text{m} \times 1\text{m}}{2.5\text{m} \times 1\text{m}} = 6.45\ \text{kN/m}^2$$

The maximum moment in the short span is 5.81 kN·m/m, which is less than the resistance provided by minimum reinforcement.

The strong band itself spans in the long direction and must be designed to carry a load of 12 + 6.45 = 18.45 kN/m². The design moment for the simply supported strong band is 57.66 kN·m and the average design moment for the long span is 27.4 kN·m/m.

CORNER SUPPORTED ELEMENTS: THE ADVANCED STRIP METHOD

There are many instances in design where a portion of a two-way slab is tributary to a corner support. Examples of this occur at column supports or reentrant wall supports, illustrated in Figure 8. While it is possible to handle such design problems with a system of simple strips and strong bands, the resulting calculations and reinforcement patterns are often anything but simple.

To avoid these complications, Hillerborg developed the corner supported load distribution element. The design method incorporating this element was dubbed the advanced strip method, an unfortunate choice of words, for while the