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# Thermal Stresses in Square Shaped Concrete Pavements

by J. Silfwerbrand

**Synopsis:**
Thermal stresses in concrete pavements might be calculated according to a procedure developed by professor J. Eisenmann. The thermal stresses are dependent on the subgrade stiffness. Soft subgrades result in lower stresses. The Eisenmann procedure has been developed to cover square shaped slabs. This procedure is presented in this paper. Two calculation examples are also presented and discussed.

Keywords: Concrete pavements; square slabs; thermal stresses



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## 1. INTRODUCTION

Concrete pavements are designed for traffic and thermal stresses. In Sweden, the thermal stresses have been calculated according to professor J. Eisenmann[1]. He bases his calculations on an effective span length. In the calculation of the effective span length, he considers the effect of the subgrade reaction using a beam element. Soft subgrades shorten the effective span length and, hence, reduce the thermal stresses. The procedure has been used to calculate thermal stresses under Swedish conditions in highway pavements, industrial pavements, and pavements in tunnels[2-4].

The temperature distribution in a concrete pavement can approximately be divided into two parts: (a) a uniform part and (b) a temperature gradient (Fig. 1). The uniform temperature distribution does not cause any major temperature stresses in jointed concrete pavement because the pavement is practically free to move horizontally at the joints. The temperature gradient causes curling or warping and - if the curling is restrained - thermal stresses. Hence, the immediate cause of the stresses is not the temperature, but the dead load that counteracts the curling tendency of the pavement.

During a warm summer day, the top surface of the pavement is heated more than the bottom surface. A positive temperature gradient develops. The pavement slab curls with the central point moving upwards. The dead load of the pavement counteracts the curling and causes flexural tensile stresses in the bottom of the pavement (Fig. 2, left). During the night, the top surface gets cool more rapidly than the bottom surface. A negative temperature gradient develops and the pavement slab edges move upwards. In this case, the dead load causes flexural tensile stresses at the top of the slab (Fig. 2, right). Traffic loading causes flexural tensile stresses in the bottom of the slab, i.e., in the same part of the slab

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as the positive gradient. Consequently, the positive temperature gradient usually is the most interesting one. In the following two sections, positive temperature gradients are dealt with in beams and slabs, respectively.

## 2. RESEARCH SIGNIFICANCE

Estimations of thermal stresses are necessary in the design of concrete pavements. Eisenmann's beam theory slightly modified to slabs is used in, e.g., Swedish and German pavement design procedures. The purpose of the research described in this paper is to develop a slab theory based on Eisenmann's assumptions. Some calculation examples presented show that the use of this slab theory leads to decreased calculated thermal stresses.

## 3. THERMAL STRESSES IN BEAMS

A thorough discussion of thermal stresses in beams is given in Eisenmann[1]. If the curling is completely restrained, but horizontal movements are possible (Fig. 3), the temperature stress $\sigma_{temp0}$ is given by the following expression:

$$\sigma_{temp0} = \frac{E\alpha\Delta\theta \cdot h}{2} \tag{1}$$

where,

E = modulus of elasticity of concrete
$\alpha$ = coefficient of thermal expansion
$\Delta\theta$ = positive temperature gradient in °C/m (top surface warmest)

A tensile stress $+\sigma_{temp0}$ develops on the bottom surface of the beam and a compressive stress
$-\sigma_{temp0}$ develops in the top surface of the beam (Fig. 3).

A beam subjected to a positive temperature gradient, resting on a subgrade, and free at the ends will curl. The central part tends to move upwards. The central point of the beam leaves the support only if the upward displacement due to temperature is greater than the downward displacement due to dead load $q=\gamma bh$. We obtain:

$$w_{temp} = \frac{\alpha\Delta\theta L^2}{8} > w_q = \frac{5(\gamma bh)L^4}{384Ebh^3/12} \tag{2}$$

where,

h = beam height

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b = beam width
L = beam length
$\gamma$ = gravity of concrete in N/m$^3$

By setting these displacements equal, Eisenmann could define a critical length $L_{cr}$:

$$L_{cr} = \sqrt{\frac{4E\alpha\Delta\theta}{5\gamma}} \cdot h \tag{3}$$

Assuming that $L \leq L_{cr}$, the maximum flexural tensile stress $\sigma_{temp}$ caused by the dead load is given by the following expressions:

$$\sigma_{temp} = \frac{(\gamma bh)L^2/8}{bh^2/6} = \frac{3\cdot\gamma L^2}{4\cdot h} = \frac{3\cdot\gamma L^2}{4\cdot h}\cdot\frac{1}{L_{cr}^2}\cdot\frac{4E\alpha\Delta\theta\cdot h^2}{5\gamma} =$$

$$1.2\cdot\frac{E\alpha\Delta\theta\cdot h}{2}\cdot\left(\frac{L}{L_{cr}}\right)^2 =$$

$$= 1.2\cdot\sigma_{temp0}\cdot\left(\frac{L}{L_{cr}}\right)^2 \tag{4}$$

The maximum stress arises at mid span on the bottom surface of the beam. We see that the maximum stress for $L=L_{cr}$ is 20 percent higher than the maximum stress $\sigma_{temp}$ for the completely restrained beam. We can limit the thermal stresses efficiently by reducing the beam length L.

If $L>L_{cr}$, $\sigma_{temp}=1.2\cdot\sigma_{temp0}$. It arises at a distance $L_{cr}/2$ from the beam end (Eisenmann[1]). In the following, let us assume that $L \leq L_{cr}$.

Eisenmann has proposed a procedure to determine the subgrade influence on the thermal stresses[1]. He substitutes the slab length L with an effective span length $L_{ef}<L$. He assumes that the beam ends sink down into the subgrade (Fig. 4). The displacement is w' at the end. The displacement is assumed to diminish linearly and is zero at a distance a' from the end. The subgrade reaction is proportional to the displacement (Winkler foundation). The following equilibrium equation can be established:

$$(\gamma bh)L = 2\cdot\frac{kw'ba'}{2} \tag{5}$$

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where, k is the modulus of subgrade reaction in $N/m^3$. The angle $\phi$ of rotation of each end consists of two parts depending on temperature gradient and dead load, respectively:

$$\phi = \frac{\alpha\Delta\theta L}{2} - \frac{(\gamma bh)L^3}{24Ebh^3/12} \tag{6}$$

Assuming small displacements, $\phi$=w'/a' and, hence, a' can be determined by combining Eqs. (5) and (6). We obtain:

$$a' = \sqrt{\frac{2\cdot\gamma h}{k\cdot\{\alpha\Delta\theta-\gamma L^2/(Eh^2)\}}} \tag{7}$$

Eisenmann gives simple expressions for certain values of the parameters. The modulus k of subgrade reaction must be known. For a two layer pavement system, Eisenmann gives the following equation for determining k:

$$k = \frac{E_u}{0.83\cdot h\cdot\sqrt[3]{E/E_u}} \tag{8}$$

where, $E_u$ is the modulus of elasticity of the subgrade. More complicated formulae are available for multi-layer systems (see, e.g., Eisenmann[1]).

The effective span length is given by:

$$L_{ef} = L - 2\cdot a'/3 \tag{9}$$

Finally, the maximum flexural tensile stress $\sigma_{temp}'$ can be calculated by using Eq. (4):

$$\sigma_{temp}' = \frac{3\cdot\gamma L_{ef}^{\ 2}}{4\cdot h} = 1.2\cdot\sigma_{temp0}\cdot\left(\frac{L_{ef}}{L_{cr}}\right)^2 \tag{10}$$

In his book, Eisenmann did not discuss the influence of varying temperature gradients and different k values on the effective span length. Considering Eq. (7), we see that for small k values (soft subgrades) and if L approaches $\sqrt{;E\alpha\Delta\theta/\gamma}\cdot h$, a' approaches infinity. Due to geometrical reasons, a' cannot exceed L/2. Consequently, $L_{ef}\leq 2L/3$. However, also a span length reduction from L to 2L/3 reduces the maximum stresses with more than 50 percent.
Eisenmann's estimation of the subgrade influence on the thermal stresses is an engineering approach, but of course, a rather rough approximation. The

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assumption that the beam edges, that have sunk down into the subgrade, are straight instead of curved will, of course, give an erroneous estimation of the support length a'. This error must, however, be compared with the error in estimating the modulus k of subgrade reaction. In many cases, the latter error will be far larger than the first one.

## 4. THERMAL STRESSES IN SLABS

Eisenmann uses roughly the same method to calculate thermal stresses in slabs. For many cases, the slab stresses can be computed simply from the beam stresses by dividing the beam stresses by ($1-\nu$), where $\nu$ is Poisson's ratio. For $\nu=0$, e.g., Eisenmann's slab stress is 25 percent higher than his beam stress. In the following, we consider that the slab will obtain a thermal curvature in two perpendicular directions.

Consider a square shaped slab subjected to a positive, linear temperature gradient $\Delta\theta$. This means that that the slab must be able to move horizontally without any restraint. It might be difficult to fulfil this demand completely in a practical case, anyhow, it is interesting to compute the thermal stresses due to a linear gradient separately. The slab thickness h is assumed to be small in comparison to the slab length a. Hence, elementary plate theory can be used. If all slab edges are built in, maximum flexural tensile stresses of the magnitude $\sigma_{temp0}$ will arise at the bottom surface. $\sigma_{temp0}$ has the value of

$$\sigma_{temp0} = \frac{E\alpha\Delta\theta\cdot h}{2(1-\nu)} \tag{11}$$

If the slab edges, on the other hand, are free to move, the slab will curl. If the dead load is neglected, the interior part of the slab will leave the subgrade and the slab will be supported at the corners only. In the real case, the dead load $q=\gamma h$ counteracts this upward deflection. If we proceed in the same manner as Eisenmann does for beams, a critical slab length $a_{cr}$ could be found by setting the upward movement due to temperature equal to the downward displacement due to dead load.

The temperature gradient causes a spherical deflection surface with radius R (see, e.g., Timoshenko and Woinowsky-Krieger[5]). The radius is given by the following equation:

$$\frac{1}{R} = \alpha\Delta\theta \tag{12}$$

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and, hence, the displacement difference $w_{temp}$ between slab center and corners is:

$$w_{temp} = \frac{a^2}{4R} = \frac{\alpha\Delta\theta \cdot a^2}{4} \tag{13}$$

The downward displacement due to dead load can be determined by solving the plate bending equation numerically. The plate bending equation is given by, e.g., Timoshenko:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{14}$$

where,

w = deflection
q = uniformly distributed load in $N/m^2$
$D = Eh^3/(12\cdot(1-\nu^2))$
x, y = rectangular coordinates

Corner-supported square shaped slabs subjected to uniformly distributed load q have been dealt with by, e.g., Silfwerbrand[6] using the finite difference method. For Poisson's ratio $\nu$=0.2 the central deflection $w_q$ has been estimated as:

$$w_q = 0.0261 \cdot \frac{qa^4}{D} = 0.0261 \cdot \frac{(\gamma h)a^4}{Eh^3/(12\cdot(1-\nu^2))} \tag{15}$$

Setting $w_{temp}=w_q$, we obtain the following expression for the critical slab length $a_{cr}$:

$$a_{cr} = \sqrt{0.798 \cdot \frac{E\alpha\Delta\theta}{\gamma(1-\nu^2)}} \cdot h = 0.893 \cdot \sqrt{\frac{E\alpha\Delta\theta}{\gamma(1-\nu^2)}} \cdot h \tag{16}$$

For $\nu$=0.2, the bending moments $m_1$ and $m_2$ at slab center and at mid span of an edge, respectively, are given by the following expressions:

$$m_1 = 0.110\cdot(\gamma h)a^2 \tag{17}$$

$$m_2 = 0.153\cdot(\gamma h)a^2 \tag{18}$$

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Corresponding flexural tensile stresses at the bottom of the slab are given by the following expressions:

$$\sigma_{temp1} = \frac{m_1}{h^2/6} = \frac{0.1101\cdot(\gamma h)a^2}{h^2/6} = \frac{0.6606\cdot\gamma a^2 h}{}\cdot\frac{1}{a_{cr}{}^2}\cdot 0.798\cdot\frac{E\alpha\Delta\theta\cdot h^2}{\gamma(-v^2)}$$

$$= 1.054\cdot\frac{E\alpha\Delta\theta\cdot h}{2(1+v)(1-v)}\cdot\left(\frac{a}{a_{cr}}\right)^2 = \frac{1.054}{1+v}\cdot\sigma_{temp0}\cdot\left(\frac{a}{a_{cr}}\right)^2 =$$

$$0.878\cdot\sigma_{temp0}\cdot\left(\frac{a}{a_{cr}}\right)^2 \qquad (19)$$

$$\sigma_{temp2} = 1.221\cdot\sigma_{temp0}\cdot\left(\frac{a}{a_{cr}}\right)^2 \qquad (20)$$

Consequently, maximum flexural tensile stress appears at mid span of the edges. The edge stress ($\sigma_{temp2}$) is about 40 percent higher than the central stress ($\sigma_{temp1}$).

If the slab is resting on a soft subgrade, the areas close to the corners will sink down into the subgrade (Fig. 5). Assume that each area has a plane, triangular shape forming the base of a pyramid. The smaller sides of the pyramid have the length a'. The height of the pyramid is w'. The gravity center of the pyramid is situated at a distance a'/(2$\sqrt{;2}$) along the diagonal from the corner. This model suffers from the same weakness as Eisenmann's beam mode, i.e., the slab parts, that have sunk down into the subgrade, are assumed to be plane. The slab model contains, however, an improvement: the slab supports are assumed to be located at the gravity center of the subgrade reactions. In Eisenmann's beam model, the resultant to the subgrade reaction is assumed to act at the beam ends when he computes the angle $\phi$ of rotation (Eq. (6)).

Assuming that the intensity of the subgrade reaction is proportional to the displacement, we can establish the following equation of equilibrium:

$$(\gamma h)a^2 = 4\cdot\frac{kw'a^2/2}{3} \qquad (21)$$

The angle of rotation along the diagonal at the corner is denoted $\phi$. Assuming small displacements, we obtain

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$$\phi = \frac{w'}{a'/\sqrt{2}} \tag{22}$$

The temperature gradient and the dead load contribute both to the angle:

$$\phi = \phi_{temp} - \phi_q \tag{23}$$

The temperature part $\phi_{temp}$ can be determined easily using the knowledge of the spherical deflection:

$$\phi_{temp} = \frac{a}{\sqrt{2} \cdot R} = \frac{\alpha \Delta \theta \cdot a}{\sqrt{2}} \tag{24}$$

If $\partial w/\partial x$ and $\partial w/\partial y$ are known, the angle of rotation along the diagonal $\phi_q = \partial w/\partial n$ is given by (see, e.g., Timoshenko[5]):

$$\phi_q = \frac{\partial w}{\partial n} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial n} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial n} = \frac{\partial w}{\partial x} \cdot \frac{1}{\sqrt{2}} + \frac{\partial w}{\partial y} \cdot \frac{1}{\sqrt{2}} \tag{25}$$

For the square, $\partial w/\partial x = \partial w/\partial y$. A value of $\partial w/\partial x$ can be estimated by solving the plate bending problem above numerically. We obtain:

$$\phi_q = \beta \cdot \frac{qa^3}{D} = \beta \cdot \frac{(\gamma h)a^3}{Eh^3/(12(1-\nu^2))} \tag{26}$$

where, β is a factor depending on the location of the corner support (Fig. 6). Combining Eqs. (21), (22), (23), (24), and (26), we obtain the following expression for a':

$$a' = \sqrt[3]{\frac{3\gamma ha/k}{\alpha\Delta\theta - 12\sqrt{2}(1-\nu^2)\beta\gamma a^2/(Eh^2)}} \tag{27}$$

where, k can be estimated using Eq. (8). If the supports move from the corners along the diagonal, the bending moment $m_1$ and $m_2$ will be reduced. We may write:

$$m_1 = \mu_1 \cdot 0.110 \cdot \gamma h a^2 \tag{28}$$

$$m_2 = \mu_2 \cdot 0.153 \cdot \gamma h a^2 \tag{29}$$

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where, $\mu_1$ and $\mu_2$ are reduction factors (Figs. 7 and 8). Hence, the flexural tensile stresses can be calculated with the following expressions:

$$\sigma_{temp1}' = 0.878 \cdot \mu_1 \cdot \sigma_{temp0} \cdot \left( \frac{a}{a_{cr}} \right)^2 \tag{30}$$

$$\sigma_{temp2}' = 1.221 \cdot \mu_2 \cdot \sigma_{temp0} \cdot \left( \frac{a}{a_{cr}} \right)^2 \tag{31}$$

$\sigma_{temp0}$ is given by Eq. (11), $\nu$=0.2.

The length a' of the support cannot be solved explicitly. A suitable calculation process is as follows:

1) assume an a' value, e.g., $a'_{assumed}=a/10$,
2) read $\beta$ for a support position at $0.25 \cdot a'_{assumed}$ from Fig. 6,
3) calculate a' from Eq. (27),
4) if $a' \neq a'_{assumed}$, repeat steps 2 and 3,
5) if $a'=a'_{assumed}$, read $\mu_1$ and $\mu_2$ from Figs. 7 and 8, respectively, and
6) calculate $\sigma_{temp1}'$ and $\sigma_{temp2}'$ from Eqs. (30) and (31).

## 5. CALCULATION EXAMPLES

Thermal stresses are computed and evaluated for four different examples (Table 1). Examples Nos. 1A and 1B are based on examples in Eisenmann[1]. Examples Nos. 2A and 2B are based on Swedish conditions.

The examples show that the resulting thermal stresses in the slab center are considerably lower than the stresses in the beams (20 %). The calculations also show that maximum tensile stress in the slab appears along the edges, at mid span. These stresses are somewhat larger than Eisenmann's beam stresses (10 %). The edge stresses are, however, considerably lower than the thermal stresses in a completely restrained slab subjected to the same thermal gradient (10 - 40 %).

## 6. CONCLUDING REMARKS

A method has been developed to calculate thermal stresses in square shaped slabs. The method forms an analogue to Eisenmann's procedure to calculate thermal stresses in beams. Calculations show that the thermal stresses in the slab center are considerably lower than thermal stresses in beams. On the other hand, the maximum slab stresses, appearing along the edges, at mid span, are

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