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RESEARCH SIGNIFICANCE

The results of these tests demonstrate the importance of shrinkage restraint and early age loading on deflection of reinforced concrete slabs. Data are provided that can be used to verify theoretical models.

INTRODUCTION

Deflection control is an important aspect of design of reinforced flexural members. Behavior during service life can be significantly affected by construction loading and curing conditions during the construction period before the concrete has attained its twenty eight day strength. To evaluate the effects of early age loading on deflection, nine one-way slab specimens were tested under short term application of live load and long term sustained dead load due to self weight. Mid-span deflection measurements were taken during live load application and removal as well as during the period of sustained load application.

Three specimens labeled B1D3, B2D3, and B3D3 were removed from the forms and loaded at 3 days, three labeled B4D7, B5D7, and B6D7 were removed from the forms and loaded at 7 days, and three labeled B7D28, B8D28, and B9D28 were removed from the forms and loaded at 28 days.

This paper describes the design and preparation of test specimens, material properties, test set-up and procedure, and results of the deflection measurements. The results demonstrate the effects of shrinkage restraint stresses on cracking and early age loading on long term deflection. Comparisons are made with calculated immediate deflections on application of live load and calculated time-dependent deflections based on code-specified time-dependent multipliers.

SPECIMEN DESIGN AND PREPARATION

All nine test specimens were fabricated with the same dimensions and flexural reinforcement. The slabs are 12 ft. (3.66 m) long, 12 in. (304.8 mm) wide, and 5 in. (127 mm) deep, reinforced with 2 - #3 Grade 60 bottom bars with an effective depth of 4 in. (101.6 mm) and simple supports located 6 in. (152.4 mm) from each end providing a simple span length of 11 ft. (3.35 m). The slabs were designed according to ACI 318-05[1] for moment capacity to resist an unfactored dead load due to self weight plus a concentrated live load of 600 lbs (2.67 kN) at midspan. The design was based on a specified concrete compressive strength of 4000 psi (27.6 MPa). Details of the concrete mix are provided in Table 1.

MATERIAL PROPERTIES

Concrete cylinders, 6 in. x12 in. (152.4 mm x 304.8 mm), were cast from the concrete batch used for the specimens following ASTM C 31. Six cylinders were made for each of the slab sets (3 day, 7 day, and 28 day loading). For each set, three cylinders were used for split cylinder tensile tests and the other three were used for compressive strength and elastic modulus using ASTM[2] test procedures (ASTM C 496, ASTM C 39, and ASTM C 469). Results are summarized in Table 2.

Time-dependent development of compressive strength and instantaneous elastic modulus up to 28 days compared with models provided by ACI 209[3], CEB-FIP[4], and Gardner and Lockman GL 2000[5] is shown in Figs. 1 and 2. The model predictions show good agreement with test results tending to slightly underestimate values at 3 and 7 days.

TEST SETUP AND PROCEDURE

Specimens were removed from forms at 3, 7, and 28 days as described above and set on simple supports on the laboratory floor as shown in Fig. 3. A dial gage was installed below each specimen at midspan immediately after the specimen was set on the supports under self weight providing the datum for all subsequent readings. In the experiment the immediate deflection due to self-weight was not measured. The initiation of deflection is measured from the application of live load. Load deflection plots and visual examination of the beams indicated that all specimens were uncracked under dead load only. Immediate deflection under dead load was therefore calculated based on an uncracked section and added to subsequent measured deflections to obtain total deflection.

Six steel blocks, each weighing an average of 105.1 lbs (0.478 kN), were placed at midspan and dial gage readings were recorded after each block was placed. The blocks were then removed one by one and dial gage readings were recorded on removal of each block. Deflection readings were taken periodically over a period of 182 days while each specimen supported its self weight.

A second application of the concentrated load at midspan was performed at age 156 days and the same procedure as before was used to record applied load and deflection. Fig 4 shows a specimen under the full applied concentrated load. The loading history is shown in Fig. 5.

IMMEDIATE DEFLECTION DUE TO LIVE LOAD

The load deflection response due to initial application and removal of the live load is shown for all specimens in Figs. 6 to 8. The average response for each set of three specimens loaded at 3, 7, and 28 days is shown in Fig. 9. For each loading age the three specimens in each set showed very similar, approximately linear, response on initial application of live load when the specimens were uncracked..

For loading at 3 and 7 days the specimens show a softening of response at approximately 400 lbs (1.779 kN) with a rapid increase in deflection under additional load indicating the onset of cracking in the midspan region. Significant differences in maximum deflection under peak load are evident in the plots indicating higher variability in deflection after cracking occurs. For loading age 28 days the softening of response begins at a lower load in the range of 200 – 300 lbs (0.890 – 1.334 kN). This trend is shown clearly in Fig. 9. The maximum deflection for the 28 day loading is higher than the maximum deflection for 3 and 7 day loading.

The difference in response between the 28-day case and the early age loading cases can be attributed to the presence of shrinkage restraint tensile stresses as a result of drying in the period 7 to 28 days for the 28 day case while the early age loading specimens were loaded immediately after the curing period. The effect of shrinkage restraint on cracking is well known[6-9] but these test data indicate that the effect can be even greater than immediate loading at early age immediately after the curing period when concrete strength is still under-developed. It should be noted that the primary source of restraint in these laboratory tests is embedded reinforcement. In actual structures additional restraint can be provided by stiff supports and adjacent slab portions placed at different times.

Fig. 10 shows the load deflection response on the second application of live load for specimen B1D3. Time dependent deflections between first and second application of live load are not included to allow comparison between loading and unloading on first and second application of live load. Second application of live load closely follows the unloading curve from first application of live load with a slight increase in both peak deflection and residual deflection on unloading. Similar trends were observed for all specimens.

LONG TERM DEFLECTION UNDER SUSTAINED LOAD

Figures 11 to 13 show the deflection histories for all specimens indicating increasing deflection with time under sustained load. A comparison of average deflection vs time for the three sets of specimens is shown in Fig. 14 which clearly shows the effect of age at loading on long-term deflection. While the slabs loaded at 28 days showed higher peak and residual deflection, the slabs loaded at 3 and 7 days show significantly larger long term deflections. However, the incremental deflection occurring after 28 days is approximately the same for all specimens. The primary impact of early age loading therefore appears to be on total deflection which could have a significant effect on perception of deflection while the incremental deflection occurring after installation of nonstructural elements appears to be much less affected by early age loading.

Variation of temperature and relative humidity with time is also shown in Fig 14. Local variations in deflection along the time axis can be attributed primarily to variations in temperature and relative humidity during the testing period. Since the specimens are located in the laboratory, variations in temperature are expected to have only a minor effect on deflections since the temperature distribution through the thickness can be expected to be approximately uniform and the roller support allowed for movement along the axis of the member. Variations in relative humidity would affect the rate at which drying occurred which would in turn affect the rate of development of shrinkage warping deflection.

ANALYSIS RESULTS

Deflection calculations were made for immediate deflection under dead plus live load based on two effective moment of inertia expressions and long-term deflections based on code specified multipliers. The Branson effective moment of inertia expression as presented in the ACI Code (ACI 318[1]) and an alternative expression proposed by Bischoff[10] were considered. Long-term deflection was calculated using the ACI 318[1] multiplier.

The immediate live load deflection at midspan is calculated as,

$$\Delta_L = \Delta_{L+D} - \Delta_D \tag{1}$$

where, Δ_L is deflection due to live load, Δ_{L+D} is the deflection due to live load plus dead load, and Δ_D is deflection due to dead load.

Dead load deflection is calculated by Eq. 2

$$\Delta_D = \frac{5 w_D \ell^4}{384 E_c I_g} \tag{2}$$

in which, E_c is elastic modulus of concrete, I_g is the gross moment of inertia when the section is not cracked under self-weight, w_D is distributed dead load where the dead load is only self-weight, ℓ is the member length.

Deflection due to dead load plus concentrated live load can be calculated by Eq. 3

$$\Delta_{L+D} = \frac{P_L \ell^3}{48 E_c I_e} + \frac{5 w_D \ell^4}{384 E_c I_e} \tag{3}$$

where, I_e is the effective moment of inertia or the gross moment of inertia if the section is not cracked, I_e is the effective moment of inertia. P_L is concentrated live load at mid-span.

In this study, the calculated deflections were obtained considering varying effective moment of inertia along the member using virtual work.

According to ACI 318[1] the effective moment of inertia is calculated by Eq.4

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \le I_g \tag{4}$$

where, M_{cr} is the cracking moment , M_a is the applied maximum service load moment, and I_{cr} is the cracked transformed moment of inertia. On the other hand, Bischoff[10] proposed the effective moment of inertia given by Eq.5

$$I_e = \frac{I_{cr}}{1 - \left(1 - I_{cr}/I_g\right)\left(M_{cr}/M_a\right)^2} \le I_g \tag{5}$$

For the long-term deflection due to creep and shrinkage under sustained load, ACI 318[1] provides multipliers for long-term deflections. The long-term deflection is determined by multiplying the immediate deflection caused by the sustained load by the factor, λ calculated by Eq.6

$$\lambda = \frac{\xi}{1 + 50\rho'} \tag{6}$$

in which, ρ' is the compression reinforcement ratio, ξ is the time-dependent factor which varies from 0.0 to 2.0 according to duration.

Time-dependent deflection, Δ_T was calculated by Eq. 7

$$\Delta_T = \lambda(t, t_o) \cdot \left(\Delta_D + \Delta_{irr}\right) \tag{7}$$

where, t_o is the age of concrete at loading of dead load. Δ_{irr} is the irrecoverable deflection after removing live load.

Immediate Deflection due to Live Load

The comparisons between analytical results and average experimental results are shown in Fig 15 to 17. In all cases Branson's equation tends to underestimate the average deflection, while Bischoff's equation tends to overestimate the average deflection. However Bischoff's equation gave a better prediction of the deflection remaining after removal of the live load. This is the calculated deflection that would be used for calculation of long time deflection using the multiplier. Calculated deflections were based on the cracking moments obtained from the load deflection plots.

Long-Term Deflection

Long-term deflections were calculated using multipliers given in the ACI 318[1] Building Code. The long-term multipliers obtained from ACI 318-05 for duration of 30, 60, 90, 120,150, and 180 days, are approximately 0.7, 0.9, 1.0, 1.1, 1.15, and 1.2 respectively. Obtained values are multiplied by initial deflections due to self-weight plus irrecoverable deflections due to removing of live load. For instance, at 90 days of duration long-term deflection of loading at 3 days, 0.076" (1.93 mm) is obtained from deflection due to self-weight 0.048" (1.22 mm) plus irrecoverable deflection 0.028" (0.71 mm) multiplied by long-term multiplier 1.0. The prediction of long-term deflection based on ACI 318 is shown in Table 3. The result of analysis shows that the long-term multipliers may not be applicable to calculation of long-term deflections for early-age loading. Figure 18 shows the comparison of long-term deflections between results of experiment and analysis. According to experimental results, the long-term deflections of loading at 3 and 7 days are much higher than prediction using the method specified in ACI 318[1].

CONCLUSIONS

Details of an experimental program to evaluate effects of loading age on immediate and time-dependent deflections of one way slabs have been presented. The results indicate that tensile stresses due to shrinkage restraint during the drying period reduce the load at which flexural cracking occurs thus tends towards increased deflection. Restraint stress showed therefore should be considered in the calculation of immediate deflection as suggested by Scanlon and Murray and among others.

While the Branson effective moment of inertia expression tended to underestimate immediate deflection, the Bischoff expression tended to overestimate peak live load deflection but resulted in a better prediction of residual deflection on removal of live load.

The results also show the significant effect of age at loading on long term deflection under sustained load. The effect is most significant in terms of total deflection. However incremental deflection after installation of non structural elements appears to be much less affected. The ACI Code long-time multiplier while being a relatively crude measure of total long time deflection provides reasonable results for loading at 28 days but significantly underestimates total long time deflection for early age loading.

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Table 1- Concrete mix used

Component	Amount /yd^3 (Amount/m^3)
Coarse Aggregate	1872 lb (1109.16 kg)
Fine Aggregate	1224 lb (725.22 kg)
Type I cement	376 lb (222.78 kg)
Slag	212 lb (125.61 kg)
Water	19.6 gal (96.88 *l*)
Air entraining agent	10 oz. (386.20 *ml*)
Superplasticizer	17.6 oz. (679.71 *ml*)

Table 2 - Concrete material properties

Details / Test Set	Compressive Strength, psi	Direct Tensile Strength, psi	Elastic Modulus, psi
Day 3	2884	306	2994100
	2729	302	3368400
	2870	354	-
Day 7	3562	416	4042000
	3690	400	3922000
	3905	269	-
Day 28	4512	448	4115500
	4796	453	4176300
	4969	362	-

Table 3 - Prediction of long-term deflection based on long-term multiplier

Day	λ	Day 3			Day 7			Day 28		
		Δ_D (in)	Δ_{irr} (in)	Δ_T (in)	Δ_D (in)	Δ_{irr} (in)	Δ_T (in)	Δ_D (in)	Δ_{irr} (in)	Δ_T (in)
0	0.00	0.048	0.028	0.000	0.039	0.022	0.000	0.037	0.051	0.000
30	0.70			0.053			0.043			0.062
60	0.90			0.069			0.055			0.080
90	1.00			0.076			0.061			0.088
120	1.10			0.084			0.067			0.097
150	1.15			0.088			0.070			0.102
180	1.20			0.091			0.073			0.106

t : duration

Δ_D :deflection due to self-weight, theoretically obtained

Δ_{irr} :irrecoverable deflection after removing live load

$\Delta_T = \lambda \cdot (\Delta_D + \Delta_{irr})$, time-dependent deflection

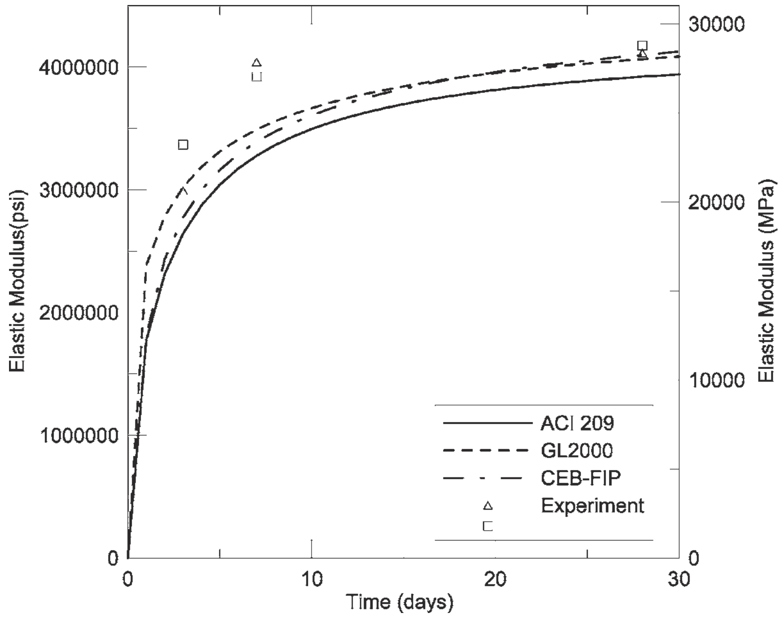

Fig. 1—Comparison of time-dependent instantaneous elastic modulus between experiment and analysis.

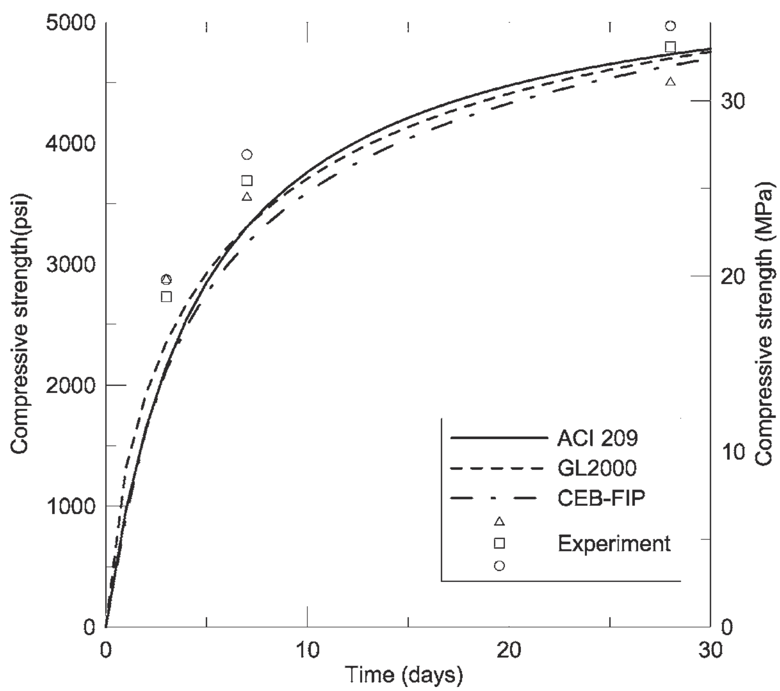

Fig. 2—Comparison of time-dependent compressive strength between experiment and analysis.

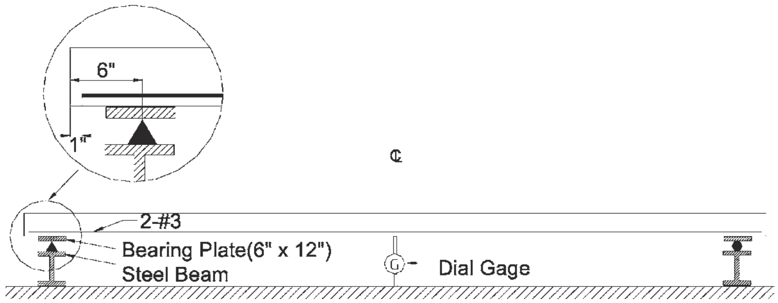

Fig. 3—Test setup.

Fig. 4—Setup for live load.

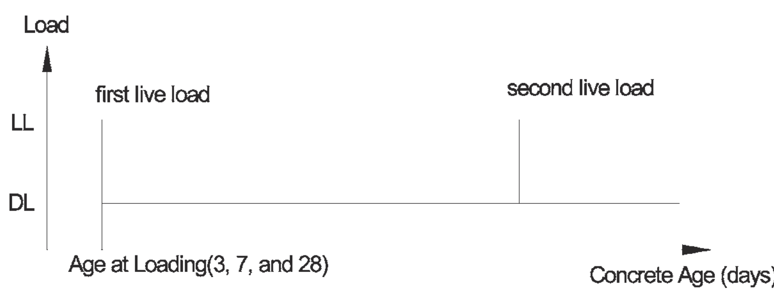

Fig. 5—Loading history.

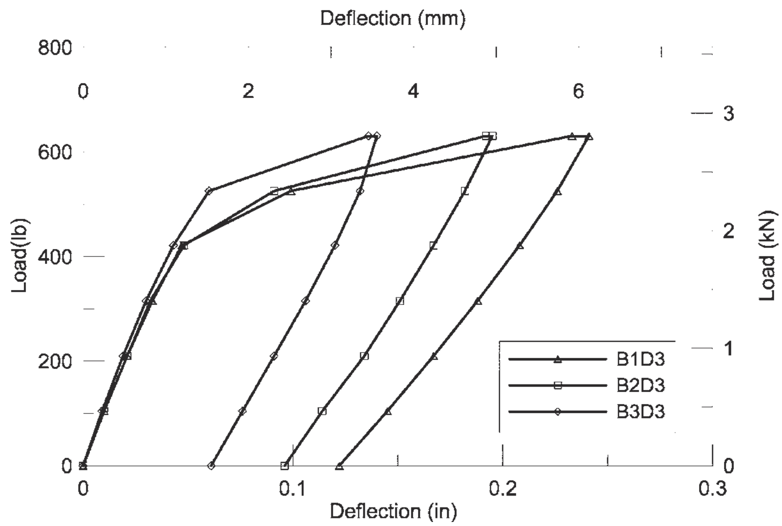

Fig. 6—Load-deflection response for loading at 3 days.

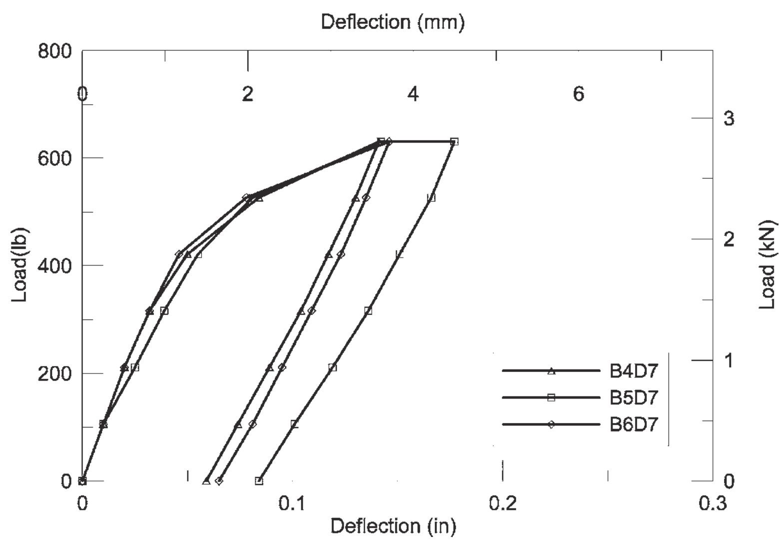

Fig. 7—Load-deflection response for loading at 7 days.

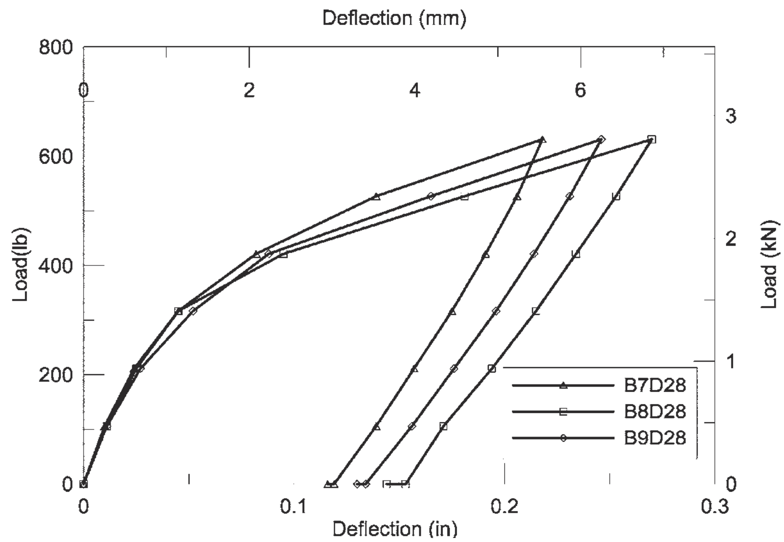

Fig. 8—Load-deflection response for loading at 28 days.

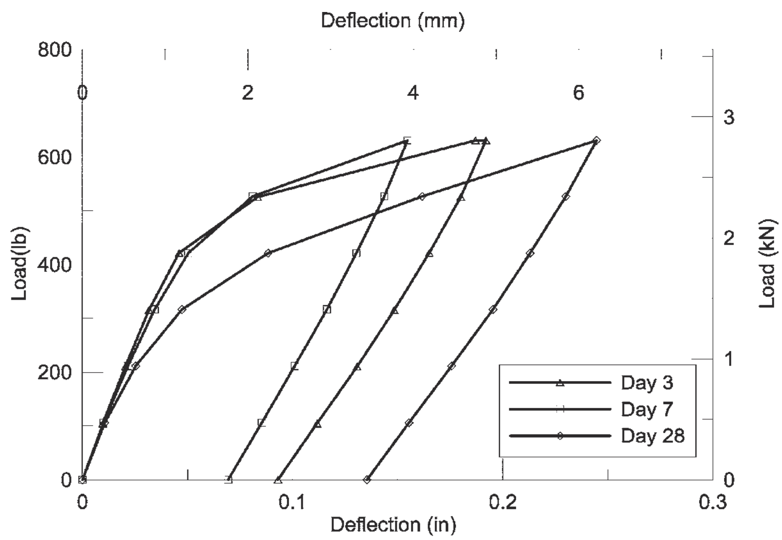

Fig. 9—Comparison of average load-deflection response of 3, 7, and 28 days.