2 Research significance

This study shows the application of the Strip Model to reinforced concrete slabs under concentrated loads failing in a combination of one-way shear, two-way shear and flexure. The resulting Extended Strip Model provides an estimate of the maximum concentrated load that can be applied to the structure without violating defined material capacities. This loading case is important in the assessment of existing bridges subjected to concentrated live loads. The presented Extended Strip Model is suitable for a loading situation that was not given much attention in the past. The validity of the model is proven by a comparison with laboratory experiments and a case study of an existing bridge tested to failure.

3 The design gap between one-way and two-way shear

Often, for the design and assessment of existing reinforced concrete bridges subjected to concentrated live loads, one-way shear will be analyzed by considering the slab as a wide beam (without taking advantage of the transverse load redistribution capacity of the slab) and two-way shear by considering the area of the load on the slab in the same way as a slab-column connection in buildings. For D-regions, strut-and-tie models can be developed, but little guidance is available on how to elaborate three-dimensional strut-and-tie models for complex loading cases such as slab bridges subjected to concentrated live loads.

Since experiments (Lantsoght et al., 2013b; Lantsoght et al., 2015b) have shown that the failure mode of slabs under concentrated loads (representing bridge decks) is a combination of one-way and two-way shear as well as flexure (see Figure 1), a method that bridges the gap between traditional one-way and two-way shear approaches was sought. In the experiments on slabs, flexural cracking was observed in the transverse and longitudinal direction. Moreover, inclined cracks on the bottom of the slab indicating shear distress were observed towards the ultimate load, as well as partial cracks showing the punching perimeter around the load. Cracks indicating torsional distress at the edges where the slab edges want to curl up were also observed for slabs loaded with a load close to the free edge. An overview of the different types of distress after one experiment (S6T4) are shown in Figure 1. Similar observations were made for slabs under a combination of loads, mimicking the typical live load models of concentrated loads and distributed loads (Lantsoght et al., 2015a). Clearly, for slabs under concentrated loads, the different failure mechanisms cannot and should not be isolated, as current design methods imply.

In experiments on elements with a smaller width (Lantsoght et al., 2014), only interaction between one-way shear and flexure was observed. When the width of specimens subjected to concentrated loads was increased, a transition from one-way shear to a combination of one-way and two-way shear was observed (Lantsoght et al., 2015c). This transition zone is typically not considered in the codes, which make a strict distinction between beam shear (one-way shear) and punching shear (two-way shear). However, measurements of the force increment in structural elements failing in one-way or two-way shear showed values that were very similar (Olonisakin and Alexander, 1999). These observations show that there is a fundamental link between one-way and two-way shear. Design solutions for loading cases in this transition zone between a beam in three- or four-point bending and a slab-column connection are not available.

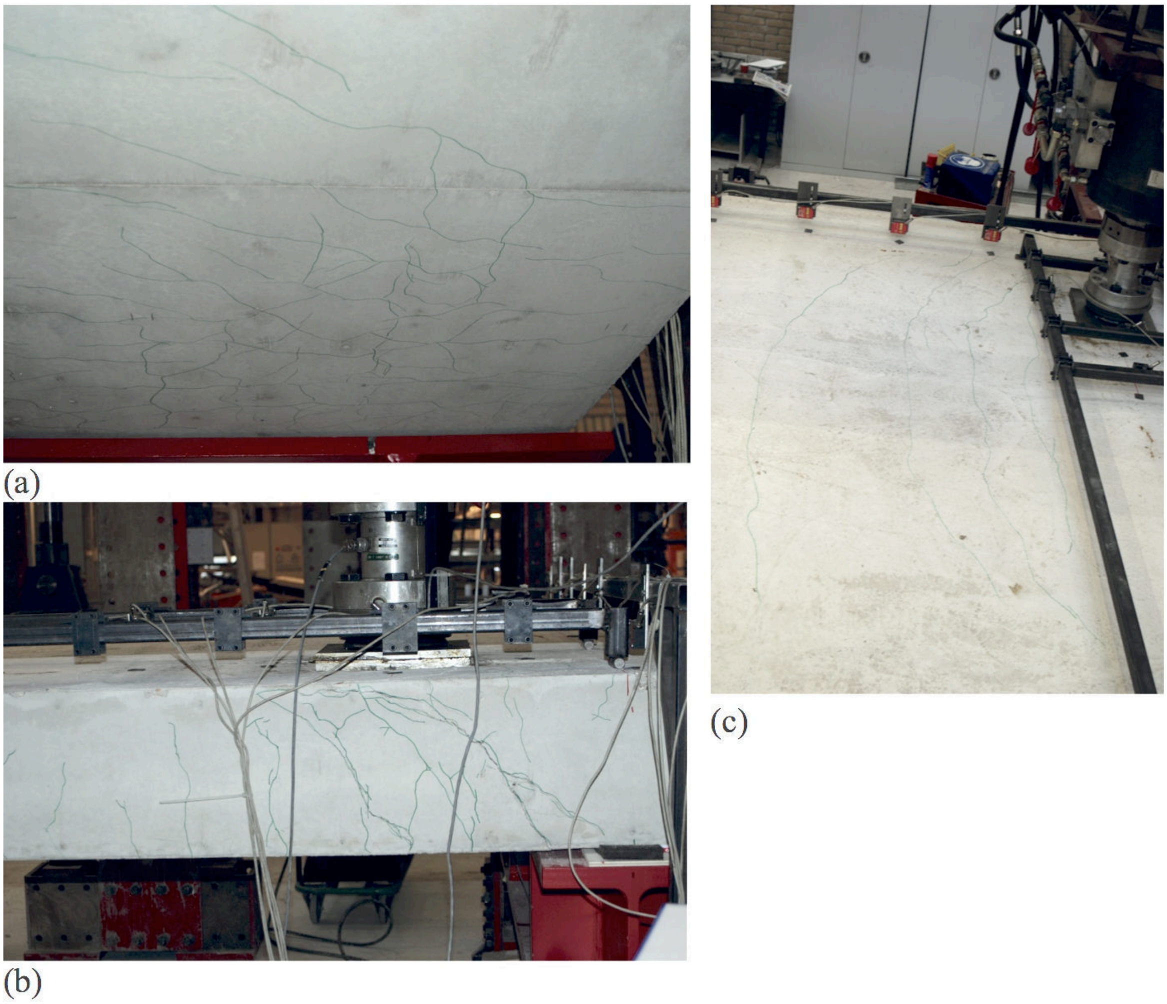


Figure 1: Observed cracking after failure of slab S6 in experiment S6T4 (Lantsoght et al., 2013b): (a) combination of one-way shear (inclined cracks on the bottom face), torsional distress at the edge and flexural distress represented by longitudinal and transverse cracks; (b) one-way shear failure crack at the edge; (c): two-way shear cracks on the top face of the slab.

4 Lower bound plasticity-based methods for slabs

4.1 Hillerborg's Strip Method for flexure

As introduced previously, Hillerborg's Strip Method (Alexander, 1999a; Hillerborg, 1975; 1982; 1996) is a lower-bound plasticity-based solution for the flexural design of slabs. The method is particularly powerful in cases where traditional methods such as direct design cannot be applied, such as slabs with openings, or slabs not supported on all edges. The Strip Method has been under development since the late 1950s, first only for slabs supported over a full edge. Later, a corner-supported element resulted in the Advanced Strip Method (Hillerborg, 1982). While the name "Advanced Strip Method" might sound rather daunting, the method itself is as easy to apply as the regular Strip Method. The ease of application of the method was shown in a book with design examples (Hillerborg, 1996).

The Strip Method is a lower bound plasticity-based method, which means that (Hillerborg, 1975) if there is a load Q_l for which it is possible to find a moment field that fulfills all equilibrium conditions and the moment is not higher than the yield moment at any point, then Q_l is a lower-bound value of the carrying capacity. The slab can certainly carry the load Q_l. The Strip Method is a simplification of the equilibrium theory, which is based on the design principle that a moment field is first determined to fulfil the equilibrium and edge conditions, after which the strength of the slab at each point is designed for this moment field. The simplification makes it possible to fulfil the lower-bound theorem and at the same time achieve a good economical arrangement of reinforcement.

The basic assumption (Alexander, 1999b) of the Strip Method is that, because of the ductility and redistribution capacity of slabs, virtually any combination of the distributed loads in the main directions q_x, q_y and under 45° q_{xy}, the load that determines the torsional reinforcement may be used as the basis for the design of a slab, as long as:

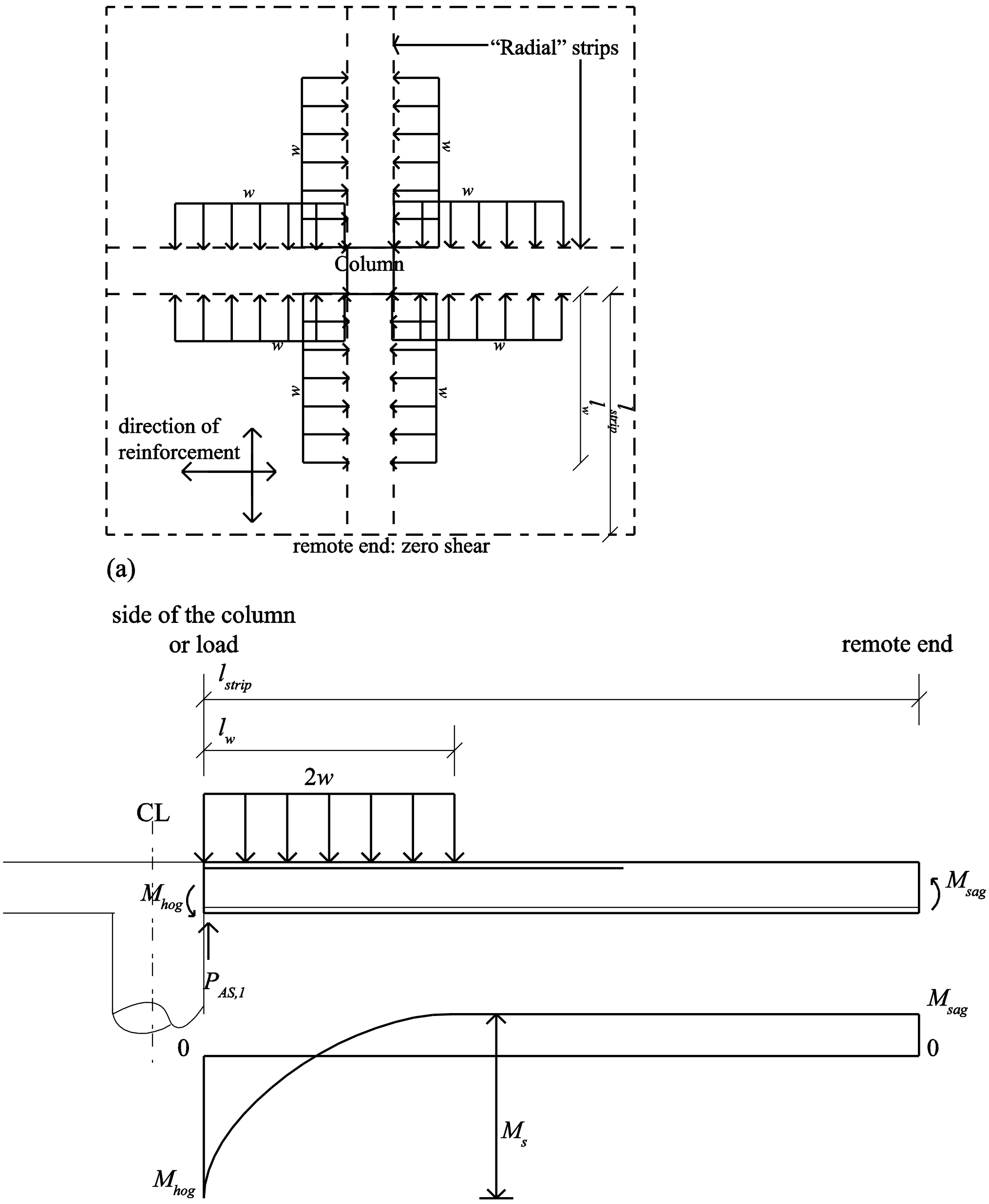
1. the slab is designed at all points to resist the moments resulting from the assumed load distribution;
2. the shear and moments resulting from the assumed load distribution do not violate any boundary condition; and
3. the assumed load distribution satisfies equilibrium: $q_x + q_y + q_{xy} = q$

In the Strip Method, the torsional moments are set to zero, so that $q_x + q_y = q$. Then, strips of the slab in the *x*- or *y*-direction can be treated as one-way beams and designed to carry the loads acting on them. The Strip Method gives a clear indication of where reinforcement should be placed to be of greatest benefit. Since complete moment diagrams are determined for the strips/beams, bar cutoffs can easily be determined.

For unsupported edges or large openings, strong bands can be used. A strong band is a strip of slab that acts like a virtual beam, on which the other strips can rest. Therefore, the strong band must be designed to carry the applied load plus the reactions of the strips carrying off to the strong band.

4.2 Strip Model for concentric punching shear

A plasticity-based solution for concentric punching shear exists as the Bond Model (Alexander, 1990; Alexander and Simmonds, 1992), later renamed the Strip Model (Afhami, 1997; Afhami et al., 1998; Ospina et al., 2003). The model subdivides the load transfer mechanism at a slab-column connection into the elementary beam shear carrying mechanism. The slab is split into strips branching out of the column, which work in arching action, and quadrants limited by the boundaries of the strips, which work in two-way flexure. An overview of the layout of the strips and quadrants is shown in Figure 2a. The interface between the quadrants working in two-way flexure and the strips working in arching action is then the weakest link in the assumed load transfer mechanism. Therefore, failure is assumed to occur at the interface between the strips and quadrants, which is governed by the limiting one-way shear capacity, working in beam shear. The loading on the strip and the resulting moment diagram is shown in Figure 2b.



(a)

(b)

Figure 2: Overview of the Bond Model for concentric punching shear: (a) layout of the slab, divided in quadrants and strips; (b) loading situation on strip and resulting moment diagram (Alexander and Simmonds, 1992).

The model was originally called the Bond Model, as it refers to the force gradients in the reinforcement in the strips working in arching action. The force gradients are transferred by bond, and originally the model was studied by defining the bond strength of the reinforcement in unconfined splitting failure (Alexander and Simmonds, 1992). For this loading case, the maximum force gradient in the reinforcement bars perpendicular to the strip determines the maximum load. Later, the equivalence between beam action shear and a limiting nominal shear stress was used to find the maximum load. The interchangeability of the beam shear capacity and a limiting force increment in the reinforcement are the main assumptions of the original Bond Model. However, later the model was renamed "Strip Model", to link the name of the model to the principle of the method in which radial strips are used, and to avoid confusion with models that describe the force transfer at the interface of a reinforcement bar and the surrounding concrete (i.e., that study only the mechanism of bond).

Experiments confirmed the arching action in the strips (Alexander, 1990). Strain measurements showed that the interaction between the arch and the adjacent quadrants determined the geometry of the curved arch, which led to the development of the failure criterion at the intersection between the strips and quadrants.

In the Strip Model, the total capacity is determined as the sum of the capacities of the four strips (as shown in Figure 2a). For the case of concentric punching shear, the capacity of each strip is equal. Analyzing a single strip, Figure 2b, shows that the strip has a length l_{strip} and is loaded over the loaded length l_w so as to maximize the occurring moments. The maximum possible moments are the moment capacities of the cross-sections M_{sag} for the sagging moment and M_{hog} for the hogging moment. The moments can be taken over the full width for simplicity. The total flexural capacity M_s of the slab is the sum of these moment capacities, $M_s = M_{hog} + M_{sag}$. The Strip Model thus assumes that yielding of the steel occurs at the ultimate. This assumption is valid for practical cases of building slabs and bridge slabs, but is sometimes violated in laboratory experiments that are designed to achieve a shear failure. The acting loading is $2w$, obtained by summing the maximum loads on both sides of the strip, which has an interface between the strip and the quadrant on both sides. Using vertical and moment equilibrium on the strips (Figure 2b) results in the following expressions:

$$M_s = \frac{2wl_w^2}{2} \tag{1}$$

$$P_{AS,1} = 2wl_w \tag{2}$$

with

M_s the total flexural capacity, combining the hogging and sagging moment capacities;

w the maximum shear at the interface between the strip and the quadrant;

l_w the loaded length, optimized to get the largest moments, as shown in Figure 2b.

Solving Eq. (1) for the unknown loaded length l_w and substituting this into Eq. (2) results in the shear capacity of a single strip, $P_{AS,1}$:

$$P_{AS,1} = 2\sqrt{M_s w} \tag{3}$$

The total capacity of the studied slab-column connection is then the sum of the capacities of four strips:

$$P_{AS} = 8\sqrt{M_s w} \tag{4}$$

Since the load w is determined by the maximum shear stress at the interface between the strips and the quadrants, the one-way shear capacity w_{ACI} from ACI 318-14 (ACI Committee 318, 2014) was proposed for use with the Bond Model (Alexander and Simmonds, 1992). This expression was first formulated in the late 1950s and determines the inclined cracking load (Morrow and Viest, 1957):

$$w_{ACI}\left[\frac{\text{kN}}{\text{m}}\right] = 0.166 \times d\left[\text{mm}\right]\sqrt{f_c^{'}\left[\text{MPa}\right]} \quad (5)$$

$$w_{ACI}\left[\frac{\text{lbf}}{\text{in}}\right] = 2.00 \times d\left[\text{in}\right]\sqrt{f_c^{'}\left[\text{psi}\right]}$$

with

$f_c^{'}$ the concrete cylinder compressive strength; and

d the effective depth.

Finite element analyses (Afhami et al., 1998) showed the validity of this simple expression for the maximum load. The shear on the side faces of the strips, when approximated by a rectangle in the region near the column, had about the same value as w_{ACI}.

5 Extended strip model applications to slab bridges

5.1 One-way slabs as used in slab bridges

The proposed method for slabs under concentrated loads is a plasticity-based method, called the Extended Strip Model. This method is based on the Strip Model for concentric punching shear, as described previously, and takes the effects of the geometry into account for describing the ultimate capacity of a slab under a concentrated load. Like the original Strip Model (Alexander and Simmonds, 1992), the Extended Strip Model consists of "strips" that work in arching action (one-way shear) and slab "quadrants" that work in two-way flexure. As such, this method is suitable for the design and assessment of elements that are in the transition zone between one-way and two-way shear, and that show significant flexural distress upon failure, such as slab bridges under concentrated loads. Slab bridges are designed so that a flexural failure (ductile failure mode) occurs before a shear failure (brittle failure mode). A full description of the theoretical background and derivations of the Extended Strip Model is provided elsewhere (Lantsoght et al., (2017)). Here, the main novelties of the Extended Strip Model as compared to the original Strip Model will be discussed.

The first difference between the slab of infinite dimensions studied by the Strip Model for concentric punching shear (i.e. a slab-column connection as used in buildings) and a bridge deck slab, is that the bridge deck slab will have different dimensions, reinforcement ratios and effective depths in the longitudinal and transverse direction. For clarity, the longitudinal direction will be named the x-direction, and the transverse direction the y-direction (see Figure 3). The effective depth of the longitudinal reinforcement (larger value) is then d_x and of the transverse reinforcement (smaller value) is d_y. For bridge deck slabs, typically a larger reinforcement ratio is used in the x-direction (main load carrying direction) than in the y-direction. As a result, the capacity of the flexural reinforcement in the x-direction $M_{s,x}$ will be larger than the capacity in the y-direction $M_{s,y}$. The load on the y-direction strips will be determined by d_x, since the cross-section of the intersection between the strip and the quadrant has the x-direction reinforcement as bending reinforcement. Similarly, the load on the x-direction strips is determined by d_y. The resulting loads are $w_{ACI,x}$ on strips in the x-direction and $w_{ACI,y}$ on strips in the y-direction. The resulting expressions for the loads on the strips then are, with f_c' the concrete compressive strength:

$$w_{ACI,x}\left[\frac{\text{kN}}{\text{m}}\right] = 0.166\times d_y\,[\text{mm}]\sqrt{f_c^{'}\,[\text{MPa}]} \tag{6}$$

$$w_{ACI,x}\left[\frac{\text{lbf}}{\text{in}}\right] = 2.00\times d_y\,[\text{in}]\sqrt{f_c^{'}\,[\text{psi}]}$$

$$w_{ACI,y}\left[\frac{\text{kN}}{\text{m}}\right] = 0.166\times d_x\,[\text{mm}]\sqrt{f_c^{'}\,[\text{MPa}]} \tag{7}$$

$$w_{ACI,y}\left[\frac{\text{lbf}}{\text{in}}\right] = 2.00\times d_x\,[\text{in}]\sqrt{f_c^{'}\,[\text{psi}]}$$

An overview of the considered loads on the *x*- and *y*-direction strips for a bridge deck slab subjected to a concentrated load is given in Figure 3.

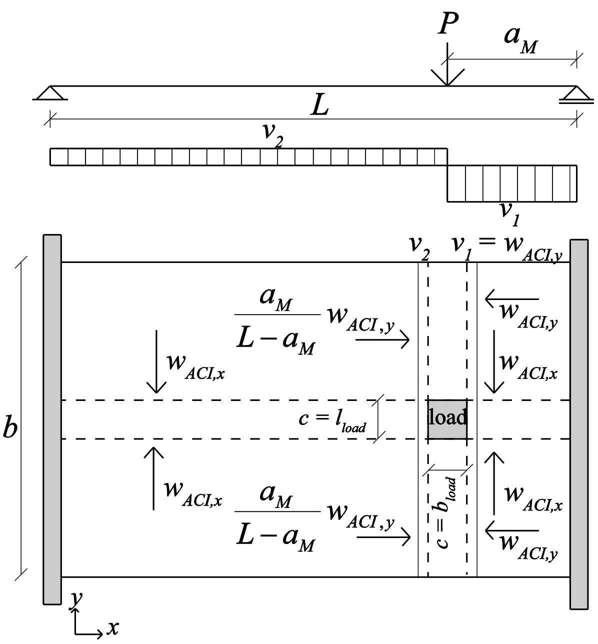


Figure 3: Overview of loads on strips in x- and y-direction and implications of static equilibrium.

5.2 Implications of static equilibrium

The second step to apply the Strip Model to one-way slabs under concentrated loads close to supports is to analyze the static equilibrium of the situation. The shear stress is not the same on each side of strips perpendicular to the main span direction. Failure will take place when the maximum shear stress is reached on the interface between the strip and the quadrants on the side of the strip that is more heavily loaded because of the shear diagram. To find the capacity of the strips, the implication is that the total load on the strip will not be $2w_{ACI}$ anymore. From the shear diagram, it is known that the averaged stresses v_1 and v_2 (as shown in Figure 3) caused by a load P are:

$$v_1 = \frac{L - a_M}{L}\frac{P}{b} \tag{8}$$

$$v_2 = \frac{a_M}{L}\frac{P}{b} \tag{9}$$

with

a_M the smallest value of the distance between the center of the load and the center of the support, and the distance between the center of the load and the point of contraflexure

L the distance between the points of contraflexure, or the span length for a simply supported slab;

P the concentrated load;

b the slab width.

The stresses are averaged over the full width, since they are used here only to find the relative maximum shear that will occur on the side of the strip that is less heavily loaded. The ratio of the stresses equals:

$$\frac{v_2}{v_1} = \frac{\frac{a_M}{L}}{\frac{L - a_M}{L}} = \frac{a_M}{L - a_M} \tag{10}$$

If the shear capacity $w_{ACI,y}$ is reached in v_1, the stress v_2 equals:

$$v_2 = \frac{a_M}{L - a_M} w_{ACI,y} \tag{11}$$

These loads are shown in Figure 3. The maximum value of the load, replacing $2w_{ACI,y}$ in the original Strip Model, then becomes:

$$q_{max,y} = \frac{L}{L - a_M} w_{ACI,y} \tag{12}$$

with

$q_{max,y}$ the total load on the y-direction strips;

$w_{ACI,y}$ the maximum shear on the interface between the quadrant and the strip in the y-direction as given by Eq. (7).

5.3 Reduction for self-weight

The maximum capacity of the strip is the total maximum stress that can occur at the position of the interface between the strip and the quadrant for all considered loads. When studying a slab subjected to a single concentrated load, the concentrated load as well as the self-weight of the structure contribute to the shear stress on the studied section. Therefore, to find the maximum value of the concentrated load, the effect of the self-weight has to be subtracted from the total available capacity. For slab-column connections, this reduction will be small because the depths of the slabs are small. For slab bridges, on the other hand, much larger cross-sections are used, and the effect of the self-weight becomes more important. For one-way spanning slabs, it is assumed that the self-weight only acts in the main span direction, and thus only affects the interface between the quadrants and the *y*-direction strips. The sectional shear at the position of the load can be transformed into a distributed load (units [kN/m] or [lbf/in] like w_{ACI}) by dividing the sectional shear by the total width of the member. For the strips in the *y*-direction, the total load then becomes:

$$q_{max,y} = \frac{L}{L - a_M}\left(w_{ACI,y} - v_{DL}\right) \tag{13}$$

The effects on the maximum loads on the strips is shown in Figure 4. The effect of torsion is represented in Figure 4 by the factor β, which will be discussed later in this paper when considering asymmetric loading conditions, where torsion reduces the available capacity further.

5.4 Size effect

The expression for the shear capacity at the interface between the strips and quadrants, as given in Eqs. (6) and (7), has one major drawback: it gives good results for small specimens, but becomes unconservative for larger specimens (Collins and Kuchma, 1999). This problem is caused by the size effect in shear (Bazant and Kim, 1984; Kani, 1967): as the depth of members increases, their shear capacity does not increase proportionally. A recommendation (Alexander, 2016) for taking into account the size effect on the shear capacity of slabs was built into the expression for the shear capacity, resulting in the following expressions:

$$w_{ACI,x}\left[\frac{\text{kN}}{\text{m}}\right] = 0.166 \times d_y\left[\text{mm}\right]\sqrt{f_c^{'}\left[\text{MPa}\right]}\left(\frac{100\text{mm}}{d\left[\text{mm}\right]}\right)^{\frac{1}{3}} \tag{14}$$

$$w_{ACI,x}\left[\frac{\text{lbf}}{\text{in}}\right] = 2.00 \times d_y\left[\text{in}\right]\sqrt{f_c^{'}\left[\text{psi}\right]}\left(\frac{3.94\text{in}}{d\left[\text{in}\right]}\right)^{\frac{1}{3}}$$

$$w_{ACI,y}\left[\frac{\text{kN}}{\text{m}}\right] = 0.166 \times d_x\left[\text{mm}\right]\sqrt{f_c^{'}\left[\text{MPa}\right]}\left(\frac{100\text{mm}}{d\left[\text{mm}\right]}\right)^{\frac{1}{3}} \tag{15}$$

$$w_{ACI,y}\left[\frac{\text{lbf}}{\text{in}}\right] = 2.00 \times d_x\left[\text{in}\right]\sqrt{f_c^{'}\left[\text{psi}\right]}\left(\frac{3.94\text{in}}{d\left[\text{in}\right]}\right)^{\frac{1}{3}}$$

with:

d the average effective depth (average of d_x and d_y).

The introduction of this size effect term leads to a good correspondence with the experimental results (Lantsoght, 2016a), as will be shown later with the case study of the Ruytenschildt Bridge.

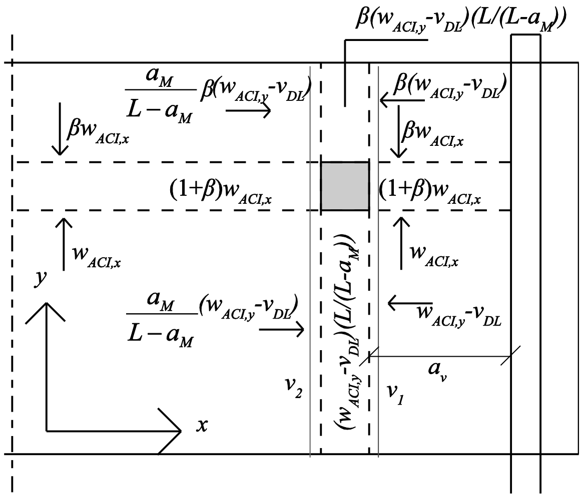


Figure 4: Overview of loads on strips in x- and y-direction and implications of static equilibrium, including reduction of self-weight and effect of torsion.

5.5 Loads close to supports

For loads close to the support, as is the governing case for the shear capacity of slab bridges under concentrated live loads, a direct compression strut can develop between the load and the support. This strut leads to an increase in the shear capacity (Grebovic and Radovanovic, 2015). For example, in Figure 4, if the load is placed close to the support (with $a_v \leq 2d_x$), the x-direction strip between the load and the support will have an enhanced capacity as a result of direct load transfer. Here, a_v is the clear shear span (face-to-face distance between the load and the support) and d_x the effective depth to the longitudinal reinforcement. To take this effect into account, the following enhancement factor is proposed:

$$1 \leq \frac{2d_x}{a_v} \leq 4 \tag{16}$$

This factor is based on the proposed enhancement of the side of the punching perimeter facing the support as proposed by Regan for slabs under concentrated loads close to supports (Regan, 1982). The effect of torsion, which will be discussed in section 6 on asymmetric loading conditions, also needs to be considered for loads close to the support.

5.6 Different longitudinal and transverse reinforcement

The original Strip Model was developed for building slabs, which have similar reinforcement layouts in both directions. For bridge slabs, on the other hand, the reinforcement in the transverse direction is typically much less than the reinforcement in the longitudinal direction. In European practice, around 20% of the longitudinal reinforcement is used for the transverse reinforcement. As a result, the moment capacities of the strips in the y-direction will be much lower than the moment capacities in the longitudinal directions. The difference in effective depth (d_x for the longitudinal reinforcement and d_y for the transverse reinforcement) needs to be taken into account for the determination of the moment capacities as well as for the maximum shear, as given by Eqs. (14) and (15).