ACI member **Richard H. Scott** is a Reader in Engineering at the University of Durham, UK. His research is mainly concerned with the behaviour of reinforced concrete structural elements with a particular interest in the measurement of reinforcement strain and bond stress distributions. He is a member of ACI Committees 435, *Deflection of Concrete Building Structures*, and 444*, Experimental Analysis for Concrete Structures*.

ACI Fellow **Andrew W. Beeby** is Professor of Structural Design at the University of Leeds, UK. He took up this post in 1991 after 27 years research experience with the Cement and Concrete Association, a research institute funded by the cement manufacturers. A particular interest has been the performance of reinforced concrete members under service loads. He has been concerned with drafting UK and European codes of practice for the design of concrete structures for many years.

INTRODUCTION

The load-deformation behaviour of a reinforced concrete member is shown schematically in Fig. 1 and can be divided into two basic phases. These are an uncracked phase and a cracked phase. The uncracked phase starts at zero load and terminates when the first crack forms (Point A in Fig. 1). During this phase, the member behaves elastically as a homogeneous section. The cracked phase starts at Point A and then extends throughout the further load history of the member with crack development during this phase resulting in a steady reduction in stiffness with increasing load. However, the concrete between the cracks continues to carry some tension (known as the *tension stiffening* effect) although this must be less, on average, than the tensile strength of the concrete. Crack development causes this tensile force to reduce but, since it cannot reduce to less than zero, the behaviour assessed assuming that the concrete carries no tension provides a lower limit on the possible stiffness.

The true response during the cracked phase starts at the Point A in Fig. 1 and then lies between the dashed line indicating the deformation assuming zero stress in the concrete and the dotted line indicating a stress in the concrete equal to the tensile strength. Since the average stress in the concrete can be expected to reduce as cracking increases, the true response can be expected to tend towards the line for zero tension in the concrete with increase in load or moment, at least until the number of cracks has stabilised, and experimental evidence confirms this. Tension stiffening is most significant in lightly reinforced members. This is particularly so with slabs which are members where deflection is most likely to be a critical design parameter.

Deflection may be calculated from the following formulae derived from elastic theory:-

$$\text{curvature, } (1/r), = M/EI \qquad [1(a)]$$
$$\text{or deflection, } \delta, = k_w WL^3/EI \text{ or } k_m ML^2/EI \qquad [1(b)]$$

where:-

δ = deflection
E = modulus of elasticity of material
I = second moment of area of section

k_w = a constant depending on the nature of the loading and support conditions
k_m = a constant depending on the shape of the bending moment diagram.
L = the span of the beam
M = the moment at the critical section
W = the total load on the beam
(1/r) = the curvature (in radians/mm)

The deflection may be calculated easily if it is possible to calculate an effective value (I_e) for the second moment of area (ie. a correct deformation will be calculated if $I = I_e$ in Equations 1(a) and 1(b)). Clearly, at Point A in Fig. 1, I_e should be calculated assuming an uncracked section (I_g) but, with increase in loading, I_e approaches a value calculated assuming that the concrete in tension carries no stress (I_{cr}). Thus, a function is needed for I_e having the following properties:-

(a) When M equals M_{cr} (the cracking moment) $I_e = I_g$

(b) As M increases, I_e approaches I_{cr} but does not reduce below this value.

Whilst there is no room for significant dispute about the prediction of deformations up to the cracking load, code provisions differ as to how best to model behaviour in the cracked phase. Reliable predictions of both short and long term behaviour in the cracked phase are obviously crucial if deflections are to be calculated with any degree of confidence. Consequently, when anomalies arose during the analysis of a flat slab test carried out at Taywood Ltd in the UK (1) this was a matter of some concern and led to the initiation of the work described in this paper. The nature of these anomalies will be indicated later but, firstly, it will be useful to summarise current code provisions for the calculation of deflections.

ACI 318

This method was first proposed in 1963 by Branson (2) and was adopted by the ACI code (3) soon afterwards.

Branson proposed the following relationship for I_e:-

$$I_e = (M_{cr}/M)^{\alpha} I_g + (1 - (M_{cr}/M)^{\alpha}) I_{cr} \qquad [2]$$

Branson suggested two values for the exponent α. Where Equation 1(a) was used $\alpha = 4$ while, if Equation 1(b) was used, $\alpha = 3$. The reason for this difference is that, where Equation 1(b) is used, the cracking state of the whole beam is being taken into account rather than the local state of cracking considered in Equation 1(a). Parts of the beam are remain uncracked and hence the decay in stiffness as load increases is less where the whole beam is considered than where a section under a particular moment is considered. ACI 318 only gives Equation 1(b) and specifies the value of α as 3.

The approach to using this formula is to calculate the short term deflection using Equations 1(b) and 2. The long term deflection is calculated simply by multiplying the short term value by a factor λ, defined as:-

$$\lambda = \xi/(1 + 50\rho'). \qquad [3]$$

ρ' is the compression reinforcement ratio (A'_s/bd) and ξ is a coefficient which depends on the time under load. The following values are given:-

3 months $\xi = 1.0$; 6 months $\xi = 1.2$; 12 months $\xi = 1.4$; 5 years or more $\xi = 2.0$

Eurocode 2

The concept behind this method (4) is very similar to the ACI method but is related to section behaviour rather than to beam behaviour. It is also more general in its formulation. Firstly, the section deformation assuming a homogeneous uncracked section (δ_1), which may be a curvature or, in the case of a member subjected to pure tension, an extension, or a combination of these, must be that calculated. With increasing load or moment, the deformation will approach that which would be calculated ignoring tension in the concrete (δ_2). A suitable function to define the deformation under the load conditions considered is thus:

$$\delta = \zeta\delta_2 + (1 - \zeta)\delta_1 \qquad [4]$$

The distribution coefficient, ζ, may conveniently defined by:

$$\zeta = 1 - \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2 \qquad [5]$$

where:-

β_1 = a coefficient taking account of the bond properties of the reinforcement. $\beta_1 = 1.0$ for ribbed bars and 0.5 for plain bars.

β_2 = a coefficient taking account of the duration of the loading or of repeated loading.
β_2 = 1 for a single, short term load;
β_2 = 0.5 for repeated or sustained loads.

δ = the deformation, which may be a curvature or a strain.

δ_1 = the deformation calculated for a homogeneous, uncracked elastic section

δ_2 = the deformation calculated ignoring concrete in tension

σ_{sr} = The stress in the reinforcement at the loading causing first cracking calculated ignoring the concrete in tension.

σ_s = the stress in the reinforcement under the loading considered calculated ignoring the concrete in tension.

ζ = a distribution coefficient taking account of the degree of cracking.

Long term deformations are calculated by: (a) taking the long term value for β_2; (b) taking a long term modulus of elasticity for the concrete ($E_{c.eff}$) which takes account of creep ($E_{c.eff} = E_c/(1 + \phi)$ where E_c is the short term modulus of elasticity and ϕ is the creep coefficient which is based on the age at loading) and (c) calculating the deformation due to shrinkage. The code gives means to establish the creep coefficient and the free shrinkage as a function of time. There is, however, no indication of how β_2 might vary as a function of time beyond the definition of a short term and long term value.

<u>British Standard BS8110</u>

BS8110 (5) recognises that, in the cracked phase, the concrete between the cracks continues to support some tension. This results in an average tensile stress in the tension zone after cracking and the assumptions made for the behaviour of a section in the cracked phase are shown in Fig. 2, the concrete tensile stress at the level of the tension steel being denoted by f^*. Values for f^* are 1 MPa and 0.55 MPa for short and long term loadings respectively. Thus BS8110 considers f^* to be independent of the strain in the tension steel.

Long term deformations are calculated by using the lower value of f^*, an effective modulus of elasticity to allow for creep and a shrinkage curvature calculated using the same formula as in Eurocode 2 (4).

<u>Comparison of the Methods.</u>

To give a simple comparison, the moment-deflection response has been calculated for a 250 mm deep slab made from concrete having an elastic modulus of 30 GPa and a tensile strength of 3 MPa. Two areas of reinforcement have been considered: 1200 mm^2/m, corresponding to 0.5% reinforcement and 600 mm^2/m, corresponding to 0.25% reinforcement. For greater areas of reinforcement, all the methods give very similar calculated deformations at around the service load because the tension stiffening effects are relatively small. Figs 3 and 4 show the moment – curvature responses calculated using the three methods for the two reinforcement ratios. In the Figures, the approximate service load corresponds to a stress in the reinforcement based on a fully cracked section of 300 MPa. Clearly, in practice, the service conditions can vary considerably from this. In general, over most of a member, the service stresses are likely to be lower than those corresponding to the indicated service load. Stresses could be higher at sections where the design ultimate moments have been reduced significantly by redistribution. The comparisons are all concerned with the prediction of the short term deformation.

Looking first at Fig. 3, it will be seen that, at the level of the service load, the Eurocode 2, ACI 318 and BS8110 methods all give very similar predictions of the deformation. In evaluating the significance of the differences, it should be noted that the CEB/FIP 1978 Model Code (6) suggests that the accuracy of the calculation should not be considered to be better than ± about 15% for the higher reinforcement ratio and ± 25% for the lower reinforcement ratio. The range of calculated deformations in Fig. 3 probably therefore lie within the accuracy of the formulae.

Fig. 4 is more interesting. Here there are very large differences between the predictions by the three methods. The ACI 318 method gives the smallest deformation. The Eurocode 2 method is next, with a calculated deformation which is about double that calculated using the ACI method and the BS8110 method is the most conservative with a deformation which is about four times that predicted by the ACI method. These differences can have real practical significance since 0.25% of reinforcement is not untypical of many solid slabs where deflection control may well be the governing factor in the design. Looking at the BS8110 approach, it has always been a perceived weakness of the method that it effectively assumes a tensile strength for the concrete of slightly above 1 MPa compared with other methods which use the actual tensile strength, which is likely to be nearer 3 - 4 MPa for normal concretes. Because of this, BS8110 is inherently more conservative than the other methods when the service loading is close to the cracking load, as it is in the case considered in Fig. 4. Considering the other methods, it is clear which is the most and which the least conservative but it is difficult to say which is actually the best without a major attempt to compare the predictions with experimental data.

There are some practical aspects of the various methods that should be noted. As far as hand calculation is concerned, the ACI and Eurocode 2 methods are relatively easy to apply. The BS8110 method is not. Equations can be set up describing the section behaviour but these can only be solved by iteration so hand calculation is time consuming. In contrast, however, the BS8110 approach may be easier to incorporate into non-linear computer methods.

A disadvantage of the ACI and Eurocode 2 methods, which is probably of only limited practical concern, is that the calculation may be sensitive to the load history in a way which the BS8110 approach is not. The calculated deformation in the Eurocode 2 and ACI methods depends upon an estimate of the cracking moment. This is no problem for a reinforced concrete beam but, if an axial load is present, a problem arises. The cracking moment will be very different depending upon whether the axial load is applied before the moment, after the moment or whether the ratio of the axial load to the moment is kept constant during loading. There is little or no experimental information to show whether the deformation is actually dependent on the load history in this way. The BS8110 method, which does not depend on the cracking load, avoids this problem.

Overall, while the ACI, Eurocode 2 and BS8110 methods can be ranked in order of the degree to which they are likely to affect the economy of design, it is very hard to judge which is technically the superior approach. However, and as mentioned earlier, the reliability of the code procedures for predicting long term deflections was questioned during the analysis of a flat slab test carried out at Taywood Ltd in the UK (1). As part of Brite EuRam Project 5480 'Economic Design and Construction with High Strength Concrete' a flat slab was constructed in high strength concrete at the Taywood Laboratories, Ruislip, UK, in 1995. The slab was 250 mm thick and was constructed on four columns at 9 m centres with 3 m cantilevers resulting in a slab 15 m square. The concrete compressive cube strength achieved by 7 days was 120 MPa. The slab was loaded by tendons tensioned to the floor to simulate a uniformly distributed load. The load-deflection results were as predicted up to and including a fully cracked state. However, the stiffness of the slab after this time did not remain at the expected value. The measured deflections were more than expected using the short term tension stiffening value of 1 MPa given in BS8110 and were, in fact, much closer to the values calculated using the long term value of 0.55 MPa.

As a consequence of the above, a programme of experimental work was initiated to investigate time dependent aspects of tension stiffening with particular emphasis on long term effects.

RESEARCH SIGNIFICANCE

The major international design codes (ACI 318, Eurocode 2, BS 8110) adopt differing approaches in their treatment of the effects of tension stiffening but they all distinguish between short and long term effects. Traditionally, both loss of tension stiffening and creep have been considered as contributing to the long term effects but this paper shows that tension stiffening decays much more rapidly than has previously been assumed. As a consequence, recommendations are made for possible modifications to the design codes.

SPECIMEN DETAILS AND TESTING

The experimental work consisted of long term tests on 47 prisms subjected to pure tension and eight large slabs subjected to bending. Testing prisms has major advantages. The average stress in the reinforcement can be obtained from measurements of the average strains while the total tension force can be measured by load cells and should therefore be known with some exactitude. The concrete area sustaining the tension stiffening is clearly defined and its line of action, in pure axial tension, is clearly coincident with that of the reinforcement. Consequently, testing square prisms reinforced with a single bar should enable unambiguous values to be obtained for the forces supported by the concrete in tension. However, it is possible that the results may not be directly applicable to flexural members but it was felt that the rate of decay in the tensile force carried by the concrete assessed from the tension tests would be applicable to flexural situations. Nevertheless, a limited series of long term slab tests was also conducted.

The 47 prisms were all 1200 mm long with a 120x120 mm cross section. Each was reinforced with a single T12 (113 mm^2), T16 (201 mm^2) or T20 (314 mm^2) axially placed rebar. Fourteen bars were internally strain gauged to obtain very detailed data concerning reinforcement strain distributions without degradation of the surface bond characteristic. A total of 81 electric resistance strain gauges (gauge length 3 mm) were installed at 15 mm centres over the full length of these specimens in a central 4x4 mm machined longitudinal duct. Three load levels, calculated to produce average concrete stresses of 3, 4 and 5 MPa, and three concrete compressive cube strengths (30, 70 and 100 MPa) were used in the test programme.

The eight one-way spanning slabs were all 3 m long, 250 mm deep and 500 mm wide and these tests were considered to be exploratory rather than a thorough investigation of tension stiffening in flexure. To make the slabs and the tension tests as closely comparable as possible, the cover for most of the slabs was specified to be the same as that used in the tension tests with the side cover and spacing of the slab bars being chosen such that the area of concrete surrounding the slab bars was 120x120 mm. The applied loading was selected to make the force in the slab reinforcement, calculated ignoring the concrete in tension, the same as the axial load applied in the equivalent tension tests. The slabs were tested in pairs, back-to-back, and loaded in four point bending with a constant moment zone between roller supports of 900 mm.

Concrete surface strains were measured on all specimens using a Demec gauge and a grillage of studs glued on the surface of the specimen. Age at the start of each test varied but was never less than 28 days. Applied loads were maintained for periods of up to three months with strain gauge and Demec readings being recorded at frequent intervals throughout the test period. Further details of the test programme are reported elsewhere (7,8)

RESULTS AND DESIGN IMPLICATIONS

The results showed very clearly that concrete stress, and hence tension stiffening, in both the prism and slab specimens reduced much more rapidly than has normally been assumed. The initial loss of concrete stress could be very rapid indeed with significant reductions within the first few hours of loading. The rate of decay of concrete stress appeared to be independent of applied load (as illustrated in Fig. 5 for a typical of prism specimen) and, as far as test results allowed, largely independent of concrete strength. There was no recovery of tension stiffening on unloading. Overall, the results consistently showed that, after loading, tension stress in the concrete reached a roughly constant level in a period ranging from as little as six hours to a maximum of thirty days with the average value being around twenty days. Since, from a practical point of view, all specimens had short decay times the possibility of relating decay times to the variables in the test programme was not considered further.

The tests also demonstrated that it can be difficult to differentiate between short and long term effects since the values of the short term and long term deformations depend on just what cracks or internal failure events (such as local bond failure or the formation of internal cracks) have occurred during the loading and what cracks and internal failure events occurred during the period of constant loading. Not infrequently, a new crack occurred within an hour or so of the load being applied and this contributed a major part of the long term deformation under this load. It is reasonable to assume that, had a slightly higher load been applied, the crack would have occurred during loading and hence would have been classified as part of the instantaneous deformation not the long-term deformation. The way these events are distributed between short term and long term is a stochastic process depending on the variation of concrete strength along the length of the member and is not uniquely predictable. Thus, calculation of the short term deformation and attempting to establish the final deformation by adding a long term increment to this is a somewhat dubious procedure. Similarly, finding the long term deformation by multiplying the short term deformation by a factor is also undesirable. Since, however, the total amount of tension stiffening loss that occurs appears to be largely independent of the loading history, the final deformation appears to be calculable with much more reliability. In addition, methods which calculate long term deflections by applying factors to the short term deflection implicitly assume that tension stiffening, creep and shrinkage all occur at the same rate whilst methods which just give a short term and long term value make no assumptions about the rate.

In view of the above, the practical consequences of the early rapid loss of tension stiffening are considerable and can be summarised as follows:-

1. Loss of tension stiffening occurs over a period which would be considered short term in contrast to creep.
2. As the loss is not recoverable, repeated short-term loads (i.e. loads of less than a day) will lead to a reduction similar to that for a longer-term load.
3. Over-loading during the life of a structure, (e.g. during construction) will leave the member with only the long-term tension stiffening effects corresponding to the maximum load reached at any time during its life for all loads up to this maximum load.

From the above, it will be seen that it is long term tension stiffening parameters which should be used for virtually all situations, including the calculation of short term effects. A consequence of this is that short term deflection calculations using current short term stiffening properties probably underestimate actual deflections. The new proposal to use the lower values of tension stiffening will increase the short-term deflection under self weight as well as increasing the deflection due to transient loads. For normal ratios of self weight to transient loads the effect of using the new proposals will be to reduce the increase in the deflection occurring after finishes are applied. However, the total long term deflection remains unchanged.

The proposed modifications to BS8110, and Eurocode 2 and ACI 318 resulting from the above are as detailed below:-

<u>BS8110</u>

It will normally be reasonable to have one value of tension stiffening for all load cases where the moment exceeds the cracking moment. i.e. to use a value of 0.55 MPa for f^* for both short and long term loading.

<u>Eurocode 2</u>

It is simply necessary to take the long term value of 0.5 for β_2 when calculating both short term and long term deformations.

ACI 318

This is the oldest of the current methods and the most difficult to modify. The problem arises because, as described earlier, the procedure for calculating long term deflections is firstly to calculate the short term deflection and then multiply this by the factor λ. In addition, the ACI approach does not introduce tension stiffening explicitly, as do BS8110 and Eurocode 2, but combines the effects of creep, shrinkage and loss of tension stiffening into this single factor. In addition, the significance of tension stiffening in the estimation of the long term deflection is heavily dependent on the tension reinforcement ratio which, at present, does not appear in the equation for λ. A substantial redrafting would be required to develop the ACI 318 provisions so that λ could be redefined to reflect tension stiffening explicitly.

CONCLUSIONS

1. The BS8110, Eurocode 2 and ACI 318 procedures for deflection calculations were reviewed. They all adopted differing approaches in their treatment of the effects of tension stiffening but it was not possible to judge which was the technically superior.

2. A programme of experimental work was undertaken to investigate time dependent aspects of tension stiffening behaviour. Both tension specimens and slabs were tested.

3. The tension forces carried by the concrete after cracking reduced within a period of up to about 20 days to about half the initial values, a much shorter timescale than was appreciated hitherto. This was found to be true for both tension and flexure.

4. Specific revisions to BS8110 and Eurocode 2 have been proposed in the light of the above. Revisions to ACI 318 are desirable but difficult to formulate due to the structure of the current provisions.

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NOTATION

k_w	=	constant depending on the nature of the loading and support conditions
k_m	=	constant depending on the shape of the bending moment diagram.
$(1/r)$	=	curvature (in radians/mm)
E	=	modulus of elasticity
E_c	=	short term modulus of elasticity
$E_{c.eff}$	=	long term modulus of elasticity
I	=	second moment of area
I_{cr}	=	second moment of area of cracked section
I_e	=	effective second moment of area
I_g	=	second moment of area of uncracked section
L	=	span
M	=	moment at the critical section
M_{cr}	=	cracking moment
W	=	total load
α	=	exponent
β_1	=	coefficient taking account of the bond properties of the reinforcement
β_2	=	coefficient taking account of the duration/type of loading
δ	=	deformation (deflection, curvature or strain)
δ_1	=	deformation calculated for a homogeneous, uncracked elastic section
δ_2	=	deformation calculated ignoring concrete in tension
ζ	=	distribution coefficient taking account of the degree of cracking.
λ	=	factor for long term loading
ξ	=	coefficient depending on time under load
ρ'	=	compression reinforcement ratio
φ	=	creep coefficient
σ_{sr}	=	stress in the reinforcement at the loading causing first cracking
σ_s	=	stress in the reinforcement under the loading considered calculated

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