frequently $(I_e)$ for $M_D$ equals $I_g$,

$$(a_t)_D = \xi_r \nu_t (a_i)_D \tag{4-5}$$

a fictitious value

$$(a_i)_{D+L} = \xi M_{D+L} \ell^2 / E_c (I_e) \text{ for } M_{D+L} \tag{4-6}$$

and then for live load,

$$(a_i)_L = (a_i)_{D+L} - (a_i)_D \frac{E_{ci}}{E_c} \tag{4-7}$$

The ACI-318 Codes (12,27) refer to $(a_t)_D + (a_i)_L$ in certain cases for example.

In general, the deflection of a non-composite reinforced concrete member at any time and including ultimate value in time is given by Eqs. (4-8) and (4-9) respectively (77).

$$a_t = \overbrace{(a_i)_D}^{(1)} + \overbrace{(a_t)_D}^{(2)} + \overbrace{a_{sh}}^{(3)} + \overbrace{(a_i)}^{(4)} \tag{4-8}$$

$a_u$ = (Eq. (4-8) except that $\nu_u$ and $(\varepsilon_{sh})_u$ shall be used in lieu of $\nu_t$ and $\varepsilon_{sh}$ when computing terms (2) and (3) respectively.)

(4-9)

where:

Term (1) is the initial dead load deflection as given by Eq. (4-4).

Term (2) is the dead load creep deflection as given by Eq. (4-5).

Term (3) is the deflection due to shrinkage warping as given by Eq. (3-12).

Term (4) is the live load deflection as given by Eq. (4-7).

4.3 Deflection of Composite Precast Reinforced Beams in Shored and Unshored Construction (48,49,77)

For composite beams, subscripts 1 and 2 are used to refer to the slab or the effect of the slab dead load and the precast beam, respectively. The effect of compression steel in the beam (with use of $\xi_r$) should be neglected when it is located near the neutral axis of the composite section.

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes, and the gross I (neglecting steel and cracking), be used in computing the composite moment of inertia, $I_c$, in Eqs. (4-10) and (4-12), with the exception as noted in term (7) for live load deflection. Note that shrinkage warping of the precast beam is not computed separately in Eqs. (4-10) and (4-12).

4.3.1 Deflection of Unshored Composite Beams

The deflection of unshored composite beams at any time and including ultimate values, is given by Eqs. (4-10) and (4-11) respectively.

$$a_t = \underbrace{(a_i)_2}_{(1)} + \underbrace{\nu_s (a_i)_2}_{(2)} + \underbrace{(\nu_{t2} - \nu_s)(a_i)_2 \frac{I_2}{I_c}}_{(3)} + \underbrace{(a_i)_1}_{(4)} + \underbrace{\nu_{t1} (a_i)_1 \frac{I_2}{I_c}}_{(5)} + \underbrace{a_\delta}_{(6)} + \underbrace{a_L}_{(7)} \qquad (4\text{-}10)$$

$$a_u = \underbrace{(a_i)_2}_{(1)} + \underbrace{\nu_s (a_i)_2}_{(2)} + \underbrace{(\nu_u - \nu_s)(a_i)^2 \frac{I_2}{I_c}}_{(3)} + \underbrace{(a_i)_1}_{(4)} + \underbrace{\nu_{us} (a_i)_1 \frac{I_2}{I_c}}_{(5)} + \underbrace{a_\delta}_{(6)} + \underbrace{a_L}_{(7)} \qquad (4\text{-}11)$$

where:

Term (1) is the initial dead load deflection of the precast beam, $(a_i)_2 = \xi M_2 \ell^2/E_{ci}I_2$. See Table 4.2.1 for $\xi$ and M values. For computing $I_2$ in Eq. (3-2), $M_{max}$ refers to the precast beam dead load and $M_{cr}$ to the precast beam.

Term (2) is the creep deflection of the precast beam up to the time of slab casting. $\nu_s$ is the creep coefficient of the precast beam concrete at the time of slab casting. Multiply $\nu_s$ and $\nu_u$ by $\xi_r$ (from Eq. 3-8) for the effect of compression steel in the precast beam. Values of $\nu_t/\nu_u = \nu_s/\nu_u$ from Eq. (2-8) are given in Table 2.4.1.

Term (3) is the creep deflection of the composite beam for any period following slab casting due to the precast beam dead load. $\nu_{t2}$ is the creep coefficient of the precast beam concrete at any time after slab casting. Multiply this term by $\xi_r$ (from Eq. 3-8) for the effect of compression steel in the precast beam. The expression, $I_2/I_c$, modifies the initial value, in this case $(a_i)_2$, and accounts for the effect of the composite section in restraining additional creep curvature after slab casting.

Term (4) is the initial deflection of the precast beam under slab dead load, $(a_i)_1 = \xi M_1 \ell^2/E_{cs}I_2$. See Table 4.2.1 for $\xi$ and M values. For computing I in Eq. (3-2), $M_{max}$ refers to the precast beam plus slab dead load and $M_{cr}$ to the precast beam.

Term (5) is the creep deflection of the composite beam due to slab dead load. $\nu_{t1}$ is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. Multiply $\nu_{t1}$ and $\nu_u$ by $\xi_r$ (from Eq. 3-8) for the effect of compression steel in the precast beam. See Term (3) for comment on $I_2/I_c$. $\nu_{us}$ is given by Eq. (2-13).

Term (6) is the deflection due to differential shrinkage. For simple spans, $a_\delta = Qy_{cs}\ell^2/8\, E_{cs}I_c$, where $Q = \delta A_1 E_{c1}/3$. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects (6,48). In the case of continuous members, differential shrinkage produces secondary moments (similar to the effect of prestressing but opposite in sign, normally) that should be included (58).

Term (7) is the live load deflection of the composite beam, which should be computed in accordance with Eq. (4-7), using $E_c I_c$. For computing $I_c$ in Eq. (3-2), $M_{max}$ refers to the precast beam plus slab dead load and the live load, and $M_{cr}$ to the composite beam.

Additional information on deflection due to shrinkage warping of composite reinforced concrete beams of unshored construction is given by Eq. (2) in Ref. 77.

4.3.2 Deflection of Shored Composite Beams

The deflection of shored composite beams at any time and including ultimate values is given by Eqs. (4-12) and (4-13), respectively.

$a_t$ = Eq. (4-10), with Terms (4) and (5) modified as follows. (4-12)

$a_u$ = Eq. (4-11), except that the composite moment of inertia is used in Term (4) to compute $(a_i)_1$, and the ratio, $I_2/I_c$, is eliminated in Term (5). (4-13)

Term (4) is the initial deflection of the composite beam under slab dead load, $(a_i)_1 = \xi M_1 \ell^2/E_{cs} I_c$.

Term (5) is the creep deflection of the composite beam under slab dead load, $\nu_{t1}(a_i)_1$. The composite section effect is already included in Term (4).

4.4 Loss of Prestress and Camber in Noncomposite Prestressed Beams (6,49-58,63)

4.4.1 Loss of Prestress in Prestressed Concrete Beams

Loss of prestress at any time and including ultimate values, in percent of initial tensioning stress, is given by Eqs. (4-14) and (4-15).

$$\lambda_t = \Big[\overbrace{(nf_c)}^{(1)} + \overbrace{(nf_c)\nu_t\left(1 - \frac{F_t}{2F_o}\right)}^{(2)} + \overbrace{(\varepsilon_{sh})_t E_s/(1 + n\rho\xi_s)}^{(3)} + \overbrace{(f_{sr})_u}^{(4)}\Big]\frac{100}{f_{si}} \qquad (4\text{-}14)$$

$$\lambda_u = \Bigg[\overbrace{(nf_c)}^{(1)} + \overbrace{(nf_c)\ \nu_u\ (1 - \frac{F_u}{2F_o})}^{(2)} +$$

$$\overbrace{(\varepsilon_{sh})_u E_s/(1+n\rho\xi_s)}^{(3)} + \overbrace{(f_{sr})_t}^{(4)}\Bigg] \frac{100}{f_{si}} \qquad (4\text{-}15)$$

where:

Term (1) is the prestress loss due to elastic shortening, in which $f_c = \frac{F_i}{A_t} + \frac{F_i e^2}{I_t} - \frac{M_D e}{I_t}$, and n is the modular ratio at the time of prestressing. Frequently $F_o$, $A_g$, and $I_g$ are used as an approximation instead of $F_i$, $A_t$, and $I_t$, being $F_o = F_i(1 - n\rho)$. Only the first two terms for $f_c$ apply at the ends of simple beams. For continuous members, the effect of secondary moments due to prestressing should also be included. Suggested values for n are given in Table 4.4.1.1.

Term (2) is the prestress loss due to the concrete creep. The expression, $\nu_t\ (1 - \frac{F_t}{2F_o})$, was used in References 50 and 53 to approximate the creep effect resulting from the variable stress history. Approximate values of $F_t/F_o$ (in the form of $F_s/F_o$ and $F_u/F_o$) for this secondary effect as given in Table 4.4.1.2. To consider the effect of nontensioned steel in the member, multiply $\nu_t$, $\nu_u$, $(\varepsilon_{sh})_t$ and $(\varepsilon_{su})_u$ by $\xi_r$ (from Eq. 3-9).

Term (3) is the prestress loss due to shrinkage (56). The expression, $(\varepsilon_{sh})_t E_s$, somewhat overestimates this loss. The denominator represents the stiffening effect of the steel and the effect of concrete creep. Additional information on Term (3) is given in Ref. 63.

Term (4) is the prestress loss due to steel relaxation. Values of $(f_{sr})_t$ and $(f_{sr})_u$ for wire and strand are given in Table 4.4.1.3, (63), where t is the time after initial stressing in hours and $f_y$ is the 0.1 percent offset yield stress. Values in Table 4.4.1.3 are recommended for most design calculations because they are consistent with the approximate nature of creep and shrinkage calculations. Relaxation of other types of steel should be based on manufacturer's recommendations supported by adequate test data. For a more detailed analysis of the interdependency between steel relaxation, creep and shrinkage of concrete see Section 3.7 of this report.

4.4.2 Camber of Noncomposite Prestressed Concrete Beams

The camber at any time, and including ultimate values, is given by Eqs. (4-16) and (4-17) respectively. It is suggested that an average of the end and midspan loss be used for straight tendons and 1-pt. harping, and the midspan loss for 2-pt harping.

$$a_t = \overbrace{-(a_i)_{F_o}}^{(1)} + \overbrace{(a_i)_D}^{(2)} - \overbrace{\left[ -\frac{F_t}{F_o} + \left(1 - \frac{F_t}{2F_o}\right)\nu_t \right] (a_i)_{F_o}}^{(3)} \overbrace{+\nu_t (a_i)_D}^{(4)} + \overbrace{a_L}^{(5)} \qquad (4\text{-}16)$$

$$a_u = \overbrace{-(a_i)_{F_o}}^{(1)} + \overbrace{(a_i)_D}^{(2)} - \overbrace{\left[ -\frac{F_u}{F_o} + \left(1 - \frac{F_u}{2F_o}\right)\nu_u \right] (a_i)_{F_o}}^{(3)} \overbrace{+ \nu_u (a_i)_D}^{(4)} + \overbrace{a_L}^{(5)} \qquad (4\text{-}17)$$

where:

Term (1) is the initial camber due to the initial prestress force after elastic loss, $F_o$. See Table 4.4.2.1 for common cases of prestress moment diagrams with formulas for computing camber, $(a_i)_{F_o}$.

Here, $F_o = F_i(1 - nf_c/f_{si})$, where $f_c$ is determined as in Term (1) of Eq. (4-14). For continuous members, the effect of secondary moments due to prestressing should also be included.

Term (2) is the initial dead load deflection of the beam, $(a_i)_D = \xi M\ell^2/E_{ci}I_g$. $I_g$ is used instead of $I_t$ for practical reasons. See Table 4.2.1 for $\xi$ and $M$ values.

Term (3) is the creep (time-dependent) camber of the beam due to the prestress force. This expression includes the effects of creep and loss of prestress; that is, the creep effect under variable stress. $F_t$ refers to the total loss at any time minus the elastic loss. It is noted that the term, $F_t/F_o$, refers to the steel stress or force after elastic loss, and the prestress loss in percent, $\lambda$ as used herein, refers to the initial tensioning stress or force. The two are related as:

$$\frac{F_t}{F_o} = \frac{1}{100}(\lambda_t - \lambda_{el})\frac{f_{si}}{f_o} \qquad (4\text{-}18)$$

and can be approximated by:

$$\frac{F_t}{F_o} = \frac{1}{100}(\lambda_t - \lambda_{el})\frac{1}{1 - n\rho} \qquad (4\text{-}18a)$$

Term (4) is the dead load creep deflection of the beam. Multiply $\nu_t$ and $\nu_u$ by $\xi_r$ (from Eq. 3-9) for the effect of compression steel (under dead load) in the member.

Term (5) is the live load deflection of the beam.

Additional information on the effect of sustained loads other than a composite slab or topping applied some time after the transfer of prestress is given by Terms (6) and (7) in Eqs. (29) and (30) in Ref. 63.

## 4.5 Loss of Prestress and Camber of Composite Precast and Prestressed Beams, Unshored and Shored Constructions (6,49-58,63,77)

### 4.5.1 Loss of Prestress of Composite Precast-Beams and Prestressed Beams

The loss of prestress at any time and including ultimate values, in percent of initial tensioning stress, is given by Eqs. (4-19) and (4-20) respectively for unshored and shored composite beams with both prestressed steel and nonprestressed steel.

$$\lambda_t = \Big[\underbrace{(nf_c)}_{(1)} + \underbrace{(nf_c)\nu_s(1-\frac{F_s}{2F_o})}_{(2)} + \underbrace{(nf_c)(\nu_{t2}-\nu_s)(1-\frac{F_s+F_t}{2F_o})\frac{I_2}{I_c}}_{(3)}$$

$$+\underbrace{(\varepsilon_{sh})_t E_s/(1+n\rho\xi_s)}_{(4)} + \underbrace{(f_{sr})_t}_{(5)} - \underbrace{(mf_{cs})}_{(6)} - \underbrace{(mf_{cs})\nu_{t1}\frac{I_2}{I_c}}_{(7)} - \underbrace{mf_{cd}}_{(8)}\Big]\frac{100}{f_{si}} \qquad (4\text{-}19)$$

$$\lambda_u = \Big[\underbrace{(nf_c)}_{(1)} + \underbrace{(nf_c)\nu_s(1-\frac{F_s}{2F_o})}_{(2)} + \underbrace{(nf_c)(\nu_u-\nu_s)(1-\frac{F_s+F_u}{2F_o})\frac{I_2}{I_c}}_{(3)}$$

$$+\underbrace{(\varepsilon_{sh})_u E_s/(1+n\rho\xi_s)}_{(4)} + \underbrace{(f_{sr})_u}_{(5)} - \underbrace{(mf_{cs})}_{(6)} - \underbrace{(mf_{cs})\nu_{us}\frac{I_2}{I_c}}_{(7)} - \underbrace{mf_{cd}}_{(8)}\Big]\frac{100}{f_{si}} \qquad (4\text{-}20)$$

where:

Term (1) is the prestress loss due to elastic shortening. See Term (1) of Eq. (4-14) for the calculation of $f_c$.

Term (2) is the prestress loss due to concrete creep up to the time of slab casting. $\nu_s$ is the creep coefficient of the precast beam concrete at the time of slab casting. See Term (2) of Eq. (4-14) for comments concerning the reduction factor, $(1 - \frac{F_s}{2F_o})$. Multiply $\nu_s$ and $\nu_u$ by $\xi_r$ (from Eq. 3-9) for the effect of nontensioned steel in the member. Values of $\nu_t/\nu_u = \nu_s/\nu_u$ from Eq. (2-8) are given in Table 2.4.1.

Term (3) is the prestress loss due to concrete creep for any period following slab casting. $\nu_{t2}$ is the creep coefficient of the precast beam concrete at any time after slab casting. The reduction factor, $(1 - \frac{F_s + F_t}{2F_o})$, with the incremental creep coefficient, $(\nu_{t2} - \nu_s)$, estimates the effect of creep under the variable prestress force that occurs after slab casting. Multiply this term by $\xi_r$ (from Eq. 3-9) for the effect of nontensioned steel in the precast beam. See Term (3) of Eq. (4-10) for comment on $I_2/I_c$.

Term (4) is the prestress loss due to shrinkage. See Term (3) of Eqs. (4-14) and (4-15) for comment.

Term (5) is the prestress loss due to steel relaxation. In this term t is time after initial stressing in hours. See Term (4) of Eqs. (4-14) and (4-15) for comments.

Term (6) is the elastic prestress gain due to slab dead load, and m is the modular ratio at the time of slab casting. $f_{cs} = \frac{(M_{S,Di})e}{I_g}$, $M_{S,Di}$ refers to slab or slab plus diaphragm dead load; e and $I_g$ refer to the precast beam section properties for unshored construction and the composite section properties for shored construction. Suggested values for n and m are given in Table 4.4.1.1.

Term (7) is the prestress gain due to creep under slab dead load. $\nu_{t1}$ is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. See Term (5) of Eq. (4-10) for comments on $\xi_r$ and $I_2/I_c$. For shored construction, drop the term, $I_2/I_c$ $\nu_{us}$ is given by Eq. (2-13).

Term (8) is the prestress gain due to differential shrinkage, where $f_{cd} = Qy_{cs}e_c/I_c$ is the concrete stress at the steel c.g.s. and $Q = (\delta A_{g1}E_{c1})/3$ in which $A_{g1}$ and $E_{c1}$ refer to the cast in-place slab. See Notation for additional descriptions of terms. Since this effect results in a prestress gain, not loss, and is normally small, it may usually be neglected (77).

### 4.5.2 Camber of Composite Beams-Precast Beams Prestressed Unshored and Shored Construction

The camber at any time, including ultimate values, is given by Eqs. (4-21), (4-22), (4-23) and (4-24) for unshored and shored composite beams, respectively. It is suggested that an average of the end and midspan loss of prestress be used for straight tendons and 1-pt. harping, and the midspan loss for 2-pt. harping (6).

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes be used. For the composite moment of inertia, $I_c$ in Eqs. (4-21) through (4-24), use the gross section $I_g$ except in Term (10) for the live load deflection.

a. Unshored Construction

$$a_t = \underbrace{-(a_i)_{F_o}}_{(1)} + \underbrace{(a_i)_2}_{(2)} - \underbrace{\left[-\frac{F_s}{F_o} + \left(1 - \frac{F_s}{2F_o}\right)\nu_s\right](a_i)_{F_o}}_{(3)}$$

$$-\underbrace{\left[-\frac{F_t - F_s}{F_o} + \left(1 - \frac{F_s + F_t}{2F_o}\right)(\nu_{t2} - \nu_s)\right](a_i)_{F_o}\frac{I_2}{I_c}}_{(4)} + \underbrace{\nu_s(a_i)_2}_{(5)}$$

$$+\underbrace{(\nu_{t2} - \nu_s)(a_i)_2\frac{I_2}{I_c}}_{(6)} + \underbrace{(a_i)_1}_{(7)} + \underbrace{\nu_{t1}(a_i)_1\frac{I_2}{I_c}}_{(8)} + \underbrace{a_\delta}_{(9)} + \underbrace{a_L}_{(10)}$$

(4-21)

$$a_u = \underbrace{-(a_i)_{F_o}}_{(1)} + \underbrace{(a_i)_2}_{(2)} - \underbrace{\left[-\frac{F_s}{F_o} + \left(1 - \frac{F_s}{2F_o}\right)\nu_s\right](a_i)_{F_o}}_{(3)}$$

$$-\underbrace{\left[-\frac{F_u - F_s}{F_o} + \left(1 - \frac{F_s + F_u}{2F_o}\right)(\nu_u - \nu_s)\right](a_i)_{F_o}\frac{I_2}{I_c}}_{(4)} + \underbrace{\nu_s(a_i)_2}_{(5)}$$

$$+\underbrace{(\nu_u - \nu_s)(a_i)_2\frac{I_2}{I_c}}_{(6)} + \underbrace{(a_i)_1}_{(7)} + \underbrace{\nu_{us}(a_i)_1\frac{I_2}{I_c}}_{(8)} + \underbrace{a_\delta}_{(9)} + \underbrace{a_L}_{(10)}$$

(4-22)