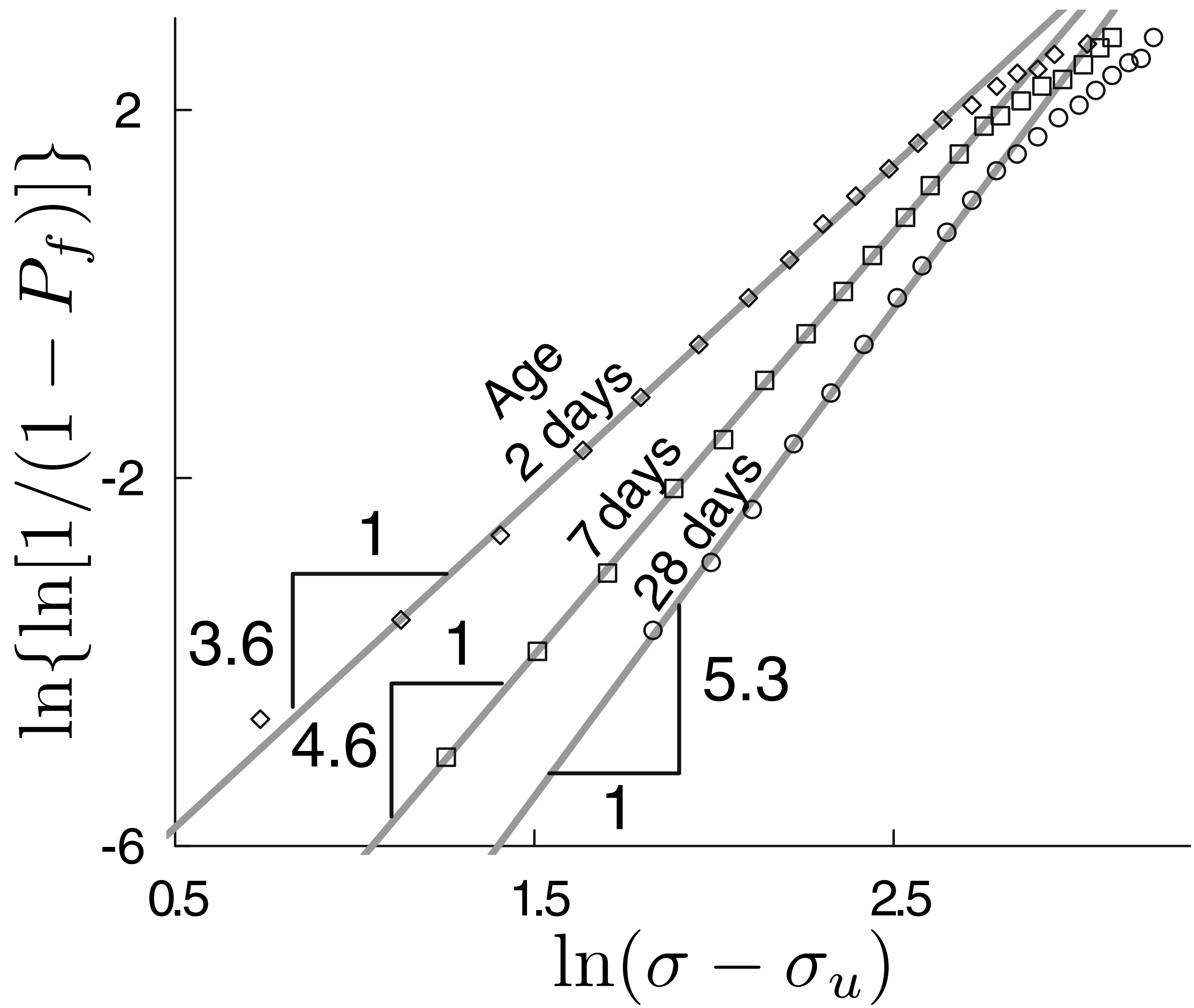

$\ln\{\ln[1/(1-P_f)]\}$
2
-2
-6
0.5
1.5
2.5
$\ln(\sigma - \sigma_u)$
Age
2 days
7 days
28 days
1
3.6
1
4.6
5.3
1


Fig. 9

# THE ROLE OF FRACTURE MECHANICS IN RELIABILITY ANALYSES

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**Abstract:** The basis of all design codes and recommendations that are endorsed by engineering societies are safety concepts which have been formulated with the intent to meet a society's safety demands. These demands are expressed in terms of failure probabilities, differentiating between structural safety and serviceability, accounting for the expected service life and the potential loss of life and assets. While in the last century safety formats were mainly based on experience, newer code developments are supported by fully probabilistic concepts and reliability engineering tools. Nonetheless, a realistic assessment of structural performance, and in consequence the expected service life, is in many cases impaired due to oversimplified design assumptions, the elastic determination of internal forces applying the principle of superposition, and a lack of understanding regarding the relevant stochastic models. While the 'elastic' design has merit in many design situations, its limitations are quickly reached if a realistic assessment of bearing capacity or serviceability are to be performed. Within this contribution the role of fracture mechanics in the reliability analyses of reinforced and pre-stressed concrete structures will be presented. After providing a review of the relevant concepts, examples are given to illustrate the significance of fracture mechanics as well as point out existing short-comings and the need for additional research.

***Keywords:*** *fracture mechanics, safety concepts, partial safety factor, global safety factor, limit state, small-sample simulation.*

## INTRODUCTION

The basis of all design codes and recommendations that are endorsed by engineering societies are safety concepts which have been formulated with the intent to meet a society's safety demands. These demands are expressed in terms of annual failure probabilities, differentiating between structural safety and serviceability. Typically a violation of serviceability related design requirements with a failure rate of 1/1000 per year is accepted, while the requirement for structural safety is $10^{-6}$ (JCSS ; EN1990). The design safety level which is the safety level of the

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virgin structure after construction is depending on the expected service life, typically 50 years but also up to 200 years for infrastructure of national importance. The potential consequences of failure due to loss of life as well as direct and indirect economic costs are accounted for through concepts that allow the adaption of the safety level such as introduced in Eurocode in terms of consequence classes. More recently, engineering communities have started to develop performance based design concepts instead of rigid normative code provisions (e.g. specifying concrete cover based on environmental classes and concrete strength is replaced by the requirement to limit the effective permeability of the concrete cover). This is essential to allow for cost-efficient design and the application of rapidly developing new technologies.

While in the last century safety codes were reflecting the experience of the engineering community, newer code developments are supported by fully probabilistic concepts and reliability engineering tools. Entire professional societies work on questions regarding the formulation of stochastic models for material properties, load actions (frequent or rare), or numerical procedures that allow the application of probabilistic concepts to real world problems.

Considering the multitude of design situations and load combinations that are to be analyzed for every structure the elastic design (determination of internal forces using the principle of superposition as well as determination of deformations) certainly has merit. Nonetheless, its limitations are quickly reached if a realistic assessment of bearing capacity or serviceability related quantities are to be performed. The accurate prediction of a structure's current and future response under environmental and mechanical loads requires models that are able to reproduce the nonlinear characteristics of construction materials and in particular concrete. These are aspects of plasticity with damage evolution and hardening in confined compression and especially softening in tension. Fracture mechanics provide the necessary formulations for a realistic description in analytical formulations and numerical calculations and, in consequence represent the basis for performance based design concepts, reliability analyses, and service life predictions.

On a structural level global response quantities such as deflections or bearing capacity can only be predicted accurately if fracture mechanics are applied. Without, the consequences of softening and cracking such as the redistribution of internal forces due to the formation of hinges and the size effect cannot be captured. Furthermore, the design of many real structures is significantly affected by a models ability to accurately represent structural stiffness. Examples are the design of jointless bridges that are large determined by the soil-structure interaction, or more generally, the design of structures in seismically active regions. The latter is especially relevant for super-tall buildings or reactor containments in the nuclear industry, in which case a nonlinear push over analysis applying fracture mechanics is quintessential.

On a smaller scale, fracture mechanics allow the accurate prediction of damage localization, and crack formation which are the basis for many serviceability considerations (this is of course the source of the global phenomena captured by fracture mechanics). In particular, questions like rebar depassivation and durability (corrosion), but also leak tightness of containers and underground structures in artesian aquifers come to mind. Naturally, the accurate prediction of the local stress state and micro-cracking has even further implications and influences the macroscopically observed concrete-steel bond, creep, and shrinkage properties of concrete.

Considering the design life of many engineering structures, long term processes and durability considerations are highly relevant. These can only be accurately predicted if the interaction with cracking, described by fracture mechanics, is accounted for. For many practically relevant

questions even a bi-directional coupling of mechanical models with transport models for heat, water and chemical agents is required.
After providing a concise review of safety concepts and reliability engineering tools, examples will be given to illustrate the significance of fracture mechanics for the performance assessment and life time prediction of concrete structures. Existing short-comings and the need for additional research will be pointed out.

## SAFETY CONCEPTS AND RELIABILITY THEORY

The subsequent sections are dedicated to a short review of essential concepts and theories related to the reliability analysis of concrete structures and are limited to the extent that they are needed for the discussion of fracture mechanics in reliability engineering.

### Reliability theory

An efficient and reliable design in engineering is associated with the proper choice of geometrical (e.g. cross-sections) and material properties so that the resulting structural properties, typically denoted as resistance $R$, are sufficient to withstand the associated requirements due to the action or load effects $Q$ with a predefined safety margin. Note, $R$ and $Q$ are stochastic variables described by distribution type and the first statistical moments (mean value, variance, and skewness). The required safety margin is typically given by design codes in terms of an admissible failure probability $p_f$ ; e.g. $p_f < 10^{-6}$ per year during the assumed service life, see e.g. (EN1990 2002).

In reliability engineering, the requirement that the resistance $R$ exceeds the acting load $Q$ is described by a limit state equation of the form (Shinozuka 1983; Schneider and Schlatter 1996)

$$g(\mathbf{X}) = R - Q = 0 \tag{1}$$

where $g(\mathbf{X}) < 0$ describes the failure region depending on a vector of random variables $\mathbf{X} = \{X_1, X_2, ...X_n\}$ and $g(\mathbf{X}) = 0$ denotes the limit state. The failure probability $p_f$ is defined as integral over the failure domain:

$$p_f = \int_{-\infty}^{\infty} f_Q(x) F_R(x) dx = 1 - \int_{-\infty}^{\infty} F_Q(x) f_R(x) dx = \int_{-\infty}^{0} f_M(x) dx \tag{2}$$

where $F_R, F_Q$ are the cumulative distribution functions, and $f_R, f_Q$ are the probability density functions of resistance and load, respectively; $f_M$ is the probability density function of the safety zone $M = R - Q$.

Under the assumption that the safety zone is (approximately) normally distributed the failure probability can be expressed by an equivalent reliability index $\beta$.

$$p_f = \Phi(-\beta) = \int_{-\infty}^{-\beta} \frac{1}{\sqrt{2\pi}} \exp^{-\frac{x^2}{2}} dx \tag{3}$$

If both action and resistance side of the limit state equation are normally distributed there exists a closed form solution for the reliability index expressed by

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{4}$$

where $\mu_M = \mu_R - \mu_Q$ is the difference of mean values between resistance and load effect and $\sigma_R, \sigma_Q$ are the respective standard deviations. The calculation of the reliability index $\beta$ is a constrained optimization problem, where in the standard normal space the closest point of the limit state surface with regard to the origin is searched (Shinozuka 1983).

For reinforced concrete structures a number of limit states can be formulated and must not violated more frequently than by a certain probability of exceedance. These typically are the equations describing the ultimate limit states, stability limit states, and serviceability limit states, where the latter encompass deflection, crack width, and stress levels under different load combinations.

## Safety concepts

As requested by the current codes ULS verifications must be performed in order to obtain a design resistance to be compared with the design loads applied to the structures. The fib Model Code 2010 (2013) proposes three different methods to obtain the design resistance from non-linear finite element analyses; the Global Resistance Factor method (GRF), the Partial Factor method (PF) and the Estimate of Coefficient of Variation of Resistance method (ECOV).

Global Resistance Factor Method (GRF) - According to this method, which is also included in the Eurocode 2, the global resistance of the structure is a random variable. The effects of various uncertainties are integrated in a global design resistance and can be expressed by a global safety factor. This global safety factor is obtained based on a single non-linear calculation using effective "mean" mechanical properties of materials. These "mean" mechanical properties in the sense of the global resistance factor method according to the fib Model Code 2010 (2013) are derived from the characteristic mechanical properties in such a way that the standard partial safety factors can be recovered. The effective mean value of the concrete compressive strength $f_{cm}$ and steel yield strength $f_{ym}$ are determined from their characteristic values $f_{ck}, f_{yk}$ by

$$f_{cm} = 0.85 f_{ck},\ f_{ym} = 1.1 f_{yk}, \tag{5}$$

The other concrete parameters can then be derived from $f_{cm}$ via the standard empirical relations. The global safety coefficient is equal to the product of safety and the model coefficient $\gamma_{GL} = 1.2 \times 1.06 = 1.27$. (It should be noted that the ratio 1.27/0.85 equals the concrete partial safety coefficient of $\gamma_c$ = 1.5 and that the ratio 1.27/1.1 equals the steel partial safety coefficient of $\gamma_s$ = 1.15.) The design resistance $P_d$ is then calculated from the ultimate load $P_u$ which is obtained from a single nonlinear analysis using the above "mean" mechanical properties.

$$P_d = \frac{P_u}{\gamma_{GL}} \tag{6}$$

Partial Safety Factor Method (PF) - According to this method the basis variables are deterministic quantities so that this method separates the treatment of uncertainties and the variability originating from various causes by means of design values assigned to variables.

Design mechanical properties of materials, derived from the characteristic mechanical properties, must be input in the analysis. The design mechanical properties are calculated as follow from the characteristic concrete compressive strength $f_{ck}$, concrete tensile strength $f_{ct}$, and steel yield strength $f_{yk}$:

$$f_{cd} = \frac{f_{ck}}{\gamma_{RD} \cdot \gamma_c}, \quad f_{ctd} = \frac{f_{ctk}}{\gamma_{RD} \cdot \gamma_c}, \quad f_{yd} = \frac{f_{yk}}{\gamma_{RD} \cdot \gamma_c} \tag{7}$$

The design value of fracture energy $G_{fd}$ and Young's modulus $E_{cd}$ are predicted from the design compressive strength according to

$$G_{fd} = G_{f0}\left(\frac{f_{cd}}{10}\right)^{0.7}, \quad E_{cd} = 22000\left(\frac{f_{cd}}{10}\right)^{0.3} \tag{8}$$

where $\gamma_{RD}$ is the model uncertainty coefficient equal to 1.06, $\gamma_c$ is the concrete partial safety coefficient equal to 1.5, $\gamma_s$ is the steel partial safety coefficient equal to 1.15, and $G_{f0}$ is the base value of fracture energy depending on the aggregate type. The ultimate load $P_u$ obtained from the analysis by inputting the design material properties is already the design resistance $P_d$.

$$P_d = P_u \tag{9}$$

The nonlinear analysis is now derived with extremely low strength parameters. This may therefore cause deviation in structural response, e.g. in failure mode. For this reason it is not advised to base conclusions only on the partial safety factor method.

Estimation of Coefficient of Variation of Resistance method (ECOV) – According to this method an estimate of mean and characteristic values of resistance shall be calculated using corresponding values of material parameters. The random distribution of resistance of reinforced concrete members can be described by a two parameter lognormal distribution, therefore this method is based on the assumption of a lognormal distribution identified by two random parameters: the mean resistance and the coefficient of variation $V_R$.

Two non-linear finite element analyses must be performed by inputting measured mechanical properties of material and characteristic mechanical properties of materials. The design resistance $P_d$ is then calculated as:

$$P_d = \frac{P_{u,m}}{\gamma_{RD} \cdot \gamma_R}, \quad \gamma_R = e^{\alpha_R \cdot \beta \cdot V_R} \tag{10}$$

where $P_{u,m}$ is the ultimate load obtained from the analysis by inputting measured mechanical properties, $\gamma_{RD}$ is the model uncertainty coefficient equal to 1.06, $\alpha_R = 0.8$, and the reliability index $\beta = 3.8$. The coefficient variation then is given based on the ultimate load obtained from the analysis inputting characteristic mechanical properties, $P_{u,c}$, as follows:

$$V_R = \frac{1}{1.65} \ln\left(\frac{P_{u,m}}{P_{u,c}}\right) \tag{11}$$

## Stochastic models for concrete

There is a high variability in the experimental testing of quasi-brittle materials, such as concrete (C) and fiber reinforced concrete (FRC) due to their inherent heterogeneity, the aggregates, additives and the mixt design. The full mechanical characterization by elastic, strength, and fracture mechanical parameters is a significant challenge. In addition to the standard parameters compressive strength $f_c$ and Young's modulus $E_c$, the direct or indirect tensile strength $f_t$ and the fracture energy ($G_f$, $G_F$) have to be determined which is much more difficult and problematic than for other engineering materials.

The practical design of quasi-brittle material-based structures requires statistical approaches, simulation and probabilistic assessment procedures for the characterization of the variability of these materials. A key parameter of nonlinear fracture mechanics modelling is certainly fracture energy of concrete and its variability, which is a subject of research of many authors, e.g. Bažant & Planas (1998), Bažant & Becq-Giraudon (2002). With the availability of stochastic models for the fracture-mechanical parameters, realistic reliability analyses in a practically feasible framework can be made possible. The target of this contribution is the characterization of stochastic fracture–mechanical properties of frequently used concretes, based experimental tests and recommendations by the Joint Committee of Structural Safety, (JCSS 2001b), which are summarized subsequently.

Basic Properties – The reference property of concrete is the compressive strength $f_{co}$ of standard test specimens (cylinder of 300 mm height and 150 mm diameter) tested according to standard conditions and at a standard age of 28 days, see ISO 2736 (1983) and ISO 3893 (1977). Other concrete properties are related to the reference strength of concrete as follows:

In situ compressive strength: $f_c = \alpha(t,\tau) \cdot f_{co}^{\lambda}$ [MPa] (12)

Tensile strength: $f_{ct} = 0.3\, f_c^{2/3}$ [MPa] (13)

Modulus of elasticity: $E_c = 10.5\, f_c^{1/3}\, (1/(1+\beta_d\, \phi(t,\tau))$ [GPa] (14)

Ultimate compression strain: $\varepsilon_u = 6 \cdot 10^{-3}\, f_c^{1/6}\, (1+\beta_d\, \phi(t,\tau))$ [m/m] (15)

The lognormal variable λ takes into account the systematic variation of in situ compressive strength and strength of standard tests with mean 0.96 and coefficient of variation of 0.005. The determinist function $\alpha(t,\tau)$ takes into account the concrete age at the loading time $t$ [days] and the duration of loading τ [days] and for most applications can be assume equal to 0.80, see also (JCSS 2001b). $\phi(t,\tau)$ is the creep coefficient which is assumed to be deterministic, and $\beta_d$ is the ratio of the permanent load to the total load and depends on the type of the structure; generally $\beta_d$ is between 0.6 and 0.8. The Joint Committee for Structural Safety (JCSS) specifies a simplified bilinear and a more advanced stress-strain relationship for concrete in compression.

The probabilistic model – The strength of concrete at a particular point $i$ in a given structure $j$ as a function of standard strength $f_{c0}$ is given as:

$$f_{c,ij} = a(t,\tau)(f_{co,ij})^{\lambda}\,\mathrm{Y}_{1,\mathrm{j}} \tag{16}$$

$$f_{co,ij} = \exp((U_{ij}\Sigma_j + M_j)) \tag{17}$$

in which

$f_{c0,ij}$ = log-normal variable, independent of $\mathrm{Y}_{1,\mathrm{j}}$, with distribution parameters $M_j$ and $\Sigma_j$
$M_j$ = the logarithmic mean at job $j$
$\Sigma_j$ = the logarithmic standard deviation at job $j$
$Y_{1,\mathrm{j}}$ = a log-normal variable representing additional variations due to the special placing, curing and hardening conditions of in situ concrete at job $j$. The variable $Y_{1,j}$ can also be taken as a spatially varying random field whose mean value function takes account of systematic influences in space.
$U_{ij}$ = a standard normal variable representing the variability within one structure
Correspondingly, for the other three basic properties:

$$f_{ct,ij} = 0.3\, f_{c,ij}^{2/3} Y_{2,j} \tag{18}$$

$$E_{c,ij} = 10.5\, f_{c,ij}^{1/3} Y_{3,j} (1+\beta_d \varphi(\mathrm{t},\tau))^{-1} \tag{19}$$

$$\varepsilon_{c,ij} = 6\cdot 10^{-3}\; f_{c,ij}^{1/6} Y_{4,j} (1+\beta_d \varphi(\mathrm{t},\tau)) \tag{20}$$

where the variables $Y_{2,j}$ to $Y_{4,j}$ mainly reflect variations due to factors not well accounted for by concrete compressive strength (e.g., gravel type and size, chemical composition of cement and other ingredients, climatic conditions). The variables $U_{ij}$ and $U_{kj}$ within one member are correlated with correlation length $d_c$ = 5 m and correlation factor ρ = 0.5 acc. to JCSS (assuming a heuristic estimation), based on sample information $r_{ij}$ and $r_{kj}$:

$$\rho(\mathrm{U}_{ij},\mathrm{U}_{kj}) = \rho + (1-\rho)\exp\left\{\frac{\left(r_{ij}-r_{kj}\right)^2}{d_c^2}\right\} \tag{21}$$

Unless direct measurements are available, the parameters of the lognormal variables $Y_{k,j}$ can be taken from Table 1. The variability of the variables $Y_{k,j}$ can further be split into a part depending only on the job under consideration and a part representing spatial variability. If direct measurements are available, the parameters in Table 1 should be considered parameters of an equivalent prior sample with size $n' = 10$.

**Table 1 - Data for parameters $Y_i$**

| Variable | Distribution type | Mean | Coefficient of variation | Related to |
|---|---|---|---|---|
| $Y_{1,j}$ | LN | 1.0 | 0.06 | compression |
| $Y_{2,j}$ | LN | 1.0 | 0.30 | tension |
| $Y_{3,j}$ | LN | 1.0 | 0.15 | E-modulus |
| $Y_{4,j}$ | LN | 1.0 | 0.15 | ultimate strain |

The distribution of $x_{ij} = \ln(f_{co,ij})$ is normal provided that its parameters M and Σ are obtained from an ideal infinite sample. In general it must be assumed that concrete production varies from production unit, site, construction period, etc. and that sample sizes are limited. Therefore, the parameters $M$ and $\Sigma$ must also be treated as random variables. Then, $x_{ij}$ follows a student distribution with:

$$F_x(x) = F_{t_{\nu''}}\left[\frac{\ln(x/m'')}{s''}(1+\frac{1}{n''})^{-0.5}\right] \tag{22}$$

where $F_{t\nu''}$ is the Student distribution for ν″ degrees of freedom. $f_{co,ij}$ can be represented as

$$f_{co,ij} = \exp(m'' + t_{\nu''}s''(1+\frac{1}{n''})^{0.5}) \tag{23}$$

The values of $m$”, $n$”, $s$” and $v$” depend on the amount of specific information. Table 2 gives the values if no specific information is available (prior information). For $n$”, $v$” > 10, the log-normal distribution with mean $m$” and standard deviation $s'' = \sqrt{\frac{n''}{n''-1}\frac{\nu''}{\nu''-1}}$ is a good approximation.

**Table 2 – Prior parameters for concrete strength distribution ($f_{co}$ in MPa) (Rackwitz 1983; Kersken-Bradley and Rackwitz 1991)**

| Concrete type | Concrete grade | Parameters | | | |
|---|---|---|---|---|---|
| | | $m'$ | $n'$ | $s'$ | $v'$ |
| Ready mixed | C15 | 3.40 | 3.0 | 0.14 | 10 |