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Fracture Mechanics of Concrete: Concepts, Models and Determination of Material Properties

Reported by ACI Committee 446, Fracture Mechanics*

Zdeněk P. Bazant[1,2]
(Chairman)

Vellore S. Gopalaratnaml[1,3]
(Secretary)

Members

Oral Buyukozturk[4]	Victor C. Li[1]	Gilles Pijaudier-Cabot[3]
Luigi Cedolin[4]	Feng-Bao Lin[1]	Victor Saouma[1,3]
David Darwin[3]	Steven L. McCabe	Surendra P. Shah[1,3]
Manuel Elices[1,3]	Sheng-Taur Mau[3]	Robert L. Sicrakowski
Shu-Jin Fang	Jacky Mazars[4]	Wimal Suaris'
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Neil M. Hawkins	Antoine E. Naaman'	Tatsuya Tsubaki
Hideyuki Horii[1]	C. Dean Norman	C. Vipulanandanl
Jeremy Isenberg	Phillip A. Pfciffer	Methi Wecharatanal

The committee wishes to recognize the contributions of the following non-voting members:

Farhad Ansari[1]	Arne Hillerborg1	Pere C. Prat[1]
Ravindra Gettu[4]	B. L. Karihaloo[1]	Hans W. Reinhardt[1]

[1]Members of Subcommittee I (chaired by Bazant) which prepared the report
[2]Principal Authors
[3]Contributing Authors

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SYNOPSIS

In the first of its series of four state-of-the-art reports under preparation, the Committee describes the basic concepts of fracture mechanics of concrete, the existing theoretical models, and the methods for determining the material fracture parameters. Chapter 1 offers five reasons for introducing fracture mechanics into certain aspects of design of concrete structures, including some code provisions: (1) a theoretical energy argument; (2) the need to achieve objectivity of finite element solutions, i.e., eliminate spurious mesh sensitivity; (3) the progressive (propagating) nature of failure, implied whenever the load-deflection diagram lacks a yield plateau; (4) the need to rationally predict ductility and energy absorption capability; and most importantly, (5) the effect of structure size on the nominal strength (i.e., nominal stress at maximum or ultimate load) as well as on ductility and energy absorption capability. The size effect is due to stored energy release into the fracture front, and is not governed by Weibull-type statistical theory. Experimental evidence on the existence of the size effect, hitherto ignored in design practice and code provisions, is documented.

Chapter 2 gives a brief review of the necessary basic results of linear elastic fracture mechanics (LEFM). In concrete, departures from this classical theory are caused by the existence of distributed cracking (or damage) in a progressively softening fracture process zone which surrounds the tip of a continuous crack. In Chapter 3 nonlinear fracture models characterizing the softening stress-displacement or stress-strain relations (such as those of Hillerborg's fictitious crack model, crack band model, nonlocal strain-softening models, etc.) are described and random particle simulation of aggregate microstructure is discussed. The principles of implementation of these models in finite element programs are also outlined. Chapter 4 presents simpler nonlinear fracture models which represent adaptations of linear elastic fracture mechanics, such as Jenq and Shah's model and the R-curve, along with determination of geometry-dependent R-curves from the size effect law proposed by Bazant. This law, describing the approximate dependence of the nominal stress at maximum load on structure size, is discussed in Chapter 5, and structural response is characterized by the brittleness number.

Chapter 6 presents in considerable detail the current methods for experimental and analytical determination of material fracture parameters, including the quasi-LEFM methods, RILEM (work-of-fracture) method, the Jenq-Shah and Karihaloo-Nallathambi methods, and the size-effect method. Experimental determination of the characteristic length for nonlocal continuum models and the strain-softening properties is then examined, and material parameters for modes II and III, shear fractures and mixed mode fracture are also discussed. Chapter 7 then proceeds to describe various influencing factors, such as the loading rate, humidity and temperature, as well as the effect of cyclic loading. Chapter 8 is devoted to the effect of reinforcing bars and their bond slip on fracture propagation, and to fracture of fiber-reinforced concrete. Chapter 9 deals with more theoretical problems of modeling systems of interacting cracks. Attention is focused on systems of parallel growing cracks. Their stability decides the spacing and width of the cracks from the mechanics viewpoint.

It is concluded that, after a decade of rapid progress in research, the time appears ripe for introducing fracture mechanics into design practice. This should not only bring about more uniform safety margins, thus improving safety and economy of design, but also pave the way for safer and more efficient use of high-performance concretes and permit design extrapolations beyond the range of previous experiments and design.

KEYWORDS: Brittleness, concrete, concrete structures, crack spacing and width, cracking, damage mechanics, design codes, ductility, failure, fiber-reinforced concrete, nonlocal continuum models, reinforced concrete, size effect, strain softening, structural design, testing methods, ultimate loads.

Introduction

Concrete structures are full of cracks. Failure of concrete structures typically involves stable growth of large cracking zones and the formation of large fractures before the maximum load is reached. Yet design is not based on fracture mechanics, even though the basic fracture mechanics theory has been available since the middle of this century. So why has not fracture mechanics been introduced into concrete design? Have concrete engineers been guilty of ignorance? Not at all. The forms of fracture mechanics which were available until recently were applicable only to homogeneous brittle materials such as glass, or to homogeneous brittle-ductile metals. The question of applicability of these classical theories to concrete was explored long ago – the idea of using the stress intensity factor appeared already in the early 1950's (e.g., Bresler and Wollack, 1952) and serious investigations started in the 1960's (e.g., Kaplan, 1961, and others). But the answer was, at that time, negative (e.g., Kesler, Naus and Lott, 1971). As is now understood, the reason was that in concrete structures one must take into account strain-softening due to distributed cracking, localization of cracking into larger fractures prior to failure, and bridging stresses at the fracture front. A form of fracture mechanics that can be applied to such structures has been developed only during the last decade.

Concrete design has already seen two revolutions. The first, which made the technology of concrete structures possible, was the developrnent of the elastic no-tension analysis during 1900-1930. The second revolution, based on a theory conceived chiefly during the 1930's, was the introduction of plastic limit analysis, which occurred during 1940-1970. There are good reasons to believe that the introduction of fracture mechanics into the design of concrete structures, both reinforced and unreinforced, might be the third major revolution. The theory, formulated mostly during the last dozen years, finally appears to be ripe.

Fracture researchers have at the present no doubt that the introduction of fracture mechanics into the design criteria for all brittle failures of reinforced concrete structures (such as diagonal shear, punching shear, torsion or pull out, or for concrete dams), can bring about significant benefits. It will make it possible to achieve more uniform safety margins, especially for structures of different sizes. This, in turn, will improve economy as well as structural reliability. It will make it possible to introduce new designs and utilize new concrete materials. Fracture mechanics will be particularly important for high strength concrete structures, fiber-reinforced concrete structures, concrete structures of unusually large sizes, and for prestressed structures. The application of fracture mechanics is most urgent for structures such as concrete dams and nuclear reactor vessels or containments, for which the safety concerns are particularly high and the consequences of a potential disaster enormous.

Surveys of concrete fracture mechanics have recently been prepared by various committees (Wittmann, 1983, and Elfgren, 1989). However, due to the rapidly advancing research, the contents of the present state-of-the-art report are quite different. A unified, systematic presentation, rather than a compilation of all the contributions by various authors, is attempted in the present state-of-art report. The report is aimed primarily at researchers, not necessarily specialists in fracture mechanics. However, it should also be of interest to design engineers because it describes a theory that is likely to profoundly influence the design practice in the near future. Subsequent reports dealing with applications in design, finite element analysis of fracture, and dynamic fracture analysis, are in preparation by ACI Committee 446.

Chapter 1. WHY FRACTURE MECHANICS?

Fracture mechanics, in a broad sense, is a failure theory which (1) uses energy criteria, possibly in conjunction with strength criteria, and (2) which takes into account failure propagation through the structure.

1.1 Five Reasons for Fracture Mechanics Approach

Since concrete structures have been designed and successfully built according to codes which totally ignore fracture mechanics theory, it might seem unnecessary to change the current practice. Nevertheless, there are five compelling reasons for doing so.

Reason 1: Energy Required for Crack Formation

From the strictly physical viewpoint, it must be recognized that while crack initiation may depend on stress, the actual formation of cracks requires a certain energy – the fracture energy – which represents the surface energy of a solid. Hence, energy criteria should be used. This argument might suffice to a physicist but not a designer. But there are other reasons.

Reason 2: Objectivity of Calculations

Any physical theory must be objective in the sense that the result of calculations made with it must not depend on subjective aspects such as the choice of coordinates, the choice of mesh, etc. If a theory is found to be unobjective, it must be rejected. There is no need to even compare it to experiments. Objectivity comes ahead of experimental verification.

A powerful approach to finite element analysis of concrete cracking is the concept of smeared cracking, introduced by Rashid (1968). According to this approach, the stress in a finite element is limited by the tensile strength of the material, f'_t, and after reaching this strength limit, the stress in the finite element must decrease. As initially practiced, the stress was assumed to decrease suddenly to zero, in a vertical drop; but soon it was realized that better and more realistic results are usually obtained if the stress is reduced gradually, i.e., the material is assumed to exhibit strain-softening (Scanlon, 1971; Lin and Scordelis, 1975); see Fig. l.la. The concept of strain-softening, though, proved to be a mixed blessing. After strain-softening had been implemented in large finite element programs and widely applied, it was discovered that the convergence properties are incorrect and the calculation results are not objective with regard to the analyst's choice of the mesh, i.e., the results significantly change if the mesh is refined (Bazant, 1976, 1982; Bazant and Cedolin, 1979, 1980, 1983; Bazant and Oh, 1983a; Darwin, 1985; Rots, Nauta, Kusters and Blaauwendraad, 1985). Similar problems are encountered when cracking is modeled as discrete interelement cracks,

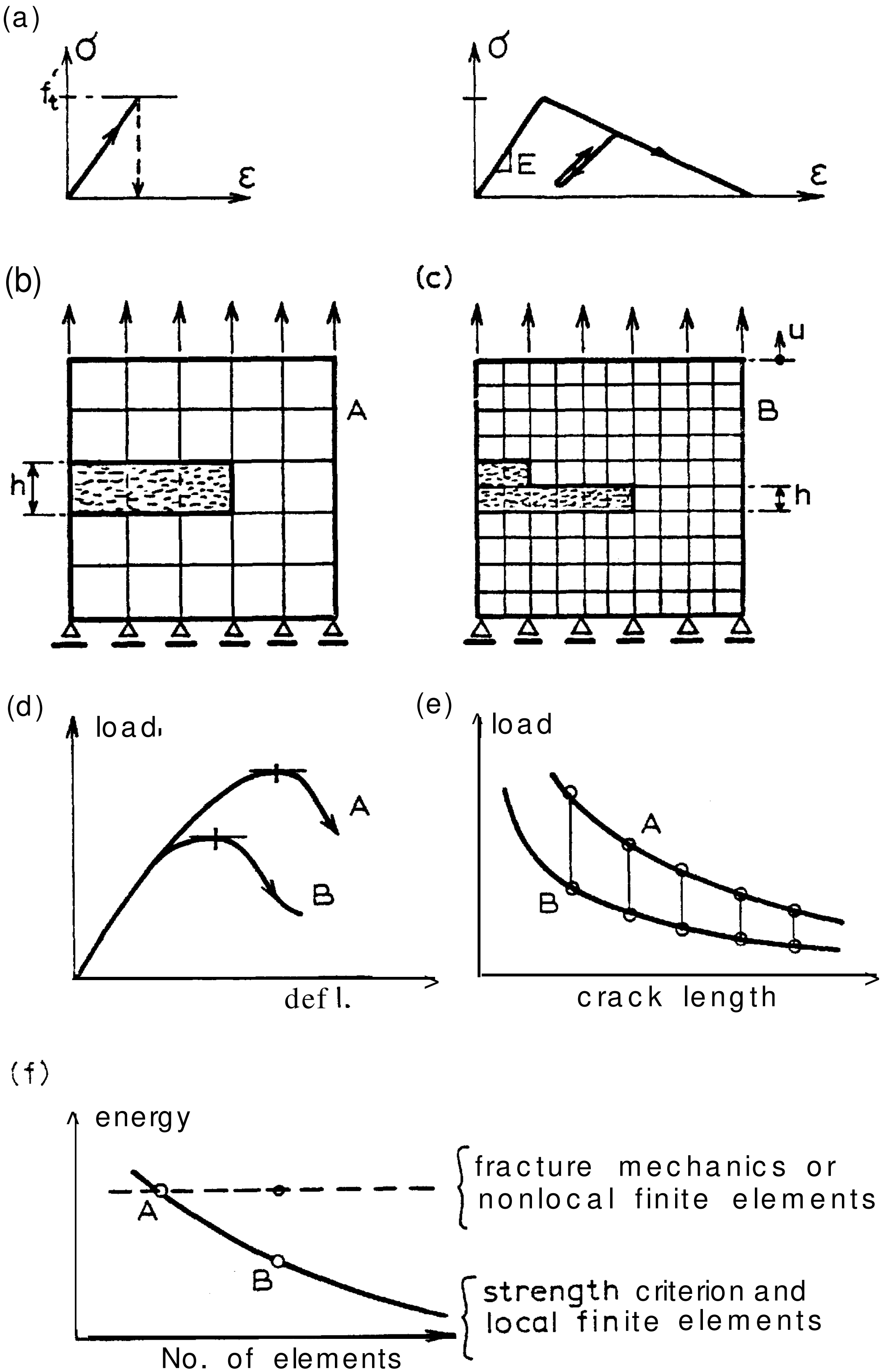

Fig. 1.1 Spurious Mesh Sensitivity

based on the strength concept (this approach was introduced into finite element analysis by Clough, 1962, and by Ngo and Scordelis, 1967).

The problem of spurious mesh sensitivity can be illustrated, for example, by the rectangular panel in Fig. 1.1b and c, which is subjected to a uniform vertical displacement at the top boundary. A small region near the center of the left side is assumed to have a slightly smaller strength than the rest of the panel, and consequently a smeared crack band starts growing from left to right. The solution is obtained by incremental loading with two finite element meshes of very different mesh sizes as shown. By stability checks it is found that the cracking must always localize into a band of single element width at the cracking front (Fig. 1.1b,c). The typical numerical results for this, as well as various other problems are illustrated in Fig. 1.1d,e,f. In the load-deflection diagram (Fig. 1.1d), it is seen that the peak load as well as the post-peak softening is strongly dependent on the mesh size, being roughly proportional to $h^{-1/2}$ where h is the element size. Plotting the load (reaction) versus the length of the crack band, large differences are again found (Fig. 1.1e).

The energy which is dissipated due to cracking decreases with the refinement of the finite element mesh (Fig. 1.1f) and converges to 0 as $h \longrightarrow 0$.

The foregoing unobjectivity is physically unacceptable. The only way to avoid it is some form of fracture mechanics. By specifying the energy dissipated by cracking per unit length of the crack or the crack band, the overall energy dissipation is forced to be independent of the element subdivision (the horizontal dashed line in Fig. 1.1f), and so is the maximum load.

Reason 3: Lack of Yield Plateau

Based on load-deflection diagrams, one may distinguish two basic types of structural failure: plastic and brittle. The typical characteristic of plastic failure is that the structure develops a single-degree-of-freedom mechanism such that failure in various parts of the structure proceeds simultaneously, in proportion to a single parameter. Such failures are manifested by the existence of a long yield plateau on the load-deflection diagram (Fig. 1.2a). If the load-deflection diagram does not have such a plateau, the failure is not plastic but brittle (or brittle-ductile) (Fig. 1.2b). If there are no significant geometric effects such as the P-Δ effect in buckling, the absence of a plateau implies the existence of softening in the material due to fracture, cracking or other damage; it implies that the failure process cannot develop a single degree-of-freedom mechanism but consists of propagation of the failure zone throughout the structure. So the failure is non-simultaneous and propagating.

To illustrate this behavior, consider the punching shear failure of a slab (Fig. 1.3). The typical (approximate) distributions of tensile stress σ along the failure surface are drawn in the figure. If the material is plastic, the cross section gradually plasticizes until all its points are at the yield limit. However, if the material exhibits softening, then the stress peak moves across the failure zone, leaving a reduced stress (softening) in its wake. The stress reduction is mild only if the structure is small, in which case the plastic limit analysis is not so far off. If the structure is large, however, the stress profile develops a steep stress drop behind the peak-stress point, and therefore the limit analysis solutions grossly over-estimate the failure

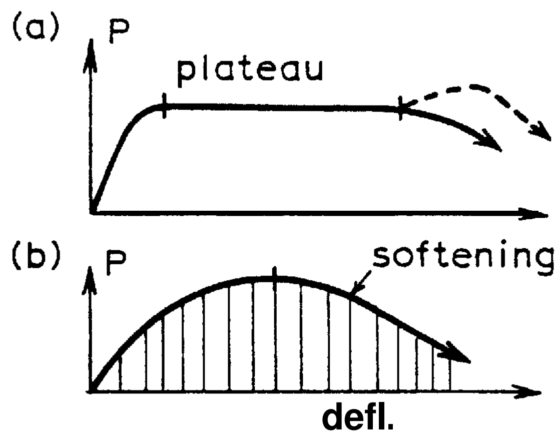

Fig.1.2 Load Deflection Diagram of Ductile and Brittle Structures

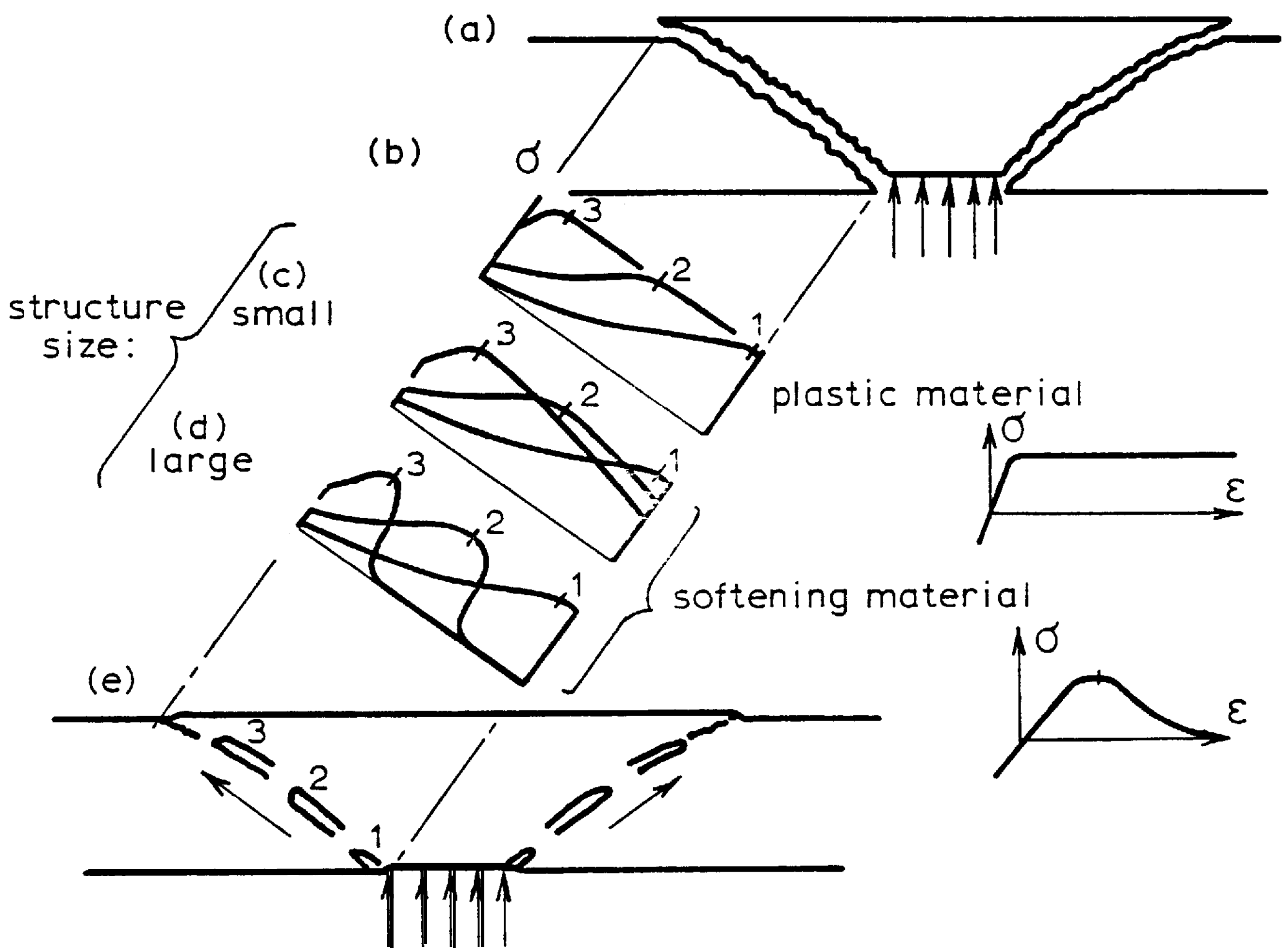

Fig.1.3 Progressive Nature of Failure Illustrated for Punching Shear of a Slab

load.

Reason 4: Energy Absorption Capability and Ductility

The area under the entire load deflection diagram represents the energy which the structure will absorb during failure and must therefore be supplied by the loads. Consideration of this energy is important especially for dynamic loading, and determines the ductility of the structure. Plastic limit analysis can give no information on the post-peak decline of the load and the energy dissipated in this process. Some form of fracture mechanics is necessary.

Reason 5: Size Effect

The size effect is, for design engineers, probably the most compelling reason for using fracture mechanics, and so a thorough discussion is in order.

The size effect is defined through a comparison of geometrically similar structures of different sizes, and is conveniently characterized in terms of the nominal stress σ_N at maximum (ultimate) load, P_u. When the σ_N -values for geometrically similar structures of different sizes are the same, we say that there is no size effect. A dependence of σ_N on the structure size (dimension) is called the size effect.

The nominal stress need not represent any actual 'stress in the structure but may be defined simply as $\sigma_N = P_u/bd$ when the similarity is two-dimensional, or as P_u/d^2 when the similarity is three-dimensional; b - thickness of the two-dimensional structure, and d characteristic dimension of the structure, which may be chosen as any dimension, e.g., the depth of the beam, or its span, since only the relative values of σ_N matter.

According to the classical theories, such as elastic analysis with allowable stress, plastic limit analysis, as well as any other theories which use some type of strength limit or failure criterion in terms of stresses (e.g., viscoelasticity, viscoplasticity), σ_N is constant, that is, independent of the structure size. This may be illustrated, e.g., by considering the elastic and plastic formulas for the strength of beams in bending, shear and torsion (regarding the definition $\sigma_N = P_u/bd$ for torsion, note that one may set $P_u = T_u/r$ where T_u= ultimate torque, P_u = force acting on an arm, r, such that r/H or r/a is constant for similar structures of different sizes; H = cross section depth, a = crack length). It is seen that these formulas are of the same form except for a factor. Thus, if we plot log σ_N vs. log d, the failure states according to a strength or yield criterion are always given by a horizontal line (dashed line in Fig. 1.4). So failures according to strength or yield criteria exhibit no size effect.

By contrast, failures governed by linear elastic fracture mechanics exhibit a rather strong size effect which in Fig. 1.4 is described by the inclined dashed line of slope $-1/2$. The reality for concrete structures is a transitional behavior illustrated by the solid curve in Fig. 1.4. This curve approaches the horizontal line for the strength criterion if the structure is very small, and the inclined straight line for linear elastic fracture mechanics if the structure is very large (the precise meaning of "very small" and "very large" will be clarified by Eq. 5.11). This size effect, which is generally ignored by current codes (with a few exceptions), is obviously important in design.

Another size effect which calls for the use of fracture mechanics is effect of size on ductility.