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# THE MECHANICS OF FIBER REINFORCEMENT OF CEMENT MATRICES

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Synopsis: The various factors which influence the fiber reinforcement of cement matrices are briefly discussed. It is shown that the existing geometrical spacing equations are inadequate in certain important respects, and that, they do not show a unique relationship with the flexural strength of steel fibrous concrete. Rather, they lead to a family of curves for different fiber lengths and fiber diameters. By considering the interfacial bond stress due to stress transfer and a crack in relation to short discontinuous fibers, a new effective spacing equation is derived. A crack control-composite mechanics approach is used to develop equations to predict the first crack and ultimate flexural strength of randomly oriented steel fibrous concrete. It is then shown that the new effective spacing equation leads to unique relationships between fiber spacing and flexural strength. The theoretical equations show excellent correlation to available test data from various sources.

Keywords: bonding; cements; cracking (fracturing); deformation; fiber reinforced concretes; flexural strength; metal fibers; reinforced concrete.

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## INTRODUCTION

The use of fibrous reinforcement to improve the strength and deformation properties of the cement matrix is now well established. The concept of fiber reinforcement is to use the deformation of the matrix under stress to transfer load to the fiber. Substantial improvements in static and dynamic strength properties could then be achieved if the fibers are strong and stiff, and loaded to fracture provided, of course, there is a minimum fiber volume fraction.

Fibers currently being used in concrete can broadly be classified into two types. Low modulus, high elongation fibers, such as nylon, polypropylene and polyethelene, are capable of large energy absorption characteristics; they do not lead to strength improvement, however - rather, they impart toughness and resistance to impact and explosive loading. High strength, high modulus fibers such as steel, glass, asbestos and carbon, on the other hand, produce strong composites. They impart primarily characteristics of strength and stiffness to the composite, and to varying degrees, dynamic properties.

In the current development of fiber reinforced concrete, short, discontinuous fibers are used. The fiber-matrix bond then becomes irregular and discontinuous. Apart from the fiber geometry and its length-diameter ratio, other factors such as fiber volume, fiber orientation and fabrication techniques profoundly influence the properties and mode of failure of the fibrous composite. The role of fibers is essentially to arrest any advancing cracks by applying pinching forces at the crack tips, thus delaying their propogation across the matrix, and creating a distinct slow crack propogation stage. The ultimate cracking strain of the composite is thus increased to many times greater than that of the unreinforced matrix.

The properties of the matrix of the fibrous composite are equally important. The mechanics of composites in which the fibers are embedded in a metal or plastic matrix is well known. With metal matrices, stress transfer between the matrix and the fiber occurs through plastic deformation of the matrix. With plastic matrices, the elastic extensibility of the matrix transfers the load to the fibers. The fundamental difference of cement-based fiber composites from other fiber reinforced materials is that the matrix failure strain is only a fraction of the fiber yield strain. Further, the matrix is viscous and porous, so that the precise mechanism of how short discrete fibers reinforce the cement matrix is not yet fully understood.

This paper discusses the mechanics of fiber reinforced cement-matrices containing uniformly distributed but randomly oriented short steel fibers. It is shown that the existing geometrical spacing equations are inadequate in certain important respects and do not lead to a unique relationship with the flexural strength of steel fibrous concrete. By considering the interfacial bond stress due to stress transfer and cracking in relation to short discontinuous fibers, a new effective spacing equation is derived. A crack control-composite mechanics approach is used to develop equations to predict the first crack and ultimate flexural strength of randomly oriented steel fibrous concrete. It is then shown that the new spacing equation leads to unique relationships between fiber spacing and flexural strength. The theoretical equations show excellent correlation to available test data from many sources on fiber reinforced paste, mortar and concrete involving numerous parameters of mix proportions, fiber geometry, fiber volume, and size and volume of aggregates.

## CRACK ARREST MECHANISM OF FIBER REINFORCEMENT

The mechanism of fiber reinforcement of cement matrices was first explained by Romualdi and Batson (1,2). They suggested that fibers act as crack arrestors by producing pinching forces which tend to close a crack. By considering a direct tensile stress field and applying the principles of linear elastic fracture mechanics they showed that the first crack tensile strength was inversely proportional to the geometrical spacing of fibers for a given fiber volume content. The geometrical spacing of fibers thus becomes a critical factor in their crack arrest mechanism. Romualdi and Mandel (3) subsequently showed that this increase in first crack strength could be obtained by mixing short fibers directly into the matrix.

## The Spacing Concept

While spacing of fibers is easily defined for systems in which fibers are arranged in a specific direction, visualising spacing in composites with randomly oriented fibers is conceptually complicated. Spacing of fibers is basically a geometrical quantity that characterises the pattern of distribution of fibers. While the concept of fiber spacing is important, a purely geometrical concept without relevance to the interaction between the fiber and matrix, and the mode of failure cannot fully explain the mechanism of fiber reinforcement or the strength and deformation properties of the composite.

The crack arrest mechanism of Romualdi and Batson (1,2) is primarily based on a geometrical fiber spacing concept which establishes a relationship between the first crack tensile strength of the composite and fiber spacing. They assume that the shear forces at the fiber-matrix interface are absent until the occurrence of a crack when the additional concrete displacements caused by the extensional strains in the neighbourhood of the crack cause a distribution of shear forces along the wires that act to close the crack. Although these assumptions are valid for long, continuous fibers where the shear stress distribution in the absence of a crack extends only up to half the critical length ($l_c/2$)(4) from each end of the fiber, thus leaving a major proportion of the fiber length free from any shear stresses, these assumptions are not valid for short fibers (of length $< l_c$) where the shear stress distribution in the absence of a crack extends along the whole length of the fiber. The calculation of "effective spacing" of short fibers, therefore, necessitates the consideration of these shear forces along the fiber-matrix interface in addition to any stresses developed at the occurrence of a crack.

The crack arrest or the geometrical fiber spacing concept (1,2) also assumes that the bond between the fiber and the matrix is intact which is not necessarily true for discrete fibers. It does not also fully recognise the influence of the geometry of the fiber (3). Further, the application of linear-elastic fracture mechanics to Portland cement pastes, mortars and concretes is questionable. Recent tests have shown that in concrete materials there is significant crack growth prior to fracture. Even in the cement paste, which is probably the nearest to an elastic homogenous cementitious material, some slow micro-cracking does occur at the crack tip (5). Similarly, the mechanism of crack propogation is not independent of specimen geometry, and the stress intensity factor also varies with the crack geometry (6). The stress intensity factor may not therefore be a valid criterion for concrete fracture. The existence of microcracking zones in concrete materials implies that in applying energy criteria to concrete fracture, energy dissipating mechanisms other than surface energy need to be considered (7).

Probably, the major drawback of the geometrical fiber spacing concept (1,2) is that it is based on a direct tensile stress field whereas the results used to prove the concept are based on flexural or indirect tension tests (1,3). Data based on direct tension tests (8,9) do not appear to follow the predictions of the theory, and data based on flexural and indirect tension tests (10) are lower than those predicted by the theory. Further, the crack arrest or fiber spacing concept recognises only the initial cracking strength and does not explain the post-cracking behavior and the ultimate strength of the composite. These are significant properties of great value in design and need to be accounted for.

## Effective Spacing

In considering composites with randomly oriented fibers, a spacing concept based entirely on a geometrical or an assumed ordered distribution of fibers is unlikely to lead to a clear understanding of the mechanism of fiber reinforcement. A formal definition of such a composite should not only relate to a statistical description of the spacing of the centroids of the fibers but also incorporate the fiber-matrix interaction and the mode of failure. This is accomplished by the concept of "effective spacing".

Even though the "spacing" of fibers is essentially a geometrical quantity (with hardly any relevance to any physical process involved such as cracking or fracture), the case is different with "effective spacing". Deducing the effective spacing with randomly oriented fibers in a composite requires information regarding the physical interaction involved, the bond stress at the fiber-matrix interface and the mode of fracture in addition to fiber geometry. Effective spacing in some sense, enables reduction of a statistically isotropic system into an equivalent transversely isotropic system (or a system strengthened with aligned fibers in a single direction) which simplifies the study of the medium under uniaxial loading conditions.

In a study of the spacing of short fibers, Kar and Pal (11) have attempted to take these factors into consideration by including a bond efficiency factor in their effective spacing equation. However, the shear stress distribution at the fiber-matrix interface in the absence of a crack has been overlooked, and the shear stress distribution due to the presence of a crack in the vicinity of a fiber alone has been considered.

## Relationship Between Existing Spacing Equations

The two spacing equations of Romualdi and Mandel(3), and Kar and Pal (11), and the centroidal spacing equation (defining the distance between centroids of fibers without taking into account the overlapping effect of long fibers) (12) are all derived for randomly oriented short fibers by assuming the fibers to be arranged in a square array. The interrelationship between these equations can best be understood by studying the factors which govern fiber spacing.

The relation between fiber length and spacing (as calculated by the three spacing equations) for a mix containing 1.91 per cent fibers by volume of matrix (excluding aggregate) and fiber diameter of 0.46 mm (0.018") is shown in FIG.1. The centroidal spacing increases as the fiber length increases since, for a constant volume of fibers the number of fibers is reduced as fiber length is increased. The overlapping effect of fibers on spacing has been considered by Romualdi and Mandel (3), and hence their spacing remains constant with variation in the length of fibers. In Kar and Pal's equation (11), spacing decreases with increasing length of fibers, since the bond efficiency of longer fibers is greater than that for short fibers.

The influence of fiber diameter on spacing is shown in FIG. 2 for a mix containing various diameters of fibers, and a constant fiber length (25 mm) and fiber volume (2%). The spacing increases with increasing fiber diameter since, to keep the fiber volume constant, the number of fibers decreases as their diameter is increased, thus increasing the spacing between them.

The relationship between the fiber spacings as calculated from the three equations for four different lengths and nine different diameters of fibers is shown in FIG. 3. The volume percentages of fibers used is shown in the figure. The results show that for each length and diameter of fiber, a different relationship exists. FIGS. 1,2, and 3, show conclusively that the three spacing equations are not generalised to include the two important factors of fiber geometry (namely, length and diameter) which influence fiber spacing.

In order to establish the application of these spacing equations (3,11,12) to various lengths and diameters of fibers, the results of an extensive series of tests, in which fibers of eight different diameters ranging from 0.27 mm to 1.82 mm (0.0108 to 0.073 in) and four different lengths from 12.5 mm to 50 mm ($\frac{1}{2}$ in to 2 in) were used, are shown in FIGS. 4,5 and 6.

In these tests, a concrete mix of 1:2:5:1.2 with a water-cement ratio of 0.55 was used. Ordinary Portland cement and crushed gravel of 10 mm ($\frac{3}{8}$ in) maximum size was used throughout. The fibers were interannealed, hard drawn mild steel wires with a plane surface.

The results (FIGS. 4,5 and 6) show that in all three cases different relationships are obtained for different lengths and diameters of fibers, and that none of the spacing equations take into account fully the different parameters involved in fiber spacing. The results also show no unique relationship between the flexural strength and the spacing equation for all the test data.

## A Combined Crack Arrest-Composite Mechanics Approach to Fiber Reinforcement

The discussion so far has shown that none of the conventional crack arrest mechanisms and spacing equations can adequately explain all the factors involved in the mechanics of fiber reinforcement of cement matrices. If the fibers are not continuous or aligned, but instead if the composite consists of short, discontinuous fibers which are randomly oriented and uniformly dispersed in the matrix, then there are three basic considerations related to the transfer of stress from matrix to fiber. These are: (1) critical fiber length or the fiber transfer length (which is analogous to the transmission length of the tendon in prestressed concrete); (2) the fiber-matrix interfacial bond, and (3) the fiber efficiency or the orientation factor for random fibers. Both the critical fiber length and the interfacial bond are interdependent and also closely related to the fiber length-diameter ratio (i.e. aspect ratio). To get a clear understanding of the mechanism of fiber reinforcement all these factors need to be considered.

In the following, the critical fiber length, the interfacial bond and the orientation factor in relation to short discontinuous fibers are briefly discussed. Based on these, three new concepts in relation to fiber reinforced concrete are presented. Firstly, a new effective fiber spacing equation is derived for randomly oriented short fibers uniformly distributed in a concrete matrix. Bond deficiency is taken into account in this spacing equation by introducing bond efficiency factors for both the length and diameter of fibers. Secondly a combined crack control-composite mechanics approach is presented to predict the first crack and the ultimate flexural strength of steel fiber reinforced concrete. These equations reflect the crack arrest criterion, the interfacial bond and the randomness of fibers in the matrix. Finally, it is shown that the concepts presented here lead to a unique relationship between effective fiber spacing and the first crack and ultimate flexural strengths of the fibrous concrete.

## MECHANISM OF FIBER REINFORCEMENT

In a general composite system comprising amatrix phase and a fiber phase, several modes of failure are possible, depending on the relative values of the failure strains of the matrix and the fibers, as well as the fiber geometry and the fiber-matrix interfacial bond strength. It is common for the fibers in a metal or plastic matrix to fracture at the failure stress of the composite whereas in steel fiber reinforced concrete, bond failure at the fiber-matrix interface usually occurs at the ultimate strength of the composite.

The mechanism of fiber reinforcement in a steel fiber reinforced concrete composite can be explained by considering a specimen under uniaxial tension. The reinforcing action by fibers occurs through the fiber-matrix interfacial bond stress. Cracks in the matrix occur when the composite strain exceeds the cracking strain of the matrix. Since the fibers are stiffer than the matrix ($E_f > E_m$), they deform less and hence exert a pinching force at the crack tips. In this respect, the role of fibers is analogous to the role of aggregates as crack arrestors in the paste or mortar matrix. The problems of interfacial bond, of size and shape of the inclusion, and of the stiffness of the inclusion relative to that of the matrix are all factors common to both systems (13).

In this manner the cracks are prevented from propogating until the composite ultimate stress is reached when failure occurs either by the simultaneous yielding of the fibers and matrix, or by fiber-matrix interface bond failure. The criterion that determines whether yielding or bond failure occurs is the fiber length and surface geometry. In general, if the fiber length is greater than the critical length, fibers should yield at the failure of the composite whereas for shorter lengths than the critical, bond failure occurs.

## Critical Fiber Length

Consider a fiber of size l x d embedded in the surrounding matrix. The load is transferred by an average interfacial shear stress $\tau$, and the longitudinal tensile stress in the fiber $\sigma_f$ varies from zero at the ends of the fiber to the fracture stress $\sigma_{fu}$, if the fiber is long enough (FIG. 7). For a short fiber $\sigma_f$ might not reach the value for fiber fracture and "pull-out" or sliding of the interface would occur. The ratio $l_c/2$ is defined as the transfer length where $l_c$ is the critical fiber length. From equilibrium considerations,

$$\frac{\pi d^2}{4} \frac{d\sigma}{dl} = \pi d \ \tau$$

or

$$\frac{dl}{d\sigma} = \frac{d}{4\tau} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

The fracture stress can be reached only when

$$\frac{l_c}{2} = \sigma_{fu} \cdot \frac{d}{4\tau}$$

or

$$l_c = \sigma_{fu} \cdot \frac{d}{2\tau} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

The aspect ratio or the shape of the fiber must then be

$$\frac{l_c}{d} \geqslant \frac{\sigma_{fu}}{2\tau} \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

The value of $l_c$ depends on $\tau$, and if bond failure occurs, $\tau$ will then represent merely the frictional force per unit area between matrix and fiber. Even then, the frictional force may be adequate to impart to the composite a measure of ductility, crack resistance and toughness. The fiber tensile stress is also not constant along the length (FIG. 8) and thus in considering the strength of a discontinuous fiber composite, the average stress must be used instead of $\sigma_{fu}$. The average fiber stress over the length $l_c$ is $\sigma_{fu}/2$ (assuming linear distribution), but if the fibers are much longer than $l_c$, the average stress will approach the maximum $\sigma_{fu}$, and the fibers will effectively act as continuous fibers. It is clear that discontinuous fibers will strengthen less effectively than continuous ones, but with longer fibers the difference in strength should become progressively less.

## Fiber-matrix Interfacial Bond

Thus both the interfacial bond stress and the fiber tensile stress are closely related to the fiber aspect ratio. The interfacial bond stress of a steel fiber in a concrete matrix can be considered to consist of two parts;

(1) the interfacial bond stress due to load transfer from matrix to fiber. FIG. 8 shows a schematic representation of the interfacial shear stress $\tau$ along the length of the fiber as well as the fiber tensile stress $\sigma$ for various fiber aspect ratios in relation to the critical value. The tensile stress in the fiber is zero at the ends and builds up towards the centre of the fiber, analogous to the development of the prestressing force in the tendon. The shear stress is maximum at the fiber ends and decays to zero at a distance $l_c/2$ from the end. For long fibers ($l > l_c$), the central zone is free from shear stresses; for short fibers ($l < l_c$), the bond stress is present throughout the fiber length.

(2) the interfacial bond stress due to the presence of a crack. This distribution occurs in the vicinity of the crack, increases to a maximum and then decreases with increasing distance from the crack edge (FIG. 9). A crack thus brings discontinuity to the bond stress distribution.

For short fibers both the above bond stress distributions need to be considered, and the combined effect is shown in FIG. 9. In the derivation of the spacing equations, the former bond stress distribution has not been considered by some investigators (3,11). However, Kar and Pal (11) considered a bond efficiency factor to fiber length alone, but did not take into account the relative bond efficiency of different diameters of fibers.

## Efficiency of Randomly Oriented Fibers

When short discontinuous fibers are randomly oriented in the matrix, there is the further problem of determining the "efficiency factor" or "effective length" or the "orientation factor" of the fibers. With random orientation only those fibers which are parallel or nearly parallel to the tensile stress trajectories are effective in crack control. For those fibers that are less effectively oriented in space, a correction must be applied. Assuming