### 16.4—Flexural strengthening of an interior reinforced concrete beam with near-surface-mounted FRP bars

An existing reinforced concrete beam (Fig. 16.4) is to be strengthened using the loads given in Table 16.3a and the near-surface-mounted (NSM) FRP system described in Table 16.4a. Specifically, three No. 3 carbon FRP (CFRP) bars are to be used at a distance 23.7 in. (602.1 mm) from the extreme top fiber of the beam.

By inspection, the degree of strengthening is reasonable in that it does meet the strengthening limit criteria put forth in Eq. (10.1). That is, the existing flexural strength without FRP, $(\phi M_n)_{w/o}$ = 266 kip-ft (361 kN-m), is greater than the unstrengthened moment limit, $(1.1M_{DL} + 0.75M_{LL})_{new}$ = 177 kip-ft (240 kN-m). The design calculations used to verify this configuration follow in Table 16.4b.

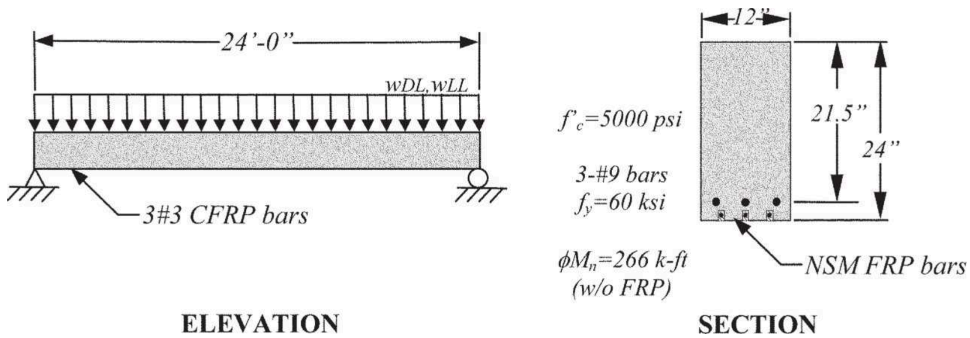


*Fig. 16.4—Schematic of the idealized simply supported beam with FRP external reinforcement.*

**Table 16.4a—Manufacturer's reported NSM FRP system properties**

| | | |
|---|---|---|
| Area per No. 3 bar | 0.10 in.$^2$ | 64.5 mm$^2$ |
| Ultimate tensile strength $f_{fu}$* | 250 ksi | 1725 N/mm$^2$ |
| Rupture strain $\varepsilon_{fu}$* | 0.013 in./in. | 0.013 mm/mm |
| Modulus of elasticity of FRP laminates $E_f$ | 19,230 ksi | 132,700 N/mm$^2$ |
| Length of the beam $\ell$ | 29 ft | 8.84 m |
| Bay width $l_2$ | 30 ft | 9.14 m |
| Width of beam $w$ | 24 in. | 610 mm |
| $d_p$ | 22.5 in. | 571 mm |
| $h$ | 25 in. | 635 mm |
| Effective flange width $b_f$ | 87 in. | 2210 mm |
| Flange thickness $h_f$ | 4 in. | 102 mm |
| $f_c'$ | 4000 psi | 27.6 N/mm$^2$ |
| Strands diameter | 1/2 in. | 12.7 mm |
| $f_{pe}$ | 165 ksi | 1138 N/mm$^2$ |
| $f_{py}$ | 230 ksi | 1586 N/mm$^2$ |
| $f_{pu}$ | 270 ksi | 1860 N/mm$^2$ |
| $E_p$ | 28,500 ksi | $1.96 \times 10^5$ N/mm$^2$ |
| $\phi M_n$ without FRP | 336 kip-ft | 455 kN-m |

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## Table 16.4b—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 1—Calculate the FRP system design material properties**<br><br>The beam is located in an interior space and a CFRP material will be used. Therefore, per Table 9.4, an environmental reduction factor of 0.95 is suggested.<br><br>$f_{fu} = C_E f_{fu}^*$<br><br>$\varepsilon_{fu} = C_E \varepsilon_{fu}^*$ | $f_{fu}$ = (0.95)(250 ksi) = 237.5 ksi<br><br>$\varepsilon_{fu}$ = (0.95)(0.013 in./in.) = 0.0123 in./in. | $f_{fu}$ = (0.95)(1725 N/mm$^2$) = 1639 N/mm$^2$<br><br>$\varepsilon_{fu}$ = (0.95)(0.013 mm/mm) = 0.0123 mm/mm |
| **Step 2—Preliminary calculations**<br><br>Properties of the concrete:<br><br>$\beta_1$ from ACI 318-14, Section 22.2.2.4.3<br><br>$E_c = 57{,}000\sqrt{f_c'}$ | $\beta_1 = 1.05 - 0.05\dfrac{f_c'}{1000} = 0.85$<br><br>$E_c = 57{,}000\sqrt{5000\text{ psi}} = 4{,}030{,}00\text{ psi}$<br><br>$A_s$ = 3(1.00 in.$^2$) = 3.00 in.$^2$<br><br>$A_f$ = (3 bars)(0.01 in.$^2$/bar) = 0.3 in.$^2$ | $\beta_1 = 1.05 - 0.05\dfrac{f_c'}{6.9} = 0.85$<br><br>$E_c = 4700\sqrt{34.5\text{ N/mm}^2} = 27{,}600\text{ N/mm}^2$<br><br>$A_s$ = 3(645.2 mm$^2$) = 1935 mm$^2$<br><br>$A_f$ = (3 bars)(64.5 mm$^2$/bar) = 194 mm$^2$ |
| **Step 3—Determine the existing state of strain on the soffit**<br><br>The existing state of strain is calculated assuming the beam is cracked and the only loads acting on the beam at the time of the FRP installation are dead loads. A cracked section analysis of the existing beam gives $k$ = 0.334 and $I_{cr}$ = 5937 in.$^4$ = 2471 × 10$^6$ mm$^4$<br><br>$\varepsilon_{bi} = \dfrac{M_{DL}(d_f - kd)}{I_{cr}E_c}$ | $\varepsilon_{bi} = \dfrac{(864\text{ kip-in.})[23.7\text{ in.} - (0.334)(21.5\text{ in.})]}{(5937\text{ in.}^4)(4030\text{ ksi})} = 0.00061$ | $\varepsilon_{bi} = \dfrac{(97.6\text{ kN-mm})[602\text{ mm} - (0.334)(546\text{ mm})]}{(2471\times10^6\text{ mm}^4)(27.6\text{ kN/mm}^2)} = 0.00061$ |
| **Step 4—Determine the bond-dependent coefficient of the FRP system**<br><br>Based on the manufacturer's recommendation, the dimensionless bond-dependent coefficient for flexure $\kappa_m$ is 0.7. | $\kappa_m$ = 0.7 | $\kappa_m$ = 0.7 |
| **Step 5—Estimate *c*, the depth to the neutral axis**<br><br>A reasonable initial estimate of $c$ is 0.20$d$. The value of the $c$ is adjusted after checking equilibrium.<br><br>$c = 0.20d$ | $c$ = (0.20)(21.5 in.) = 4.30 in. | $c$ = (0.20)(546 mm) = 109 mm |

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## Table 16.4b (cont.)—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 6—Determine the effective level of strain in the FRP reinforcement** | | |
| The effective strain level in the FRP may be found from Eq. (10.2.5). | | |
| $\varepsilon_{fe} = \left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \leq \kappa_m \varepsilon_{fd}$ | $\varepsilon_{fe} = 0.003\left(\frac{23.7\text{ in.} - 4.3\text{ in.}}{4.3\text{ in.}}\right) - 0.00061 = 0.0129$ | $\varepsilon_{fe} = 0.003\left(\frac{602\text{ mm} - 109\text{ mm}}{109\text{ mm}}\right) - 0.00061 = 0.0129$ |
| Note that for the neutral axis depth selected, FRP debonding would be the failure mode because the second expression in this equation controls. If the first expression governed, then concrete crushing would be the failure mode. Because FRP controls the failure of the section, the concrete strain at failure, $\varepsilon_c$, may be less than 0.003 and can be calculated using similar triangles: | $\kappa_m\varepsilon_{fd} = 0.7(0.0123) = 0.00865$<br>Hence, $\varepsilon_{fe} = 0.00865$<br>(Mode of failure is FRP debonding) | $\kappa_m\varepsilon_{fd} = 0.7(0.0123) = 0.00865$<br>Hence, $\varepsilon_{fe} = 0.00865$<br>(Mode of failure is FRP debonding) |
| $\varepsilon_c = (\varepsilon_{fd} + \varepsilon_{bi})\left(\frac{c}{d_f - c}\right)$ | $\varepsilon_c = (0.00865 + 0.00061)\left(\frac{4.3}{23.7 - 4.3}\right) = 0.0020$ | $\varepsilon_c = (0.00865 + 0.00061)\left(\frac{109}{602 - 109}\right) = 0.0020$ |
| **Step 7—Calculate the strain in the existing reinforcing steel** | | |
| The strain in the reinforcing steel can be calculated using similar triangles according to Eq. (10.2.10a). | | |
| $\varepsilon_s = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d - c}{d_f - c}\right)$ | $\varepsilon_s = (0.00865 + 0.00061)\left(\frac{21.5 - 4.3}{23.7 - 4.3}\right) = 0.0082$ | $\varepsilon_s = (0.00865 + 0.00061)\left(\frac{546 - 109}{602 - 109}\right) = 0.0082$ |
| **Step 8—Calculate the stress level in the reinforcing steel and FRP.** | | |
| The stresses are calculated using Eq. (10.2.10b) and Hooke's Law. | | |
| $f_s = E_s\varepsilon_s \leq f_y$ | $f_s = (29{,}000\text{ ksi})(0.0082) \leq 60\text{ ksi}$<br>$f_s = 238\text{ ksi} > 60\text{ ksi}$<br>therefore, $f_s = 60\text{ ksi}$ | $f_s = (200\text{ kN/mm}^2)(0.0082) \leq 0.414\text{ kN/mm}^2$<br>$f_s = 1.64\text{ kN/mm}^2 \leq 0.414\text{ kN/mm}^2$<br>therefore, $f_s = 0.414\text{ kN/mm}^2$ |
| $f_{fe} = E_f\varepsilon_{fe}$ | $f_{fe} = (19{,}230\text{ ksi})(0.00865) = 166\text{ ksi}$ | $f_{fe} = (132{,}700\text{ N/mm}^2)(0.00865) = 1147\text{ N/mm}^2$ |

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## Table 16.4b (cont.)—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 9—Calculate the internal force resultants and check equilibrium**<br><br>Concrete stress block factors may be calculated using ACI 318. Approximate stress block factors may also be calculated based on the parabolic stress-strain relationship for concrete as follows: | | |
| $\beta_1 = \frac{4\varepsilon_c - \varepsilon'_c}{6\varepsilon_c - 2\varepsilon'_c}$ | $\beta_1 = \frac{4(0.0021) - 0.002}{6(0.0021) - 2(0.002)} = 0.743$ | $\beta_1 = \frac{4(0.0021) - 0.002}{6(0.0021) - 2(0.002)} = 0.743$ |
| $\alpha_1 = \frac{3\varepsilon'_c\varepsilon_c - \varepsilon_c^2}{3\beta_1\varepsilon'^2_c}$<br>where $\varepsilon_c'$ is strain corresponding to $f_c'$ calculated as | $\alpha_1 = \frac{3(0.0021)(0.002) - (0.002)^2}{3(0.743)(0.0021)^2} = 0.870$ | $\alpha_1 = \frac{3(0.0021)(0.002) - (0.002)^2}{3(0.743)(0.0021)^2} = 0.870$ |
| $\varepsilon'_c = \frac{1.7f'_c}{E_c}$ | $\varepsilon'_c = \frac{1.7(5000)}{4030 \times 10^6} = 0.0021$ | $\varepsilon'_c = \frac{1.7(34.5)}{27,606} = 0.0021$ |
| Force equilibrium is verified by checking the initial estimate of $c$ with Eq. (10.3.1.6g). | | |
| $c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 f_c' \beta_1 b}$ | $c = \frac{(3.00\text{ in.}^2)(60\text{ ksi}) + (0.3\text{ in.}^2)(166\text{ ksi})}{(0.87)(5\text{ ksi})(0.743)(12\text{ in.})}$<br><br>$c = 5.92$ in. $\neq$ 4.30 in. n.g.<br><br>∴ revise estimate of $c$ and repeat Steps 6 through 9 until equilibrium is achieved. | $c = \frac{(1935\text{ mm}^2)(414\text{ N/mm}^2) + (194\text{ mm}^2)(1147\text{ N/mm}^2)}{(0.87)(34.5\text{ N/mm}^2)(0.743)(305\text{ mm})}$<br><br>$c = 150$ mm $\neq$ 109 in. n.g.<br><br>∴ revise estimate of $c$ and repeat Steps 6 through 9 until equilibrium is achieved. |
| **Step 10—Adjust *c* until force equilibrium is satisfied**<br><br>Steps 6 through 9 were repeated several times with different values of $c$ until equilibrium was achieved. The results of the final iteration are<br><br>$c$ = 5.26 in.; $\varepsilon_s$ = 0.0082; $f_s = f_y$ = 60 ksi; $\varepsilon_{fe}$ = 0.00865; $\varepsilon_c$ = 0.0027; $\beta_1$ = 0.786; $\alpha_1$ = 0.928; and $f_{fe}$ = 166 ksi | $c = \frac{(3.00\text{ in.}^2)(60\text{ ksi}) + (0.3\text{ in.}^2)(166\text{ ksi})}{(0.928)(5\text{ ksi})(0.786)(12\text{ in.})}$<br>$c = 5.25$ in. $\approx$ 5.26 in.<br><br>∴ the value of $c$ selected for the final iteration is correct. | $c = \frac{(1935\text{ mm}^2)(414\text{ N/mm}^2) + (193\text{ mm}^2)(1147\text{ N/mm}^2)}{(0.928)(34.5\text{ N/mm}^2)(0.786)(305\text{ mm})}$<br>$c = 133$ mm $\approx$ 134 mm<br><br>∴ the value of $c$ selected for the final iteration is correct. |

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## Table 16.4b (cont.)—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 11—Calculate flexural strength components.**<br><br>The design flexural strength is calculated using Eq. (10.2.10d). An additional reduction factor, $\psi_f = 0.85$, is applied to the contribution of the FRP system.<br>Steel contribution to bending:<br>$M_{ns} = A_s f_s\left(d - \frac{\beta_1 c}{2}\right)$ | $M_{ns} = (3.0\text{ in.}^2)(60\text{ ksi})\left(21.5\text{ in.} - \frac{0.786(5.25\text{ in.})}{2}\right)$<br>$M_{ns} = 3498$ kip-in. = 291 kip-ft | $M_{ns} = (1935\text{ mm}^2)(414\text{ N/mm}^2)\left(546\text{ mm} - \frac{0.786(133\text{ mm})}{2}\right)$<br>$M_{ns} = 394$ kN-m |
| FRP contribution to bending:<br>$M_{nf} = A_s f_{fe}\left(d_f - \frac{\beta_1 c}{2}\right)$ | $M_{nf} = (0.3\text{ in.}^2)(166\text{ ksi})\left(23.7\text{ in.} - \frac{0.786(5.25\text{ in.})}{2}\right)$<br>$M_{nf} = 1077$ kip-in. = 90 kip-ft | $M_{nf} = (194\text{ mm}^2)(1147\text{ N/mm}^2)\left(602.1\text{ mm} - \frac{0.786(133\text{ mm})}{2}\right)$<br>$M_{nf} = 122$ kN-m |
| **Step 12—Calculate design flexural strength of the section**<br><br>The design flexural strength is calculated using Eq. (10-1) and (10.2.10d). Because $\varepsilon_s = 0.0082 > 0.005$, a strength reduction factor of $\phi = 0.90$ is appropriate per Eq. (10.2.7).<br><br>$\phi M_n = \phi[M_{ns} + \psi_f M_{nf}]$ | $\phi M_n = 0.9[291\text{ kip-ft} + 0.85(90\text{ kip-ft})]$<br>$\phi M_n = 331\text{ kip-ft} \geq M_u = 294\text{ kip-ft}$<br><br>∴ the strengthened section is capable of sustaining the new required flexural strength. | $\phi M_n = 0.9[394\text{ kN-m} + 0.85(122\text{ kN-m})]$<br>$\phi M_n = 448\text{ kN-m} \geq M_u = 398\text{ kN-m}$<br><br>∴ the strengthened section is capable of sustaining the new required flexural strength. |
| **Step 13—Check service stresses in the reinforcing steel and the FRP**<br><br>Calculate the elastic depth to the cracked neutral axis. This can be simplified for a rectangular beam without compression reinforcement as follows:<br>$k = \sqrt{\left(\rho_s\frac{E_s}{E_c} + \rho_f\frac{E_f}{E_c}\right)^2 + 2\left(\rho_s\frac{E_s}{E_c} + \rho_f\frac{E_f}{E_c}\left(\frac{d_f}{d}\right)\right)} - \left(\rho_s\frac{E_s}{E_c} + \rho_f\frac{E_f}{E_c}\right)$ | *<br>$k = 0.345$<br>$kd = (0.345)(21.5\text{ in.}) = 7.4\text{ in.}$ | ‡<br>$k = 0.345$<br>$kd = (0.345)(546\text{ mm}) = 188\text{ mm}$ |
| Calculate the stress level in the reinforcing steel using Eq. (10.2.10.1) and verify that it is less than the recommended limit per Eq. (10.2.8a).<br>$f_{s,s} = \frac{\left[M_s + \varepsilon_{bi}A_f E_f\left(d_f - \frac{kd}{3}\right)\right](d - kd)E_s}{A_s E_s\left(d - \frac{kd}{3}\right)(d - kd) + A_f E_f\left(d_f - \frac{kd}{3}\right)(d_f - kd)}$<br>$f_{s,s} \leq 0.80 f_y$ | †$f_{s,s} = 40.3\text{ ksi} \leq (0.80)(60\text{ ksi}) = 48\text{ ksi}$<br><br>∴ the stress level in the reinforcing steel is within the recommended limit. | §$f_{s,s} = 278\text{ N/mm}^2 \leq (0.80)(410\text{ N/mm}^2) = 330\text{ N/mm}^2$<br><br>∴ the stress level in the reinforcing steel is within the recommended limit. |

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## Table 16.4b (cont.)—Procedure for flexural strengthening of an interior reinforced concrete beam with NSM FRP bars

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 14—Check creep rupture limit at service of the FRP**<br><br>Calculate the stress level in the FRP using Eq. (10.2.10.2) and verify that it is less than creep-rupture stress limit given in Table 10.2.9. Assume that the full service load is sustained. | | |
| $f_{f,s} = f_{s,s}\left(\frac{E_f}{E_s}\right)\left(\frac{d_f - kd}{d - kd}\right) - \varepsilon_{bi}E_f$ | $f_{f,s} = 40.3\text{ ksi}\left(\frac{19{,}230\text{ ksi}}{29{,}000\text{ ksi}}\right)\left(\frac{23.7\text{ in.} - 7.4\text{ in.}}{21.5\text{ in.} - 7.4\text{ in.}}\right) - (0.00061)(19{,}230\text{ ksi})$ | $f_{f,s} = 0.278\text{ kN/mm}^2\left(\frac{133\text{ kN/mm}^2}{200\text{ kN/mm}^2}\right)\left(\frac{602\text{ mm} - 188\text{ mm}}{546\text{ mm} - 188\text{ mm}}\right) - (0.00061)(133\text{N/mm}^2)$ |
| For a carbon FRP system, the sustained plus cyclic stress limit is obtained from Table 10.2.9: | $f_{f,s} = 19\text{ ksi} \leq (0.55)(85\text{ ksi}) = 50\text{ ksi}$ | $f_{f,s} = 134\text{ N/mm}^2 \leq (0.55)(590\text{ N/mm}^2) = 324.5\text{ N/mm}^2$ |
| Sustained plus cyclic stress limit = $0.55f_{fu}$ | ∴ the stress level in the FRP is within the recommended sustained plus cyclic stress limit. | ∴ the stress level in the FRP is within the recommended sustained plus cyclic stress limit. |

*$$k = \sqrt{\left[0.0116\left(\frac{29{,}000}{4030}\right) + 0.0012\left(\frac{19{,}230}{4030}\right)\right]^2 + 2\left[0.0116\left(\frac{29{,}000}{4030}\right) + 0.0012\left(\frac{19{,}230}{4030}\right)\left(\frac{23.7\text{ in.}}{21.5\text{ in.}}\right)\right]} - \left[0.0116\left(\frac{29{,}000}{4030}\right) + 0.0012\left(\frac{19{,}230}{4030}\right)\right]$$

†$$f_{s,s} = \frac{\left(\left[2424\text{ kip-in.} + \left[(0.00061)(0.3\text{ in.}^2)\times(19{,}230\text{ ksi})\left(23.7\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\right]\right]\times\left[(21.5\text{ in.} - 7.4\text{ in.})(29{,}000\text{ ksi})\right]\right)}{\left(\left[(3.00\text{ in.}^2)(29{,}000\text{ ksi})\left(21.5\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\times(21.5\text{ in.} - 7.4\text{ in.})\right] + \left[(0.3\text{ in.}^2)(19{,}230\text{ ksi})\left(23.7\text{ in.} - \frac{7.4\text{ in.}}{3}\right)\times(23.7\text{ in.} - 7.4\text{ in.})\right]\right)}$$

‡$$k = \sqrt{\left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\right]^2 + 2\left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\left(\frac{602\text{ mm}}{546\text{ mm}}\right)\right]} - \left[0.0116\left(\frac{200}{27.6}\right) + 0.0012\left(\frac{133}{27.6}\right)\right]$$

§$$f_{s,s} = \frac{\left(\left[273{,}912\text{ kN-mm} + \left[(0.00061)(194\text{ mm}^2)\times(132.7\text{ kN/mm}^2)\times\left(602\text{ mm} - \frac{188\text{ mm}}{3}\right)\right]\right]\times\left[(546\text{ mm} - 188\text{ mm})(200\text{ kN/mm}^2)\right]\right)}{\left(\left[(1935\text{ mm}^2)(200\text{ kN/mm}^2)\times\left(546\text{ mm} - \frac{188\text{ mm}}{3}\right)(546\text{ mm} - 188\text{ mm})\right] + \left[(194\text{ mm}^2)(132.7\text{ kN/mm}^2)\times\left(602\text{ mm} - \frac{188\text{ mm}}{3}\right)(602\text{ mm} - 188\text{ mm})\right]\right)}$$

In detailing the FRP reinforcement, FRP bars should be terminated at a distance equal to the bar development length past the point on the moment diagram that represents cracking.

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## 16.5—Flexural strengthening of an interior prestressed concrete beam with FRP laminates

A number of continuous prestressed concrete beams with five 1/2 in. (12.7 mm) diameter bonded strands (Fig. 16.5) are located in a parking garage that is being converted to an office space. All prestressing strands are Grade 270 ksi (1860 N/mm$^2$) low-relaxation seven-wire strands. The beams require an increase in their live-load-carrying capacity from 50 to 75 lb/ft$^2$ (244 to 366 kg/m$^2$). The beams are also required to support an additional dead load of 10 lb/ft$^2$ (49 kg/m$^2$). Analysis indicates that each existing beam has adequate flexural capacity to carry the new loads in the negative moment region at the supports but is deficient in flexure at midspan and in shear at the supports. The beam meets the deflection and crack control serviceability requirements. The cast-in-place beams support a 4 in. (100 mm) slab. For bending at midspan, beams should be treated as T-sections. Summarized in Table 16.5a are the existing and new loads and associated midspan moments for the beam. FRP system properties are shown in Table 16.3b.

By inspection, the degree of strengthening is reasonable in that it does meet the strengthening limit criteria put forth in Eq. (10.1). That is, the existing flexural strength without FRP, $(\phi M_n)_{w/o}$ = 336 kip-ft (455 kN-m), is greater than the unstrengthened moment limit, $(1.1M_{DL} + 0.75M_{LL})_{new}$ = 273 kip-ft (370 kN-m). The design calculations used to verify this configuration follow. The beam is to be strengthened using the FRP system described in Table 16.3b. A one-ply, 24 in. (610 mm) wide strip of FRP is considered for this evaluation.

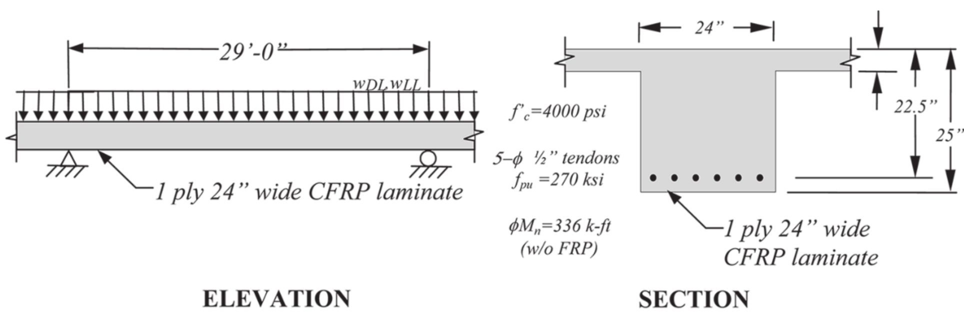


*Fig. 16.5—Schematic of the idealized continuous prestressed beam with FRP external reinforcement.*

### Table 16.5a—Loadings and corresponding moments

| Loading/moment | Existing loads | | Anticipated loads | |
|---|---|---|---|---|
| Dead loads $w_{DL}$ | 2.77 kip/ft | 40.4 N/mm | 3.09 kip/ft | 45.1 N/mm |
| Live load $w_{LL}$ | 1.60 kip/ft | 23.3 N/mm | 2.4 kip/ft | 35 N/mm |
| Unfactored loads ($w_{DL} + w_{LL}$) | 4.37 kip/ft | 63.8 N/mm | 5.49 kip/ft | 80.2 N/mm |
| Unstrengthened load limit ($1.1w_{DL} + 0.75w_{LL}$) | NA | NA | 5.2 kip/ft | 75.9 N/mm |
| Factored loads ($1.2w_{DL} + 1.6w_{LL}$) | 5.88 kip/ft | 85.9 N/mm | 7.55 kip/ft | 110.2 N/mm |
| Dead-load moment $M_{DL}$ | 147 kip-ft | 199 kN-m | 162 kip-ft | 220.2 kN-m |
| Live-load moment $M_{LL}$ | 85 kip-ft | 115 kN-m | 126 kip-ft | 171.1 kN-m |
| Service-load moment $M_s$ | 232 kip-ft | 314 kN-m | 288 kip-ft | 391.3 kN-m |
| Unstrengthened moment limit $(1.1M_{DL} + 0.75M_{LL})_{new}$ | NA | NA | 273 kip-ft | 371 kN-m |
| Factored moment $M_u$ | 312 kip-ft | 423 kN-m | 397 kip-ft | 538 kN-m |

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## Table 16.5b—Procedure for flexural strengthening of an interior prestressed concrete beam with FRP laminates

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 1—Calculate the FRP-system design material properties** | | |
| The beam is located in an interior space and a CFRP material will be used. Therefore, per Table 9.4, an environmental reduction factor of 0.95 is suggested. | | |
| $f_{fu} = C_E f_{fu}^*$ | $f_{fu}$ = (0.95)(90 ksi) = 85 ksi | $f_{fu}$ = (0.95)(621 N/mm²) = 590 N/mm² |
| $\varepsilon_{fu} = C_E \varepsilon_{fu}^*$ | $\varepsilon_{fu}$ = (0.95)(0.015 in./in.) = 0.0142 in./in. | $\varepsilon_{fu}$ = (0.95)(0.015 mm/mm) = 0.0142 mm/mm |
| **Step 2—Preliminary calculations** | | |
| Properties of the concrete: | | |
| $\beta_1$ from ACI 318-14, Section 22.2.2.4.3 | $\beta_1 = 1.05 - 0.05\dfrac{f_c'}{1000} = 0.85$ | $\beta_1 = 1.05 - 0.05\dfrac{f_c'}{6.9} = 0.85$ |
| $E_c = 57{,}000\sqrt{f_c'}$ | $E_c = 57{,}000\sqrt{4000\text{ psi}} = 3{,}605{,}00\text{ psi}$ | $E_c = 4700\sqrt{27.6\text{ N/mm}^2} = 24{,}700\text{ N/mm}^2$ |
| Properties of the existing prestressing steel: | $A_{ps}$ = 5(0.153 in.²) = 0.765 in.² | $A_{ps}$ = 5(99 mm²) = 495 mm² |
| Area of FRP reinforcement: | | |
| $A_f = nt_f w_f$ | $A_f$ = (1 ply)(0.040 in./ply)(24 in.) = 0.96 in.² | $A_f$ = (1 ply)(1.0 mm/ply)(610 mm) = 610 mm² |
| Cross-sectional area: | | |
| $A_{cg} = b_f h_f + b_w(h - h_f)$ | $A_{cg}$ = (87 in.)(4 in.) + (24 in.)(25 in. – 4 in.) = 852 in.² | $A_{cg}$ = (2210 mm)(102 mm) + (610 mm)(612 mm – 102 mm) = 5.5 × 10⁵ mm² |
| Distance from the top fiber to the section centroid: | | |
| $y_t = \dfrac{b_f \dfrac{h_f^2}{2} + b_w(h-h_f)\left(h_f + \dfrac{(h-h_f)}{2}\right)}{A_{cg}}$ | $y_t = \dfrac{87\text{ in.}\times\dfrac{4\text{ in.}^2}{2} + 24\text{ in.}\times 21\times 14.5}{852} = 9.39\text{ in.}$ | $y_t = \dfrac{2210\text{ mm}\times\dfrac{102\text{ mm}^2}{2} + 610\text{ mm}\times 533\times 368}{5.5\times 10^5} = 238\text{ mm}$ |
| Gross moment of inertia: | | |
| $I_g = \dfrac{b_f h_f^3}{12} + b_f h_f\left(y_t - \dfrac{h_f}{2}\right)^2 + \dfrac{b_w(h-h_f)^3}{12} + b_w(h-h_f)\left(y_t - \dfrac{h-h_f}{2}\right)^2$ | $I_g = \dfrac{87\text{ in.}\times 4\text{ in.}^3}{12} + 87\text{ in.}\times 4\text{ in.}(9.39\text{ in.} - 2)^2 + \dfrac{24\text{ in.}\times 21^3}{12} + 24\text{ in.}\times 21(9.39 - 10.5)^2 = 38{,}610\text{ in.}^4$ | $I_g = \dfrac{2210\text{ mm}\times 102\text{ mm}^3}{12} + 2210\text{ mm}\times 102\text{ mm}(238-51)^2 + \dfrac{610\text{mm}\times 533^3}{12} + 610\text{ mm}\times 533(238 - 266.7)^2 = 1.61\times 10^{10}\text{mm}^4$ |
| Radius of gyration: | | |
| $r = \sqrt{\dfrac{I_g}{A_{cg}}}$ | $r = \sqrt{\dfrac{38{,}610}{852}} = 6.73\text{ in.}$ | $r = \sqrt{\dfrac{1.61\times 10^{10}}{5.5\times 10^5}} = 171\text{ mm}$ |
| Effective prestressing strain: | | |
| $\varepsilon_{pe} = \dfrac{f_{pe}}{E_p}$ | $\varepsilon_{pe} = \dfrac{165}{28{,}500} = 0.00578$ | $\varepsilon_{pe} = \dfrac{1138}{1.96\times 10^5} = 0.00579$ |
| Effective prestressing force: | | |
| $P_e = A_{ps} f_{pe}$ | $P_e$ = 0.765 × 165 = 126.2 kip | $P_e$ = 495 × 1138 = 563,310 N |
| Eccentricity of prestressing force: | | |
| $e = d_p - y_t$ | $e$ = 22.5 – 9.39 = 13.1 in. | $e$ = 571 – 238 = 333 mm |

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**Table 16.5b (cont.)—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer laminates**

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 3—Determine the existing state of strain on the soffit**<br><br>The existing state of strain is calculated assuming the beam is uncracked and the only loads acting on the beam at the time of the FRP installation are dead loads. Distance from extreme bottom fiber to the section centroid:<br><br>$y_b = h - y_t$<br><br>Initial strain in the beam soffit:<br><br>$\varepsilon_{bi} = \frac{-P_e}{E_c A_{cg}}\left(1+\frac{ey_b}{r^2}\right)+\frac{M_{DL}y_b}{E_c I_g}$ | $y_b = 25 - 9.39 = 15.61$ in.<br><br>$\varepsilon_{bi} = \frac{-126.2}{3605\times 852}\left(1+\frac{13.1\times 15.6}{6.73^2}\right)+\frac{147\times 12\times 15.6}{3605\times 38,610}$<br><br>$\varepsilon_{bi} = -2.88\times 10^{-5}$ | $y_b = 635 - 238 = 397$ mm<br><br>$\varepsilon_{bi} = \frac{-563,310}{24,700\times 5.5\times 10^5}\left(1+\frac{333\times 397}{171^2}\right)+\frac{199\times 10^6\times 397}{24,700\times 1.61\times 10^{10}}$<br><br>$\varepsilon_{bi} = -2.88\times 10^{-5}$ |
| **Step 4—Determine the design strain of the FRP system**<br><br>The design strain of FRP accounting for debonding failure mode $\varepsilon_{fd}$ is calculated using Eq. (10.2.1.1)<br><br>Because the design strain is smaller than the rupture strain, debonding controls the design of the FRP system. | $\varepsilon_{fd} = 0.083\sqrt{\frac{4000 \text{ psi}}{1(5,360,000 \text{ psi})(0.04 \text{ in.})}}$<br>$= 0.0113 \le 0.9(0.0142) = 0.0128$ | $\varepsilon_{fd} = 0.042\sqrt{\frac{27.6 \text{ N/mm}^2}{1(37,000 \text{ N/mm}^2)(1.016 \text{ mm})}}$<br>$= 0.0113 \le 0.9(0.0142) = 0.0128$ |
| **Step 5—Estimate *c*, the depth to the neutral axis**<br><br>A reasonable initial estimate of *c* is 0.1*h*. The value of the *c* is adjusted after checking equilibrium.<br><br>$c = 0.1h$ | $c = (0.1)(25 \text{ in.}) = 2.50$ in. | $c = (0.1)(635 \text{ mm}) = 63.5$ mm |
| **Step 6—Determine the effective level of strain in the FRP reinforcement**<br><br>The effective strain level in the FRP may be found from Eq. (10.3.1.6c).<br><br>$\varepsilon_{fe} = 0.003\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \le \varepsilon_{fd}$<br><br>Note that for the neutral axis depth selected, FRP debonding would be the failure mode because the second expression in this equation controls. If the first (limiting) expression governed, then FRP rupture would be the failure mode. | $\varepsilon_{fe} = 0.003\left(\frac{25-2.5}{2.5}\right) - 0.00003 = 0.027$<br>$> \varepsilon_{fd} = 0.0113$<br><br>Failure is governed by FRP debonding<br><br>$\varepsilon_{fe} = \varepsilon_{fd} = 0.0113$ | $\varepsilon_{fe} = 0.003\left(\frac{635-63.5}{63.5}\right) - 0.00003 = 0.027$<br>$> \varepsilon_{fd} = 0.0113$<br><br>Failure is governed by FRP debonding<br><br>$\varepsilon_{fe} = \varepsilon_{fd} = 0.0113$ |

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**Table 16.5b (cont.)—Procedure for flexural strengthening of an interior prestressed concrete beam with fiber-reinforced polymer laminates**

| Procedure | Calculation in in.-lb units | Calculation in SI metric units |
|---|---|---|
| **Step 7—Calculate the strain in the existing prestressing steel**<br><br>The strain in the prestressing steel can be calculated using Eq. (10.3.1.6c) and (10.3.1.6a). | | |
| $\varepsilon_{pnet} = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{d_p - c}{d_f - c}\right)$ | $\varepsilon_{pnet} = (0.0113 + 0.00003)\left(\frac{22.5 - 2.5}{25 - 2.5}\right)$<br>$\varepsilon_{pnet} = 0.01$ | $\varepsilon_{pnet} = (0.0113 + 0.00003)\left(\frac{571 - 63.5}{635 - 63.5}\right)$<br>$\varepsilon_{pnet} = 0.01$ |
| $\varepsilon_{ps} = \varepsilon_{pe} + \frac{P_e}{A_{cg}E_c}\left(1 + \frac{e^2}{r^2}\right) + \varepsilon_{pnet} \le 0.035$ | $\varepsilon_{ps} = 0.00589 + \frac{126.2}{852 \times 3605}\left(1 + \frac{13.1^2}{6.73^2}\right) + 0.01$<br>$\varepsilon_{ps} = 0.016 \le 0.035$ | $\varepsilon_{ps} = 0.00589 + \frac{563{,}310}{5.5 \times 10^5 \times 24{,}700}\left(1 + \frac{333^2}{171^2}\right) + 0.01$<br>$\varepsilon_{ps} = 0.016 \le 0.035$ |
| **Step 8—Calculate the stress level in the prestressing steel and FRP**<br><br>The stresses are calculated using Eq. (10.3.1.6e) and Hooke's Law. | | |
| $f_{ps} = \begin{cases} 28{,}500\varepsilon_{ps} & \text{for } \varepsilon_{ps} \le 0.0086 \\ 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} & \text{for } \varepsilon_{ps} > 0.0086 \end{cases}$ | $f_{ps} = 270 - \frac{0.04}{0.016 - 0.007} = 265.6 \text{ ksi}$ | $f_{ps} = 1860 - \frac{0.276}{0.016 - 0.007} = 1831 \text{ N/mm}^2$ |
| $f_{fe} = E_f\varepsilon_{fe}$ | $f_{fe}$ = (5360 ksi)(0.0113) = 60.6 ksi | $f_{fe}$ = (37,000 N/mm$^2$)(0.0113) = 418 N/mm$^2$ |
| **Step 9—Calculate the equivalent concrete compressive stress block parameters $\alpha_1$ and $\beta_1$.**<br><br>The strain in concrete at failure can be calculated from strain compatibility as follows: | | |
| $\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{c}{d_f - c}\right)$ | $\varepsilon_c = (0.0113 + 0.00003)\left(\frac{2.5}{25 - 2.5}\right) = 0.0013$ | $\varepsilon_c = (0.0113 + 0.00003)\left(\frac{63.5}{635 - 63.5}\right) = 0.0013$ |
| The strain $\varepsilon_c'$ corresponding to $f_c'$ is calculated as | | |
| $\varepsilon_c' = \frac{1.7f_c'}{E_c}$ | $\varepsilon_c' = \frac{1.7(4000)}{3605 \times 10^6} = 0.0019$ | $\varepsilon_c' = \frac{1.7(27.6)}{24{,}700} = 0.0019$ |
| Concrete stress block factors can be estimated using ACI 318. Approximate stress block factors may be calculated from the parabolic stress-strain relationship for concrete and is expressed as follows: | | |
| $\beta_1 = \frac{4\varepsilon_c' - \varepsilon_c}{6\varepsilon_c - 2\tilde{\varepsilon}_c}$ | $\beta_1 = \frac{4(0.0019) - 0.0013}{6(0.0019) - 2(0.0013)} = 0.716$ | $\beta_1 = \frac{4(0.0019) - 0.0013}{6(0.0019) - 2(0.0013)} = 0.716$ |
| $\alpha = \frac{\varepsilon_c'\varepsilon_c - \varepsilon_c}{\beta\,\varepsilon'}$ | $\alpha_1 = \frac{3(0.0019)(0.0013) - (0.0013)^2}{3(0.716)(0.0019)^2} = 0.738$ | $\alpha_1 = \frac{3(0.0019)(0.0013) - (0.0013)^2}{3(0.716)(0.0019)^2} = 0.738$ |

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