

EXPERIMENTAL RESULTS VERSUS THEORETICAL PREDICTIONS

Experimental pullout tests on straight stainless steel fibers embedded in normal strength matrix were conducted. The mix proportion used was - water:cement:sand:aggregate = 0.50:1.0:2.5:2.0. Cement used was of CSA Type 10 specification. The above mix proportion gave an average 28-day compressive strength of 36 MPa. Calibration procedure for interfacial properties described in reference (31) (not described here due to space limitation) yielded the following values: $\tau_s = -2.4$ MPa, $\sigma_c = -29.3$ MPa, and $\mu = 0.085 e^{-0.7 p_d} + 0.035$. Note that the coefficient of friction, μ takes the form of the following evolution law: $\mu = (\mu_i - \mu_{ss}) e^{-cp_d} + \mu_{ss}$, where μ_i is initial coefficient of friction, μ_{ss} is steady state value of coefficient of friction attained at large pullout distances, and c is a constant that governs the rate at which coefficient of friction decays with increase in pullout distance. This equation depicts that the coefficient of friction, μ decreases exponentially with increase in pullout distance. The decrease in coefficient of friction is attributable to the matrix wear and consequent smoothing of interface layer taking place during the process of fiber pullout. It must also be pointed here that both the contact stress and the coefficient of friction, among other parameters are also a function of Poisson's ratio of fiber.

Figure 2.1 compares the experimental pullout response with the theoretical - a good correspondence between the theoretical prediction and the experimental curves is noticeable. Theoretically predicted peak pullout load and the displacement corresponding to the peak pullout load are compared with the experimental results in the Table 1.1 - model predicted results agree well with the experimental results.

Figure 2.2 compares the theoretical prediction with the experimental fiber pullout response reported by Naaman and Shah (18) for a steel fiber. Embedded fiber length and fiber diameter were 12.5 mm and 0.25 mm, respectively. Following interfacial properties are assumed to obtain theoretical prediction: $\tau_s = 2.4$ MPa, $\sigma_c = 29.3$ MPa, and $\mu = 0.085 e^{-0.7 p_d} + 0.035$ (i.e., $\mu_i = 0.12$ and $\mu_{ss} = 0.035$). In the figure, it can be noticed that good agreement between the theoretical and the experimental response is obtained.

Figure 2.3 compares the theoretical prediction with the experimental fiber pullout response reported by Wang et al. (7) for a straight, smooth polypropylene fiber. Embedded fiber length and fiber diameter were 50 mm and 0.508 mm, respectively. Following interfacial properties are assumed to obtain the theoretical prediction: $\tau_s = 0.61$ MPa, $\sigma_c = 11.0$ MPa, and $\mu = 0.02 e^{-0.3 p_d} + 0.03$ (i.e., $\mu_i = 0.05$ and $\mu_{ss} = 0.03$). In the figure it can be seen that the theoretical prediction matches quite well with the experimental response. It must be noted that the above values of interfacial properties were calibrated using the

experimental pullout test results for polypropylene fibers. These tests were carried out as part of the ongoing research program.

PARAMETRIC STUDIES

From the viewpoint of optimizing properties of fiber reinforced concrete composites, it is critical to identify the relative significance of various interfacial properties on the fiber pullout response. Thus, in the following, influence of interfacial properties on progressive debonding behavior and fiber pullout response is investigated using the proposed progressive debonding model. In this context, parametric studies are carried out for the following interfacial properties:

- Adhesional bond strength, τ_s
- Interfacial contact stress, σ_c
- Interfacial coefficient of friction, μ

Influence of the aforementioned interfacial properties is investigated for two fiber types - the first fiber has an elastic modulus of 210 GPa (\cong steel) and that for the second fiber is 3.5 GPa (\cong polypropylene). For both fiber types, the total fiber length, L is taken as 50 mm (i.e., embedded length=25 mm), and the fiber diameter, d is taken as 1.0 mm.

Influence of Adhesional Bond Strength, τ_s

Fiber Elastic Modulus, $E_f = 210$ GPa -- Parametric studies are carried out for three different values of adhesional bond strength, τ_s : -1 MPa, -5 MPa and -10 MPa. Assumed values of the other interfacial properties are: $\sigma_c = -15.0$ MPa and $\mu = 0.065 e^{-0.7 P_d} + 0.035$ (i.e., $\mu_t = 0.1$ and $\mu_{ss} = 0.035$). Mechanical properties of fiber are assumed as: Elastic modulus, $E_f = 210$ GPa and Poisson's ratio, $\nu_f = 0.20$; the same for matrix are assumed as 30 GPa and 0.30, respectively.

Figure 3.1a compares the fiber pullout response for three different values of adhesional bond strength, τ_s . The initial linear part of the pullout curves seen in the figure depicts elastic loading of fibers. In the linear region, the interfacial shear stress at any point along the embedded fiber length remains below the interfacial adhesional bond strength, τ_s (consequently, interface along the entire embedded length remains fully bonded). In the non-linear region of pullout curve (following the linear region), fiber is partially debonded and its pullout is resisted by adhesional shear stresses acting over the bonded interface and frictional shear stresses acting over the debonded interface. This non-linear region is termed as the region of progressive debonding, since, interfacial debonding initiates and continues in a progressive fashion, (i.e., pullout load increases with increase in

debond length). The reason for increase in pullout load during progressive debonding is that the rate of increase in frictional component of pullout load with change in debond length is greater than the corresponding rate of decrease in adhesional component of the pullout load, i.e., $d\sigma_{o,fric} / dl_d \geq d\sigma_{o,bond} / dl_d$. Peak pullout load is attained when the rate of increase in frictional component of pullout load equals the corresponding rate of decrease in adhesional component of pullout load. Beyond the peak pullout load, the remaining bonded portion of the interface debonds in a catastrophic manner, i.e., no increase in pullout load is required to debond the interface. The reason for this drop in pullout load is that the rate of increase in frictional component of pullout load with change in debond length becomes smaller than the corresponding rate of decrease in the adhesional component of pullout load. It can also be noted that the drop in pullout load is accompanied by decrease in pullout displacement. The theoretically predicted decrease in pullout displacement is not observed in the experiments, since, the pullout tests are normally carried out at a constant rate of pullout displacement. After the completion of interfacial debonding, pullout curves are identical for different magnitudes of adhesional bond strength, τ_s . Particularly noteworthy is the fact that, the variation in the area under the pullout curve (representing energy absorbed during fiber pullout) for the different cases of adhesional bond strength are insignificantly small. This observation is important since it depicts that composite toughness can not be significantly improved solely by increasing the adhesional bond strength, τ_s . On the other hand, the objective function of increasing composite strength can be achieved by increasing the adhesional bond strength, τ_s in the case of high modulus fibers.

In the Figure 3.1b, the pullout load at initial debonding and the peak pullout load are plotted as a function of adhesional bond strength, τ_s . Both the pullout load at initial debonding and the peak pullout load increase with increase in adhesional bond strength, however, the former increases at a greater rate than the latter. In the Figure 3.1c, pullout displacement at the peak pullout load is plotted as a function of adhesional bond strength, τ_s . It can be noted that the pullout displacement at the peak pullout load increases with increase in adhesional bond strength, τ_s .

After initial debonding, further interfacial debonding requires the applied pullout load to overcome the interfacial frictional shear stresses at the debonded interface and adhesional shear stresses at the bonded interface. As a result, the pullout load required to further debond the interface depends upon the extent of prior debonding. Figure 3.1d shows variation in pullout load as a function of debond length for the case when $\tau_s = -5$ MPa. In the same figure, components of pullout load, i.e., the adhesional and the frictional components are also plotted. Following points can be noted in the figure:

- Interfacial debonding initiates at the location where fiber enters the matrix. At initiation of debonding the fiber pullout load (i.e., the initial debonding load) is equal to the adhesional component of pullout load, since, the frictional component of pullout load is equal to zero.

- With increase in debond length, fiber pullout load continues to increase until debond length corresponding to peak pullout load is attained, and thereafter, fiber pullout load begins to decrease. Moreover, the adhesional component of pullout load decreases and the frictional component of pullout load increases with increase in debond length.
- The peak pullout load on the pullout load vs. debond length curve corresponds to the point at which the slope of the curve becomes zero. This condition is satisfied when the slope of the adhesional component of pullout load vs. debond length curve becomes equal and opposite to that of the frictional component of pullout load vs. debond length curve. And, the debond length corresponding to this point is termed as the catastrophic debond length, $l_{d,cat}$, since the debonding process turns catastrophic upon further debonding.

Figure 3.1e shows pullout load and its components as a function of debond length at different values of adhesional bond strength, τ_s . From this figure following important observations can be made:

- Pullout load corresponding to any given debond length increases with increase in adhesional bond strength, τ_s . However, at complete debonding, magnitude of pullout load is independent of adhesional bond strength, τ_s .
- Prior to complete debonding, the adhesional component of pullout load increases with increase in adhesional bond strength, on the other hand, the frictional component of pullout load decreases with increase in adhesional bond strength.
- Catastrophic debonding takes place at $\tau_s = -5.0$ MPa and -10 MPa, on the other hand, at $\tau_s = -1.0$ MPa debonding process is completely stable, i.e., pullout load continues to increase until the fiber is completely debonded.
- For a given fiber length, catastrophic debond length, $l_{d,cat}$ decreases with increase in the adhesional bond strength, τ_s .

Figure 3.1f shows variation in axial load distribution at completion of debonding for different values of adhesional bond strength, τ_s . It can be noted that the axial load distribution along the fiber length is independent of adhesional bond strength, τ_s . It can also be noted that the fiber axial load is maximum at the loaded fiber end and it decreases almost linearly to a value of zero at the embedded fiber end. Figure 3.1g shows interfacial shear stress distribution at completion of interfacial debonding. It can be noted that the interfacial shear stress distribution along the embedded fiber length is independent of adhesional bond strength, τ_s . Moreover, interfacial shear stress is maximum at the embedded fiber end and it gradually decreases towards the exit fiber end. Poisson's contraction of fiber is responsible for the observed interfacial shear stress distribution.

Fiber Elastic Modulus, $E_f = 3.5$ GPa -- Parametric studies are carried out for three different values of adhesional bond strength, τ_s : -1 MPa, -5 MPa and -10 MPa. The assumed values of other interfacial properties are chosen as: $\sigma_c = -15.0$

MPa and $\mu = 0.02 e^{-0.3 p_d} + 0.03$ (i.e., $\mu_i = 0.05$ and $\mu_{ss} = 0.03$). Mechanical properties of fiber are assumed as: Elastic modulus, $E_f = 3.5$ GPa and Poisson's ratio, $\nu_f = 0.35$; the same for matrix are assumed as 30 GPa and 0.30, respectively.

Figure 3.2a compares the pullout response of fibers with $E_f = 3.5$ GPa at three different values of adhesional bond strength, τ_s . In this figure it can be seen that the difference between the pullout responses at three different values of adhesional strength, τ_s is relatively insignificant. At $\tau_s = -1$ MPa the debonding process is completely stable. On the other hand, debonding process turns catastrophic at $\tau_s = -5$ MPa and -10 MPa, however the load drop during catastrophic debonding is insignificant in comparison to that observed with the high modulus fibers.

In the Figure 3.2b, the pullout load at initial debonding and the peak pullout load are plotted as a function of adhesional bond strength, τ_s . Both the pullout load at initial debonding and the peak pullout load increase with increase in adhesional bond strength, and similar to high modulus fibers, the rate of increase of the latter is smaller to that of the former. Comparing Figures 3.1b and 3.2b it can be noted that for any given adhesional bond strength, the initial debonding load for low modulus fibers is considerably smaller than that for high modulus fibers, and this disparity increases with increase in adhesional bond strength. In the Figure 3.2c, displacement at the peak pullout load is plotted as a function of adhesional bond strength, τ_s , and it can be noted that the displacement at peak pullout load increase with increase in adhesional bond strength, τ_s . Moreover, the displacement at peak pullout load for low modulus fibers are considerably greater than that for high modulus fibers.

Figure 3.2e shows pullout load and its components as a function of debond length at different values of adhesional bond strength, τ_s . Variation in the pullout load and its components with increase in debond length is similar to that for high modulus fibers. Following important observations can be made from this figure:

- Pullout load corresponding to any given debond length increases with increase in adhesional bond strength, τ_s . However, at complete debonding, magnitude of pullout load is independent of adhesional bond strength, τ_s . Prior to complete debonding, the adhesional component of pullout load increases with increase in adhesional bond strength, on the other hand, the frictional component of pullout load decreases with increase in adhesional bond strength.
- When adhesional bond strength is increased from -1 MPa to -10 MPa, increase in the peak pullout load for high modulus fiber is about 49%, on the other hand, the same for low modulus fiber is only about 7%. The reason for this disparity is that for low modulus fibers much of the increase in the adhesional component of pullout load obtained with increase in adhesional bond strength is compensated by the corresponding decrease in the frictional component of pullout load. From the viewpoint of optimization of interfacial

properties this observation is significant, since, it demonstrates that efficiency of low modulus fibers cannot be improved significantly solely by increasing adhesional bond strength.

- Other parameters remaining same, catastrophic debond length, $l_{d,cat}$ decreases with increase in the adhesional bond strength, τ_s . Also, catastrophic debond length is much smaller for low modulus fibers in comparison to their high modulus counterparts.

Figure 3.2f shows variation in axial load distribution at completion of debonding for different values of adhesional bond strength, τ_s . In the figure it can be seen that axial load distribution along the fiber length is independent of adhesional bond strength, τ_s . It can also be noticed that fiber axial load is maximum at the loaded fiber end and it decreases to a value of zero at the embedded fiber end. Figure 3.2g shows interfacial shear stress distribution at completion of interfacial debonding. It can be seen that interfacial shear stress is maximum at the embedded fiber end, and it decreases towards the exit fiber end. Poisson's contraction of fiber is responsible for the observed variation in shear stress distribution. Comparing Figures 3.1g and 3.2g it can also be observed that for low modulus fiber the rate of decrease in interfacial shear stress along the fiber length is much greater than that in the case of high modulus fibers.

Influence of Interfacial Contact Stress, σ_c

Fiber Elastic Modulus, $E_f = 210$ GPa -- Parametric studies are carried out for three different values of interfacial contact stress, σ_c : -5 MPa, -15 MPa and -30 MPa. Assumed values of other interfacial properties are: $\tau_s = -1.0$ MPa and $\mu = 0.065 e^{-0.7 p_i} + 0.035$ (i.e., $\mu_i = 0.1$ and $\mu_{ss} = 0.035$). Mechanical properties of fiber and matrix are same as assumed earlier. In cement based composites, the magnitude of normal contact stress acting at the fiber-matrix interface is dependent upon the degree of matrix shrinkage and the magnitude of external confining stresses present, if any.

The prepeak part of pullout curves and the complete pullout curves for different values of interfacial contact stress, σ_c are shown in Figures 3.3a and 3.3b, respectively. It can be noticed that the prepeak pullout curves become nonlinear at very small value of pullout loads. It can also be noticed that both the peak pullout load and the displacement at peak pullout load increase with increase in interfacial contact stress, σ_c . In the latter figure it can be noticed that the postpeak pullout response varies greatly at different values of interfacial contact stress, σ_c . In particular, both the postpeak pullout loads and the area under the pullout curve increase with increase in interfacial contact stress, σ_c . Increase in the latter depicts that the energy absorbed during the process of fiber pullout increases with increase in interfacial contact stress, σ_c .

Given the dependence of pullout performance on interfacial contact stress, σ_c , two approaches can be resorted for improving fiber efficiency:

- Using matrix that shrinks more during curing, setting and hardening so that contact stress, σ_c of greater magnitude is generated at the interface.
- Intelligently designing fiber such that interfacial contact stress, σ_c increases during the process of fiber pullout

Figure 3.3c shows pullout load and its components as a function of debond length at different values of interfacial contact stress, σ_c . In the figure it can be seen that pullout load corresponding to any given debond length increases with increase in interfacial contact stress, σ_c . And, this increase in pullout load is attributable to the increase in the frictional component of pullout load.

Figure 3.3d shows variation in axial load distribution at the completion of debonding for different values of interfacial contact stress, σ_c . It can be seen that the axial load along the embedded fiber length increases with increase in interfacial contact stress, σ_c . Figure 3.3e shows the interfacial shear stress distribution at completion of interfacial debonding, and it can be seen that the magnitude of interfacial shear stress increases with increase in interfacial contact stress, σ_c .

Elastic Modulus, $E_f = 3.5$ GPa -- Parametric studies are carried out for three different values of interfacial contact stress, σ_c : -5 MPa, -15 MPa and -30 MPa. Assumed values of other interfacial properties are: $\tau_s = -1.0$ MPa and $\mu = 0.02 e^{-0.3 P_d} + 0.03$ (i.e., $\mu_i = 0.05$ and $\mu_{ss} = 0.03$). Mechanical properties of fiber and matrix are same as assumed earlier.

The prepeak part of pullout curves and the complete pullout curves for different values of interfacial contact stress, σ_c are shown in Figures 3.4a and 3.4b, respectively. In the former figure it can be noticed that the prepeak pullout curves become nonlinear at very small value of pullout loads. It can also be noticed that both the peak pullout load and the displacement at peak pullout load increase with increase in interfacial contact stress, σ_c . In the latter figure it can be noticed that the postpeak pullout response varies greatly at different values of interfacial contact stress, σ_c . Similar to high modulus fibers, both the postpeak pullout loads and the area under the pullout curve increase with increase in interfacial contact stress, σ_c . Increase in the latter depicts that the energy absorbed during the process of fiber pullout increases with increase in interfacial contact stress, σ_c . It can also be noticed that at any given interfacial contact stress, σ_c , the peak pullout load for high modulus fibers is considerably greater than that for low modulus fibers. On the contrary, the displacement at the peak pullout load for low modulus fibers are much greater than those for high modulus fibers.

Figure 3.4c shows pullout load and its components as a function of debond length at different values of interfacial contact stress, σ_c . In the figure it can be seen that pullout load corresponding to any given debond length increases with increase in interfacial contact stress, σ_c . And, similar to high modulus fiber, this increase in pullout load is due to the increase in the frictional component of pullout load.

Figure 3.4d shows variation in axial load distribution at completion of debonding for different values of interfacial contact stress, σ_c . It can be seen that axial load along the embedded fiber length increases with increase in interfacial contact stress, σ_c . Figure 3.4e shows the interfacial shear stress distribution at completion of interfacial debonding, and it can be seen that interfacial shear stress increases with increase in interfacial contact stress, σ_c . It can also be observed that the rate of decay of interfacial shear stress along the embedded fiber length increases with increase in interfacial contact stress. Moreover, for low modulus fiber the rate of decay of interfacial shear stress along the embedded length is much greater than that in the case of high modulus fibers. High Poisson's contraction of low modulus fibers is responsible for the aforementioned observed interfacial shear stress distribution.

Influence of Interfacial Coefficient of Friction, μ

Fiber Elastic Modulus, $E_f = 210$ GPa -- Parametric studies are carried out by varying the initial coefficient of friction, μ_i . The three chosen values of μ_i are: 0.1, 0.25 and 0.50, and the corresponding evolution laws for coefficient of friction selected are: $\mu = 0.065 e^{-0.7 P_d} + 0.035$, $\mu = 0.215 e^{-0.7 P_d} + 0.035$, $\mu = 0.465 e^{-0.7 P_d} + 0.035$, respectively. Assumed values of the other interfacial properties are: $\tau_s = -1.0$ MPa and $\sigma_c = -15.0$ MPa. Mechanical properties of fiber and matrix are same as assumed earlier.

The prepeak part of pullout curves and the complete pullout curves for different values of interfacial coefficient of friction, μ are shown in Figures 3.5a and 3.5b, respectively. In the former figure it can be noticed that the prepeak pullout curves become nonlinear at very small value of pullout loads, and also that both the peak pullout load and the displacement at peak pullout load increase with increase in interfacial coefficient of friction, μ . In the latter figure it can be noticed that the postpeak pullout response varies greatly at different values of interfacial coefficient of friction, μ . In particular, postpeak pullout loads increase with increase in interfacial coefficient of friction, μ , and this increase translates into increased energy absorbed during the process of fiber pullout. From the viewpoint of optimization of interfacial properties this observation is important, since, it demonstrates that efficiency of high modulus fibers can be significantly

improved by increasing interfacial coefficient of friction.

Figure 3.5c shows variation in axial load distribution at completion of debonding for different values of interfacial coefficient of friction, μ . In the figure it can be seen that the axial load along the embedded fiber length increases with increase in interfacial coefficient of friction, μ . Figure 3.5d shows the interfacial shear stress distribution at completion of interfacial debonding. In the figure it can be noticed that interfacial shear stress increases with increase in interfacial coefficient of friction, μ . However, the rate of decay of interfacial shear stress along the embedded fiber length increases with increase in interfacial coefficient of friction, μ .

Fiber Elastic Modulus, $E_f = 3.5$ GPa -- Parametric studies are carried out by varying the initial coefficient of friction, μ_i . The three chosen values of μ_i are: 0.05, 0.25 and 0.50, and the corresponding evolution laws for coefficient of friction selected are: $\mu = 0.02 e^{-0.3P_i} + 0.03$, $\mu = 0.22 e^{-0.3P_i} + 0.03$, $\mu = 0.47 e^{-0.3P_i} + 0.03$, respectively. Assumed values of the other interfacial properties were: $\tau_s = -1.0$ MPa and $\sigma_c = -15.0$ MPa. Mechanical properties of fiber and matrix are same as assumed earlier.

The prepeak part of pullout curves and the complete pullout curves for different values of interfacial coefficient of friction, μ are shown in Figures 3.6a and 3.6b, respectively. In the former figure it can be noticed that the prepeak pullout curves become nonlinear at very small value of pullout loads. Also, increase in interfacial coefficient of friction, μ beyond a certain value does not produce any significant increase in peak pullout load. From the viewpoint of optimization of interfacial properties this observation is important, since, it demonstrates that efficiency of low modulus fibers cannot be significantly improved by solely increasing coefficient of friction.

Figure 3.6c shows variation in axial load distribution at completion of debonding for different values of interfacial coefficient of friction, μ . In the figure it can be seen that at large values of interfacial coefficient of friction, μ axial load remains almost constant along the major portion of embedded fiber length. Figure 3.6d shows the interfacial shear stress distribution at completion of interfacial debonding for different values of interfacial coefficient of friction, μ . It can be noticed that the peak value of interfacial shear stress increases with increase with increase in interfacial coefficient of friction, μ . In addition, the rate of decrease in shear stress increases with increase in the interfacial coefficient of friction, μ . Moreover, for low modulus fibers the interfacial shear stress along the embedded length decays more rapidly in comparison to that in the case of high modulus fibers.

CONCLUSIONS

1. A new mathematical model for the problem of fiber pullout is introduced in this paper. This model eliminates the major limitations of the earlier models and captures features such as, progressive interfacial debonding, Poisson's effect, and variation of interfacial properties with increase in pullout distance. Analysis is divided into three stages and for each stage closed-form solutions are derived for fiber pullout stress, fiber displacement, fiber axial load distribution and interfacial shear stress distribution. It is shown that coefficient of friction decays exponentially with increase in pullout distance. Furthermore, using this model it is possible to predict the entire pullout load versus displacement response.
2. Parametric studies are carried out by varying interfacial properties for fibers with two different elastic modulus (3.5 GPa and 210 GPa). Salient conclusions drawn from this study are:
 - Both the pullout load at initial debonding and the peak pullout load increase with increase in adhesional bond strength. For low modulus fibers these increases are insignificant relative to the high modulus fibers, which suggests that efficiency of the former cannot be improved significantly by the means of increasing adhesional bond strength alone. Moreover, for both high as well as low modulus fibers, increase in area under the pullout curve (representing energy absorbed during fiber pullout) brought about by increase in adhesional bond strength remains insignificant.
 - The peak pullout load increases with increase in interfacial contact stress. In addition, the pullout loads on the descending branch of pullout curve increase with increase in interfacial contact stress. This means that the energy absorbed during the process of fiber pullout increases with increase in interfacial contact stress. The above observations are found to be valid for fibers of different elastic modulus. Given the dependence of pullout performance on interfacial contact stress, two approaches can be resorted to improve fiber efficiency. First, using a matrix that shrinks more during curing, setting and hardening such that a higher value of interfacial contact stress is generated at the interface, and secondly, intelligently designing fiber such that interfacial contact stress increases during the process of fiber pullout.
 - Both the peak pullout load and the area under the pullout curve for high modulus fibers increase with increase in interfacial coefficient of friction. From the viewpoint of optimization of interfacial properties this observation is important, since, it demonstrates that efficiency of high modulus fibers can be improved significantly by increasing the interfacial coefficient of friction. On the other hand, for low modulus fibers, the peak pullout load initially increases and then it becomes constant with increase in interfacial coefficient of friction. Also, increase in area under the pullout curve is not significant. Again, from the viewpoint of optimization