Table 4–Ultimate torsional moment - Analytical predictions and comparisons

Beam	$T_{u,exp}$ (kN-m)	$T_{u,calc.SP}$ (kN-m)	$T_{u,calc.SP}$ (kN-m)	$T_{u,calc.FRP}$ (kN-m)	$T_{u,calc.SFRC}$ (kN-m)	$T_{u,exp}/T_{u,calc.}$
P-0(a)	–	–	–	–	–	–
P-0(b)	–	–	–	–	–	–
P-0(c)	–	–	–	–	–	–
P-200(a)	2.39	2.54	–	–	–	0.94
P-200(b)	2.31	2.76	–	–	–	0.84
P-150	2.65	2.94	–	–	–	0.90
SP-200	2.82	–	2.64	–	–	1.07
SP-150	3.07	–	3.02	–	–	1.02
F1-0	4.87	–	–	4.79	–	1.02
F2-0	6.65	–	–	6.78	–	0.98
SF1-0	2.41	–	–	–	1.40	1.72
SF3-0	2.73	–	–	–	2.01	1.36
SF1-200	2.73	–	–	–	2.65	1.03
SF3-200	3.15	–	–	–	3.21	0.98

– = null.
[1 kN-m = 8.851 in.-kip]

The comparisons presented in Tables 3 and 4 reveal that expressions concerning the torsional moment at cracking and at ultimate for SFRC beams require further refinement since noticeable discrepancies between predictions and test results are observed.

Furthermore, the analytical calculations of the torsional moment at cracking of the strengthened beams with C-FRP sheets are substantially lower than the experimentally obtain values. Although the conservative character of design criteria is rather anticipated, it is noted that the consideration of safety factors would further increase this difference.

DESIGN METHODOLOGY AND EXAMPLES

Proposed design procedure

The ability of the previously described expressions to be used for design purposes is investigated. The proposed design methodology aims to provide simple and safe calculation of the required non-conventional transverse torsional reinforcement as an alternative of the common steel stirrups. The purpose of this torsion design is the total or the partial replacement of closed stirrups with (a) continuous rectangular steel spirals, or (b) externally bonded C-FRP sheets, or (c) short steel fibers. The procedure is based on the ultimate torsional moment capacity of a solid RC beam with rectangular cross-section and includes the following steps for the three examined cases of non-conventional reinforcement:

Data: Cross-section dimensions, material properties, longitudinal bars and stirrups calculated by the design of a solid, rectangular RC beam subjected to a given imposed pure torsional moment according to the known code provisions of ACI 318-19[37] or EC2[38].

Step 1: Calculation of the actual crack angle of inclination, $a_{act.}$, based on the given longitudinal steel bars and transverse stirrups:

$$\tan\alpha_{act.} = \sqrt{\frac{A_{T,st.} f_{Ty,st.} p_o}{A_L f_{Ly} s}} \quad (13)$$

Step 2: Calculation of the actual ultimate torsional moment capacity, $T_{u,act.}$, based on the provided longitudinal reinforcement:

$$T_{u,act.} = \frac{2A_o A_L f_{Ly}}{p_o} \tan a_{act.} \tag{14}$$

Step 3a: Calculation of the required spacing of the continuous rectangular steel spiral reinforcement for total replacement of the given common individual closed steel stirrups:

$$T_T \geq T_L = T_{u,act.} \rightarrow \frac{2A_o F_{T,l}}{p_o} \tan a_{act.} + \frac{2A_o F_{T,t}}{s} \cot a_{act.} \geq \frac{2A_o F_L}{p_o} \tan a_{act.} \Rightarrow$$
$$\Rightarrow \frac{F_{T,l}}{p_o} \tan a_{act.} + \frac{F_{T,t}}{s} \cot a_{act.} \geq \frac{F_L}{p_o} \tan a_{act.} \Rightarrow s \leq \frac{p_o}{\tan^2 a_{act.}} \frac{A_{T,SP} f_{Ty,SP}}{\left(A_L f_{Ly} - A_{T,SP} f_{Ty,SP} \frac{\cos\varphi}{\sin\varphi}\right)} \tag{15}$$

Step 3b: Calculation of the required externally bonded C-FRP sheets for total replacement of the given common stirrups:

$$\left(n_{FRP} \cdot t_{FRP}\right) f_{T,\mathrm{FRP}} \geq \frac{T_{u,act.}}{2A_o \cot a_{act.}} = \frac{A_L f_{Ly}}{p_o} \tan^2 a_{act.} \tag{16}$$

Step 3c: Calculation of the required short steel fibers in terms of the steel fiber factor, F, for total (Eq. 17a) or partial (Eq. 17b) replacement of the given common stirrups:

$$F \geq \frac{T_{u,act.} - 0.13b^2 h\sqrt{f_c}}{0.22\frac{2A_o}{p_o} bh\sqrt{f_c}} \quad , \quad F \geq \frac{T_{u,act.} - 0.13b^2 h\sqrt{f_c} - k_2 \frac{b_{st.} h_{st.}}{s} A_{T,st.} f_{Ty,st.}}{0.22\frac{2A_o}{p_o} bh\sqrt{f_c}} \tag{17a and b}$$

Design example

An example of a solid reinforced concrete rectangular beam under pure torsion from the literature[49] is selected to illustrate the steps involved in torsion design. As shown in Fig. 6, the cross-sectional dimensions of the beam are b = 300 mm (12 in.) and h = 500 mm (20 in.). The concrete compressive strength is f_c = 20 MPa (2900 psi) and the steel yield strength of both bars and stirrups is f_y = 420 MPa (60,000 psi). The imposed design torsional moment is T_u = 30 kN·m (266 in.-kip) without taking into account the strength reduction factor. Concrete cover is 40 mm (1.5 in.) from exterior face to stirrup centerline, thus x_o = 220 mm (9 in.) and y_o = 420 mm (17 in.) are the horizontal and vertical dimension of the centerline of outermost closed transverse torsional reinforcement, respectively.

The required and provided amount of longitudinal steel bars and vertical closed steel stirrups based on the design solution using the following code provisions are[49]:

ACI 318:

Longitudinal bars:	Required:	780.8 mm^2 (1.18 in.^{2})
	Provided:	6∅14 (6 No. 5) 924 mm^2 (1.88 in.^{2})
Closed stirrups:	Required:	0.61 mm^2/mm (0.0227 in.^{2}/in.)
	Provided:	∅8/80 mm (No. 3 at 4.50 in.) 0.625 mm^2/mm (0.0245 in.^{2}/in.) or: No. 2 at 2.00 in. (0.0245 in.^{2}/in.)

EC2:

Longitudinal bars:	Required:	880 mm^2 (1.37 in.^{2})
	Provided:	6∅14 (6 No. 5) 923 mm^2 (1.86 in.^{2})
Closed stirrups:	Required:	0.36 mm^2/mm (0.0140 in.^{2}/in.)
	Provided:	∅8/125 mm (No. 3 at 7.10 in.) 0.40 mm^2/mm (0.0155 in.^{2}/in.) or: No. 2 at 3.10 in. (0. 0155 in.^{2}/in.)

Step 1: Calculation of the actual crack angle of inclination, $a_{act.}$, based on the given longitudinal steel bars and transverse stirrups:

ACI 318:
$$\tan\alpha_{act.} = \sqrt{\frac{50\times420\times1280}{924\times420\times80}} = 0.93 \rightarrow \alpha_{act.} = 43°$$

EC2:
$$\tan\alpha_{act.} = \sqrt{\frac{50\times420\times1225}{924\times420\times125}} = 0.73 \rightarrow \alpha_{act.} = 36°$$

Step 2: Calculation of the actual ultimate torsional moment capacity, $T_{u,act.}$, based on the provided longitudinal reinforcement:

ACI 318:
$$T_{u,act.} = \frac{2\times(0.85\times92400)\times924\times420}{1280}0.93\text{N·mm} = 44.4 \text{ kN·m (393 in.-kip)}$$

EC2:
$$T_{u,act.} = \frac{2\times83789\times924\times420}{1225}0.73 \text{ N·mm} = 38.8 \text{ kN·m (343 in.-kip)}$$

Step 3a: Calculation of the required spacing of the continuous rectangular steel spiral reinforcement for total replacement of the given common individual closed steel stirrups:

ACI 318:
$$s \leq \frac{1280}{0.93^2}\frac{50\times420}{\left(924\times420-50\times420\frac{\cos 45°}{\sin 45°}\right)} = 85 \text{ mm} = (3.35 \text{ in.})$$

Spirals with inclination 45°: ∅8/85 mm (No. 2 at 2.10 in.) 0.59 mm²/mm (0.0234 in.²/in.)

EC2:
$$s \leq \frac{1225}{0.73^2}\frac{50\times420}{\left(924\times420-50\times420\frac{\cos 45°}{\sin 45°}\right)} = 132 \text{ mm} = (5.20 \text{ in.})$$

Spirals with inclination 45°: ∅8/132 mm (No. 2 at 3.30 in.) 0.38 mm²/mm (0.0150 in.²/in.)

Step 3b: Calculation of the required externally bonded C-FRP sheets for total replacement of the given common stirrups:

ACI 318:
$$(n_{FRP}\cdot t_{FRP})f_{T,FRP} \geq \frac{924\times420}{1280}0.93^2 = 264 \text{ N/mm (18.09 kip/ft)}$$

One ply (n_{FRP} = 1) of unidirectional C-FRP sheets with thickness t_{FRP} = 0.22 mm (0.0087 in.) per ply as external transverse reinforcement with elastic modulus E_{FRP} = 230 GPa (33359 ksi), ultimate elongation of the fibers at failure $\varepsilon_{u,FRP}$ = 1.5% and stress of the sheets at failure due to the fiber rupture $f_{T,FRP}$ = 1.2 GPa (175 ksi).

EC2:
$$(n_{FRP}\cdot t_{FRP})f_{T,FRP} \geq \frac{924\times420}{1225}0.73^2 = 169 \text{ N/mm (11.58 kip/ft)}$$

One ply (n_{FRP} = 1) of unidirectional C-FRP sheets with thickness t_{FRP} = 0.14 mm (0.0055 in.) per ply as external transverse reinforcement with elastic modulus E_{FRP} = 230 GPa (33359 ksi), ultimate elongation of the fibers at failure $\varepsilon_{u,FRP}$ = 1.5% and stress of the sheets at failure due to the fiber rupture $f_{T,FRP}$ = 1.2 GPa (175 ksi).

Step 3c: Calculation of the required short steel fibers in terms of the steel fiber factor, F, for partial replacement of the given common stirrups for the example according to ACI 318-19[37] or for total replacement of the stirrups for the example according to EC2[38]:

ACI 318:
$$F \geq \frac{44.4\times10^6-0.13\times300^2\times500\sqrt{20}-1.588\frac{220\times420}{280}50\times420}{0.22\frac{2\times(0.85\times92400)}{1319}300\times500\sqrt{20}} = 0.374$$

Closed steel stirrups: ∅8/280 mm (No. 2 at 6.90 in.) 0.180 mm²/mm (0.0071 in.²/in.) combined with short hooked-ended steel fibers with length 30 mm (1.18 in.) and diameter 0.8 mm (0.03 in.) in volume fraction ρ_{SF} = 1.0 % and F = 1×1.0%×30/0.80 = 0.375.

<u>EC2:</u> $$F \geq \frac{38.8\times10^6 - 0.13\times300^2\times500\sqrt{20}}{0.22\frac{2\times83789}{1225}300\times500\sqrt{20}} = 0.624$$

Short hooked-ended steel fibers with length 30 mm (1.18 in.) and diameter 0.8 mm (0.03 in.) in volume fraction ρ_{SF} = 1.7 % and F = 1×1.7%×30/0.80 = 0.6375 without steel stirrups.

The data and the derived design results for all the aforementioned cases with the examined reinforcement configurations are summarized and compared in Table 5.

Table 5–Numerical example - Design results

Case	Data and reinforcement arrangements	ACI 318	EC2
	Data		
Common for all cases 1, 2, 3 and 4	b =	300 mm (12 in.)	
	h =	500 mm (20 in.)	
	f_c =	20 MPa (2900 psi)	
	f_y =	420 MPa (60,000 psi)	
	$a_{act.}$	43°	36°
	$T_{u,act.}$	55.7 kN·m (493 in.-kip)	38.8 kN·m (343 in.-kip)
	Longitudinal reinforcement		
Common for all cases 1, 2, 3 and 4	Steel reinforcing bars	6∅14 (6 No. 5)	6∅14 (6 No. 5)
	Transverse reinforcement		
Case 1	Common closed steel stirrups	∅8/80 mm (No. 2 at 2.00 in.)	∅8/125 mm (No. 2 at 3.10 in.)
Case 2	Continuous rectangular steel spirals with 45° inclination	∅8/85 mm (No. 2 at 2.10 in.)	∅8/132 mm (No. 2 at 3.30 in.)
Case 3	C-FRP sheets (One ply, E_{FRP} = 230 GPa (33359 ksi), $\varepsilon_{u,FRP}$ = 1.5%)	t_{FRP} = 0.22 mm (0.0087 in.)	t_{FRP} = 0.14 mm (0.0055 in.)
Case 4	Short hooked-ended steel fibers l_{SF} = 30 mm (1.18 in.) and d_{SF} = 0.8 mm (0.03 in.)	ρ_{SF} = 1.0 % and stirrups ∅8/280 mm (No. 2 at 6.90 in.)	ρ_{SF} = 1.7 %

CONCLUSIONS

The results of this study indicate the following concluding remarks:

- The overall torsional response of non-conventionally reinforced concrete beams is strongly influenced by the type of the provided transverse reinforcement. The application of epoxy bonded C-FRP sheets in the transverse direction proved a very effective external reinforcement against torsion. Strengthened beams with C-FRP sheets wrapping around the cross-section along their entire length exhibited significantly higher strength with compared to the corresponding pilot specimens and the other beams of the test program.
- The use of continuous steel spirals with rectangular shape instead of common closed steel stirrups is a promising alternative transverse reinforcement configuration for beams under torsion. Spirals with locking effect due to the

favorably imposed twist provided increased torsional capacity. Further, spirals are more beneficial than stirrups in construction since proper bending formation, anchorage by hook on both ends and installation of every individual closed stirrup is a labor-intensive and time-consuming process, whereas spiral reinforcement is easy-to-apply. Application of continuous spirals also increases confinement, anchorage efficiency and reinforcement cage stability.

- The addition of short steel fibers with high volume fraction (3 %) in the concrete mass proved to be essential for the beam without stirrups since fibers as the only transverse reinforcement improved torsional response and increased strength especially after concrete cracking, whereas the corresponding pilot specimens did not exhibit post-cracking behavior. Beams with steel fibers and stirrups showed even more enhanced twist and ductility capabilities.
- Analytical relationships for the prediction of the torsional moment at cracking and at ultimate of all the examined beams have also been proposed in this study. The different type of the provided non-conventional transverse reinforcement and their influence on the torsional response has been considered. Known design equations have been modified properly to take into account the contribution of spirals, C-FRP sheets and steel fibers to the torsional strength. Comparisons between predicted results yielded from the proposed expressions proved to be in good compliance with the experimental ones.
- A feasible analytical procedure has been proposed for the design of concrete beams with non-conventional transverse reinforcement under torsion and numerical examples have also been presented to illustrate the application of the methodology.

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TORSION OF RECTANGULAR CONCRETE SECTIONS

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Synopsis: The paper deals with torsion of rectangular concrete sections. The pre-cracking stage and post-cracking stage are discussed. The various design procedures are briefly mentioned and compared. The deficiencies of some methods are identified and discussed. The major part of the paper deals with the results of an experimental program executed at the Czech Technical University. The large-scale elements were tested under loading by torsion and by interaction of torsion and compression. The results showed that the effect of the compression force on the load carrying capacity of the elements in torsion differs according to the stage of performance. While at the pre-cracking stage the contribution of the compression is rather significant, when approaching the failure, it becomes reduced. Simplified technical methods of design of reinforcement were also discussed. It has been proved that the effect of the angle of the compressed diagonal in code models is rather important. The study showed that this effect is sometimes overestimated. Finally, in conclusions, some recommendations for future research are proposed.

Keywords: arch bridge, concrete, cracking, rectangular section, reinforcement, torsion.

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INTRODUCTION

Torsion of concrete sections is not usually a primary problem solved in the design of concrete structures. However, there are some cases where torsion became a governing problem for the entire design. The authors were involved in design of a highway bridge crossing a valley close to the capital of the Czech Republic, Prague. The arch bridge with the span of the arch 120 m (393.7 ft) was built between 1939 and 1949. There are two identical arch bridge structures, each for one direction of the highway (Fig. 1). The bridge was not used for many years, since the highway was not completed. In 1969 the highway was started to be built and the earlier built bridges were slightly reconstructed so that they could serve for traffic starting in 70th. The conditions for the layout and for the width of the highway changed since 1949, and the bridges had to be widened, so that it could accommodate two regular lanes in each direction. Now almost after 50 years of exploitation of the bridge, there is a necessity to extend the width of the bridge from two to three lanes. The bridge decks are rather near each other (Fig. 2), it is possible to widen the bridge deck only to the external sides, which results in eccentric loading of the arches. The contemporary width of the two lanes is 9.75 m (32 ft) and for three lanes 13.25 m (43.5 ft) is required as a minimum. If such a widening was designed, the arches would be subjected to significant loading in torsion. It was necessary to check, if the torsion can be taken by the arches.

The transversal reinforcement in the arch was rather weak. It was not surprising, since the lack of steel in 40th led to high prices of steel and the designers were forced to keep the costs as low as possible. On the other hand, good quality of concrete was observed, and replacement of the arches was considered as unnecessary. The high ultimate load carrying capacity of the arch in torsion was achieved when the full section was active, i.e. prior to concrete cracking. If the twisting angle increased, the torsional moment carrying capacity dropped down and never reached the capacity of the plain concrete section in torsion (a similar response is plotted in Fig. 9).