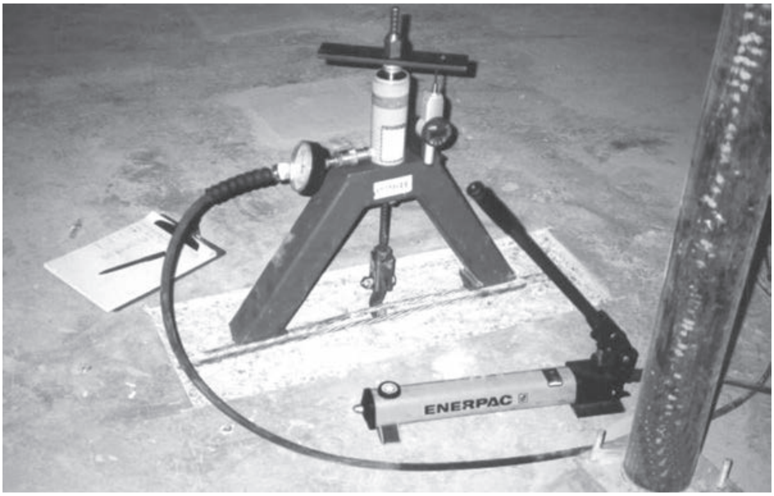


Figure 8— In-situ tension test set-up in field

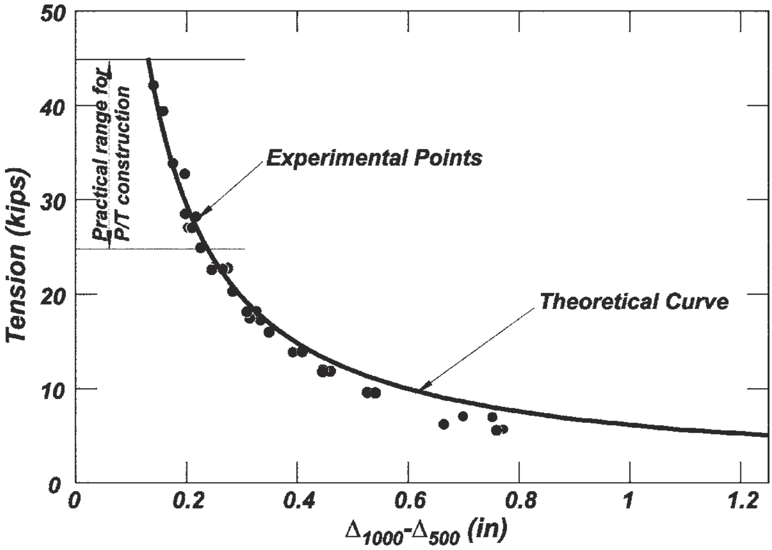
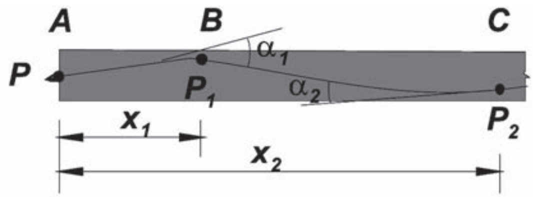


Figure 9— Calibration of in-situ tension tester



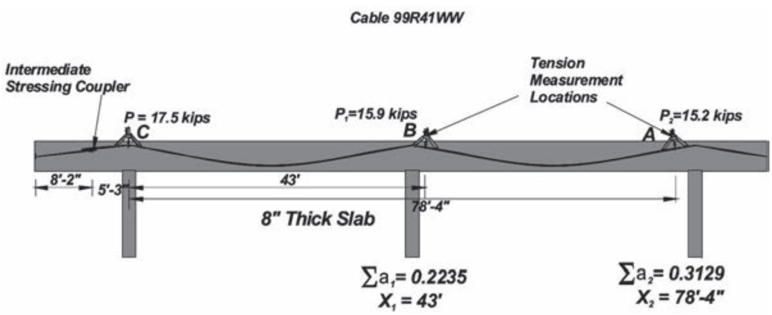
$$P = P_1 e^{(\mu\alpha_1 + Kx_1)} \qquad P = P_2 e^{(\mu(\alpha_1+\alpha_2)+Kx_2)}$$

$$\log_e \frac{P}{P_1} = \mu\alpha_1 + Kx_1 \qquad \log_e \frac{P}{P_2} = \mu(\alpha_1+\alpha_2)+Kx_2$$

Field Measure P, P_1 & P_2

Solve for μ & K

Figure 10— Procedure for calculation of curvature and wobble coefficients



Location	Δ_{1000}-Δ_{500}* (in.)	Force (kips)	Location**	Comments
A	0.3881	15.2	83'-7"	Stressing jack maintained at 3500 psi
B	0.3704	15.9	48'-3"	
C	0.3368	17.5	5'-3"	

* Average of three readings

** Distance of measuring point from stressing jack

Figure 11— Field determination of curvature and wobble coefficients

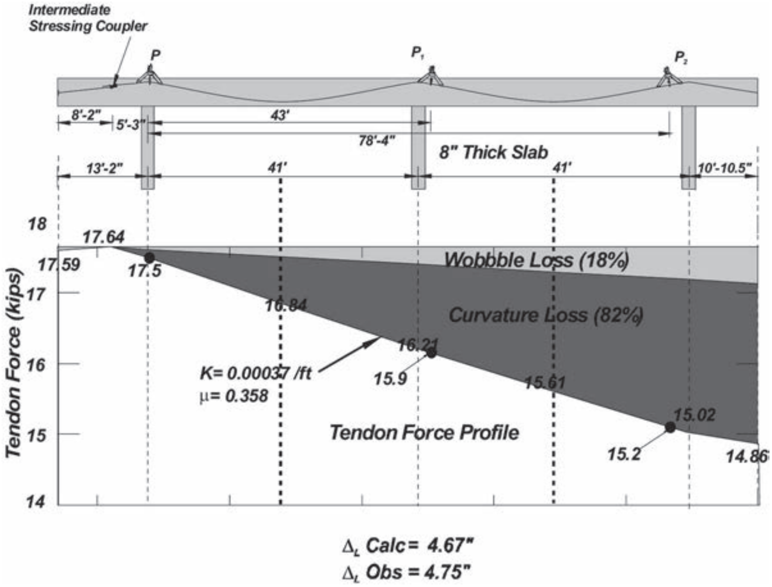


Figure 12— Measured friction and wobble losses

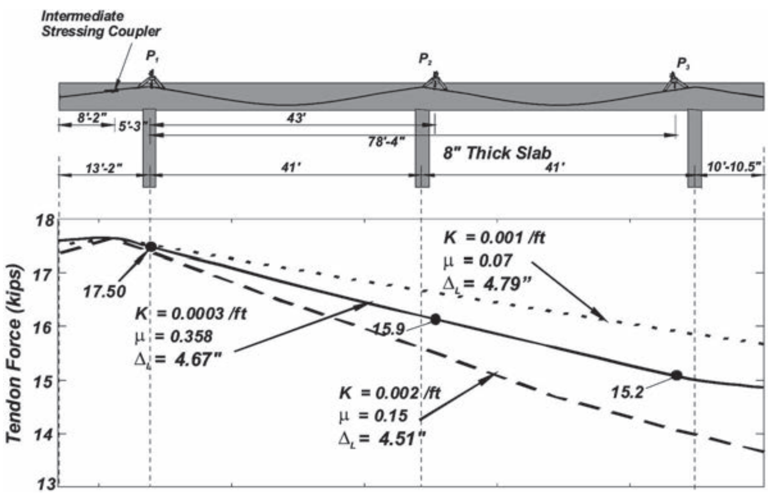


Figure 13— Range of friction losses

SP-231—9

Effects of Friction and Slip-Back on Stresses in Post-Tensioning Tendons

by J.F. Stanton

Synopsis: In post-tensioned systems, friction during stressing and slip-back due to setting the wedge anchors cause loss of prestress. If the tendon is long or contains sharp curvatures, these losses can be significant. This paper summarizes methods for calculating the losses and provides an evaluation of the numerical coefficients suggested by ACI 318-05 for friction. Equations are provided where closed form methods are possible, and numerical methods are outlined for other cases.

Keywords: anchorage set; friction; post-tension; prestress loss

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INTRODUCTION

When a tendon is post-tensioned, the stress along it varies because of friction along the length and slip-back at the anchorage. These behaviors represent instantaneous losses, which must be added to the subsequent changes in tendon stress due to creep, shrinkage, etc. The purpose of this paper is to summarize the effects of friction and slip-back, to present methods of computing them and to offer some suggestions for stressing strategies that take account of their effects. The information presented here presupposes the availability of reliable values of the friction properties of the system.

DEVELOPMENT OF EQUILIBRIUM EQUATIONS

Friction in a tendon is usually characterized as having one component due to curvature and another due to "wobble." The first is obtained as the friction caused by the normal force induced when a cable passes round a curved path, as illustrated in Figure 1. Equilibrium of a segment of length *dz*, in the directions perpendicular and parallel to the tendon axis respectively, gives

$$N = T d\theta \tag{1a}$$

$$dT = -F = -\mu |N| = -\mu T |d\theta| \tag{1b}$$

The absolute value sign arises because friction reduces the tendon force regardless of the sense of the change in angle. Wobble friction describes the loss in force per unit length of a nominally straight tendon. It may be caused by either adhesion of the tendon to the sides of the duct or, more likely, to accidental curves in the tendon path. It is expressed as K, a loss rate per unit length. When combined with Equations 1a and b, the result is

$$\frac{dT}{dz} = -\left(K + \mu \left| \frac{d\theta}{dz} \right| \right) T = -\lambda T \tag{2}$$

where $\lambda = K + \mu \left| \frac{d\theta}{dz} \right|$ (3)

In Equation 2, the term $d\theta/dz$ is the change in slope per unit length, which can be interpreted as the curvature of the tendon profile.

EXACT AND APPROXIMATE SOLUTIONS FOR FRICTIONAL EFFECTS

If the tendon path consists of a single parabolic curvature, as might be the case for a draped tendon in a simply supported beam, the exact solution of Equation 2 is given by

$$T(z) = T_0 \exp\{-\lambda z\} \tag{4}$$

where T_0 is the force at the jacking end.

If $\lambda z << 1.0$, Equation 4 may be expanded in a Taylor Series and truncated to give the linear approximation

$$T(z) \approx T_0 (1 - \lambda z) \tag{5}$$

For a tendon profile that can be represented by a general, *n*th order, polynomial, $p_n(z)$, the curvature, $\phi(z)$, is given by $p_n''(z)$, and the distribution of tendon force is

$$T(z) = T_0 \exp\left\{-\int_0^z |\phi(s)| ds\right\} = T_0 \exp\left\{-\int_0^z |p_n''(s)| ds\right\} \tag{6}$$

For example, Figure 2 shows the end span of a continuous beam, in which the tendon profile is a cubic. It passes through the cgc at the end, is horizontal over the first interior support, and maintains 2” cover to the cgp at both the high and the low points. The profile is given by

$$y(\zeta) = 104.1\left(\zeta - 2.173\zeta^2 + 1.115\zeta^3\right) \tag{7}$$

where $\zeta = z/L$

The curvature is

$$y''(\zeta) = \frac{104.1}{L^2}(-4.346 + 6.690\zeta) \tag{8}$$

which changes sign at $\zeta = 0.6494$. The integral of Equation 6 must be evaluated separately for $0 < \zeta < 0.6494$ and $0.6494 < \zeta < 1.0$ in order to include the absolute value. The slopes of the profile at $\zeta = (0, 0.6494, 1.0)$ are found to be (0.2890, -0.1189, 0.0), from which the total absolute change in angle over the span is 0.5267 radians, and

$$\frac{T(L)}{T(0)} = \exp(-0.0001 * 360 + 0.2 * 0.5267) = 0.8682 \tag{9}$$

T is plotted as a function of ζ in Figure 3. The equivalent load, w_{eq}, can be obtained from the tendon force and profile. It is a concept that is widely used in designing continuous systems, because it avoids the need for computing secondary moments when considering service load behavior. It is given by

$$w_{eq} = \frac{d(e_p T)}{dz} = Te_p'' + T'e_p' \tag{10}$$

where the prime indicates differentiation with respect to z. w_{eq} is plotted against z in Figure 4. In the absence of friction losses, it would be exactly linear. The effects of the losses are manifested in both terms in Equation 10, because T changes with z. In this example, as in most others, the second term is quite small compared with the first (< 9% in this case), and is often neglected.

FRICTION VALUES

The Commentary to ACI 318-05 gives values for wobble and curvature friction, which have been unchanged since the 1972 code. Their consistency may be examined based on the assumption that the friction attributed to wobble really arises from accidental curvature. The implied curvature is then

$$\phi(z) = \frac{d\theta}{dz} = \frac{K}{\mu} \tag{11}$$

Values of K, μ and the implied ϕ are given in Table 1 for various tendon types. The lack of a consistent pattern in the implied curvatures suggests that uncertainty exists in the suggested friction values.

The tendon types in most common use today are 7-wire strands either in metal ducts that are grouted after stressing and 7-wire strands in greased sheaths. The Commentary advises that for strands in rigid or semi-rigid ducts, the wobble coefficient may be taken as zero. This may be justified on the basis that the duct is sufficiently rigid that the accidental curvatures will be very small. Such ducts are often used in elements such as box girders, wherein the ducts are essentially constrained to exist in the plane of the web. This in itself permits lower accidental curvature values than are likely to occur in a greased mono-strand placed in a slab. There, the strands often curve in both the horizontal plane (to avoid obstructions such as openings) and the vertical plane (to provide upward load to balance gravity loads). The Commentary is silent on the question of whether the wobble coefficient should be used to represent both known horizontal curvatures and accidental curvatures, or only the latter. For thin slabs with numerous openings, the horizontal curvatures may be larger than the vertical ones, in which case it is rational to evaluate both curvatures explicitly, and to use the wobble coefficient to address only the truly accidental curvatures.

EFFECTS OF SLIP-BACK

Slip-back occurs when the wedges of a strand anchorage are set. In most multi-strand rams, the slip-back distance is a compromise between two opposing goals, and can be adjusted. If it is set too small, the strand drags on the wedge during stressing and can be damaged. If it is set too large, excessive strand stress is lost when the wedge is set. Values between about 3/16” and 3/8” are common.

Slip-back at the wedge causes the strand to slip back into the duct, and the stress drops below the jacking stress. In the absence of friction, the slip would extend along the whole strand, and the stress would drop by the same amount everywhere. However, friction prevents this from occurring, so the slip may penetrate over only part of the span. Two cases occur. In so-called “short beams”, the slip extends over the whole length, and the stress drops at the dead end anchorage. In “long beams”, the slip penetrates over only part of the span, and the stress at the dead end does not change.

Long beam

Single segment profile - Consider the simplest case of a tendon with parabolic profile. Directly after jacking, and prior to any slip back, the tendon force varies with z as

$$T(z) = T_j \exp\{-\lambda z\} \tag{12}$$

When slip occurs, the friction reverses and the force distribution in the slipped region is

$$T(z) = T_i \exp\{\lambda z\} \tag{13}$$

where T_i = the initial tension at the live end directly after setting the wedges.

If the slip penetrates to a point $z = b$, the stress force just to the right and left of $z = b$ must be the same. Therefore

$$T_j \exp\{-\lambda z\} = T(b) = T_i \exp\{\lambda z\} \tag{14}$$

so

$$\frac{T_i}{T_j} = \exp\{-2\lambda b\} \tag{15}$$

The drop in force at any point $z < b$ is given by

$$\begin{aligned} \Delta T &= T_j \exp\{-\lambda z\} - T_i \exp\{\lambda z\} \\ &= T_j\left(\exp\{-\lambda z\} - \exp\{-2\lambda b + \lambda z\}\right) \end{aligned} \tag{16}$$

Thus the slip back distance, u_{slip}, can be expressed as

$$u_{slip} = \int_0^b \frac{\Delta T}{A_p E_p} dz = \frac{f_j}{E_p \lambda}\left(1 - \exp\{-\lambda b\}\right)^2 \tag{17}$$

where f_j = the jacking stress in the tendon.

Since u_{slip} is known and the distance b is not, Equation 17 may be inverted to give

$$b = \frac{-1}{\lambda}\ln\left(1-\sqrt{\frac{u_{slip}E_p\lambda}{f_j}}\right) = \frac{-1}{\lambda}\ln(1-\eta) \quad (18)$$

where $$\eta = \sqrt{\frac{u_{slip}E_p\lambda}{f_j}} \quad (19)$$

The drop in stress at the jacking end, from T_j to T_i, is of primary interest. Equations 16, 18 and 19 can be combined to give

$$\frac{T_i}{T_j} = (1-\eta)^2 \quad (20)$$

and $$\frac{T(b)}{T_j(0)} = \frac{T_i(0)}{T(b)} = (1-\eta) \quad (21)$$

At $z = b$ the stress is the same at jacking (before slip-back) and under initial conditions (after slip-back), so no subscript is used.

Because $\eta << 1.0$, Equation 18 may be linearized by expanding the *ln* term and truncating the series, to give

$$b \approx \frac{\eta}{\lambda} = \sqrt{\frac{u_{slip}E_p}{\lambda f_j}} \quad (22)$$

However, this approximation is not much simpler than the exact value (Equation 18), so, although several authors present it, it is of little real interest.

Example

Let $\lambda = 0.001$ in^{-1}, $E_p = 28{,}500$ ksi, $f_j = 200$ ksi and $u_{slip} = 1/8$". Then Equations 18 and 19 predict slip-back penetration distances of 143.25" and 133.46" respectively. Figure 5 illustrates the distribution of stress before and after slip-back, using both the exact and approximate procedures. The slip back distance, u_{slip}, can be interpreted as $(1/E_p)$ times the triangular area.

Multi-segment profile - In a continuous beam or slab, the tendon profile is likely to be made up from a number of individual segments. Use of parabolic segments offers the advantage that the equivalent load is very nearly uniform, thereby counteracting closely what is likely to be a uniformly distributed gravity load. Figure 6 shows the end spans of a multi-span slab, which is supported on beams. The tendon profile consists of upward curving parabolas in the spans, and down-curving parabolas over the beams. Figure 7 shows the corresponding stress distribution in the tendon before and after slip-back. (The data used was: clear span = 180”, beam width = 20”, $\lambda = 0.00015$ in^{-1} in the spans, $\lambda = 0.0006$ in^{-1} over the beams, $u_{slip} = 0.375$ in., $f_j = 200$ ksi, $E_p = 28{,}500$ ksi). The slip-back penetrates for a total distance of 570 in, or nearly to the end of the third clear span.

The slip-back distance is found using the same principles as for a single segment profile, but the numerical computations have to be modified to deal with the local changes in λ. The area of the approximately triangular region between the jacking and initial stress curves is once again equal to the product $E_p * u_{slip}$. However, the value of b cannot easily be found in closed form, and is most easily determined numerically. This is done by computing the areas between f_j and f_i within each segment and adding the results. If the jacking stresses at the start and end of segment n are $f_{j,n-1}$ and $f_{j,n}$, and the initial stresses (after slip-back) are $f_{i,n-1}$ and $f_{i,n}$, then the area within segment n is

$$\begin{aligned} A_n &= \frac{f_{j,n-1}}{\lambda_n}\left[1 - e^{-\lambda_n L_n}\right] + \frac{f_{i,n-1}}{\lambda_n}\left[1 - e^{\lambda_n L_n}\right] \\ &= \frac{1}{\lambda_n}\left\{\left(f_{j,n-1} - f_{j,n}\right) - \left(f_{i,n} - f_{i,n-1}\right)\right\} \end{aligned} \tag{23}$$

Equation 23 gives the exact value. An approximate value may be obtained by treating the region as a trapezoid, which leads to

$$A_n \approx \frac{L_n}{2}\left\{\left(f_{j,n-1} - f_{i,n-1}\right) + \left(f_{j,n} - f_{i,n}\right)\right\} \tag{24}$$

The jacking and initial stresses at each point (at the start and end of each segment) are obtained by assuming a specific value of b and using, for each segment, the exact relationships

$$\frac{f_{j,n}}{f_{j,n-1}} = \exp\left\{-\lambda_n L_n\right\} \tag{25a}$$

$$\frac{f_{i,n-1}}{f_{i,n}} = \exp\left\{-\lambda_n L_n\right\} \tag{25b}$$

In the segment in which the slip-back ends, the distance from the left end of the segment to the end of the slip-back region is substituted for L_n. The total area is then computed by adding the areas A_n, and then the u_{slip} value calculated from Equation 18 is compared with the true (known) value. The value of b is then adjusted until the target and calculated values of u_{slip} agree within an acceptable tolerance. This can easily be done on a spreadsheet or other program.

In some cases the segments may consist of profiles that are not parabolic. Then, solving for the exact area of the segment becomes more difficult, and the approximation of Equation 24 may be used. The error is likely to be small unless the curvatures or friction values are unusually high. If necessary, the (exact) stresses may be computed at