# SP 50-1
# A RE-EXAMINATION OF THE EI VALUE FOR SLENDER COLUMNS

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Synopsis: An incremental rate-of-creep analysis of reinforced concrete columns subjected to sustained loads is developed and used to generate data for a statistical evaluation of the stiffness, EI, of slender concrete columns subjected to short time and sustained loads. The resulting equations are more accurate than ACI Eqn. 10.7 and are as easy to use. The possibility of column failure under very high sustained loads is discussed.

Keywords: building codes; columns (supports); computers; creep properties; deflections; loads (forces); modulus of elasticity; moment-curvature relationships; reinforced concrete; reinforcing steels; statistical analysis; stiffness; stiffness methods; structural analysis; structural design; structural engineering; tied columns.

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## INTRODUCTION

The 1971 ACI Building Code (1) presents a moment-magnifier method for analysing the effects of slenderness on the strength of columns. This design procedure is strongly affected by the effective flexural stiffness, EI, used in the calculations and it has been shown that the EI values given in the ACI Code are inaccurate in many cases(2).

The knowledge of the effect of sustained loads on the load capacity of concrete columns has been improved in the past 15 years by experiments (3-8) and analyses of the problem (6-12). Although the analyses now available are reasonably accurate in predicting the behavior of columns in tests, they are generally too complex for design use. However, such analyses are useful in carrying out computer experiments to study the effects of a wide range of variables to help in the preparation of design equations.

The first part of this paper briefly describes a step-by-step Rate-of-Creep analysis of columns subjected to sustained loads. A series of computer experiments, carried out using this analysis are described in the second part. The results of these have been used to derive design equations for flexural stiffness, EI, for use in the ACI Code moment magnifier method for designing slender columns.

## ANALYSIS OF CAPACITY OF SLENDER HINGED COLUMNS

### Material Properties

The reinforcing steel is assumed to be an idealized elasto-plastic material. The instantaneous stress-strain relationship for concrete is assumed to be:

1. linear in the strain range:

$$\frac{f'_t}{E_c} \leq \varepsilon \leq 0,$$

where $f'_t = 6\sqrt{f'_c}$

$E_c$ = Modulus of elasticity as defined in Section 8.3.1 of ACI 318-71;

2. quadratic parabola in the strain range:

$$0 < \varepsilon < \frac{2f'_c}{E_c}$$

3. perfectly plastic in the range:

$$\frac{2f'_c}{E_c} \leq \varepsilon \leq \varepsilon_u \quad .$$

The slope of unloading and reloading branches is taken equal to the modulus of elasticity, $E_c$. Under short time loads $\varepsilon_u$ was taken as 0.0035.

Although the tensile strength of concrete is generally ignored in the design of reinforced concrete members, this analysis is intended to represent reasonably closely the true behavior of slender columns so that data from the computer analysis can be used in the same manner as data from real column tests in studying column stability. For this reason the tensile strength of concrete was included in the analysis. The value of $f'_t$ was chosen to agree with Section 11.5.2 of ACI 318-71 (1) rather than Section 9.5.2.2 to give a conservative estimate of the column behavior.

The properties of concrete are affected by time and vary over a wide range depending on the curing and loading conditions. In this investigation, concrete is assumed to be a non-linear, visco-elasto-plastic material. The non-linear, stress dependent creep function used in the analysis was derived from test data in Ref. 11 for plain concrete prisms subjected to different levels of stress which were held constant for each test specimen during the test period. Shrinkage strains were measured on separate unloaded test specimens. The creep strain at time t, $\varepsilon_{cr}(t)$, was found by subtracting the initial strain, $\varepsilon_o$, and the shrinkage strain at time t, $\varepsilon_s(t)$, from the measured strain at time t, $\varepsilon(t)$:

$$\varepsilon_{cr}(t) = \varepsilon(t) - \varepsilon_o - \varepsilon_s(t) \qquad (1)$$

$\varepsilon_o$ is given by the equation of the short time stress-strain curve:

$$\varepsilon_o = \frac{2f'_c}{E_c}\left(1 - \sqrt{1 - \frac{f_c}{f'_c}}\right) \qquad (2)$$

where $E_c$ is the initial tangent modulus of elasticity of the concrete at the time under consideration. The creep function $\phi(t)$ is defined as the ratio between the creep strain and initial strain:

$$\phi(t) = \frac{\varepsilon_{cr}(t)}{\varepsilon_o} \tag{3}$$

One creep function was obtained from each of twenty test specimens subjected to different constant stress levels in the range of $f_c/f'_c$ from 0.185 to 0.808. By plotting $\phi(t)$ versus $f_c/f'_c$ at different times it was found that the stress dependency of the creep function at any time t can be described by the function:

$$\phi(t) = C_1(t) + C_2(t)\,\frac{f_c}{f'_c} \tag{4}$$

The coefficients $C_1(t)$ and $C_2(t)$ determined by linear regression analysis carried out for different times after load application are shown in Fig. 1. Since only the effect of $f_c/f'_c$ on the creep function was investigated in Ref. 11 the other variables will be accounted for using the CEB creep functions (13).

For analytical purposes it was assumed that the coefficients $C_1(t)$ and $C_2(t)$ could be represented by a function of the general type:

$$C_n(t) = \bar{C}_n \left[1 - \exp\,(-\,\alpha_n t^{\delta n})\right] \tag{5}$$

where $C_n$, etc. refers to $C_1$ or $C_2$, etc.

The values of the coefficients used in the analysis were based on those derived in Ref. 11 and were taken as:

$$\bar{C}_1 = \bar{C}_n \;\frac{[\phi\;(\infty\;,\;0.40)\;-\;0.8]}{3} \tag{6}$$

and

$$\bar{C}_2 = 2.5\;[\phi\;(\infty\;,\;0.40)\;-\;\bar{C}_1] \tag{7}$$

In these equations $\phi\;(\infty\;,\;0.40)$ refers to the value of the creep function $\phi$ at time $t = \infty$ and $f_c/f'_c = 0.40$ and is taken equal to the value of $\phi$ given by the CEB Recommendations (13).

The total strain can now be represented by:

$$\varepsilon(t) = \varepsilon_o \left[1 + C_1(t) + C_2(t)\left(\frac{f_c}{f'_c}\right)\right] + \varepsilon_s(t) \tag{8}$$

For columns with an equal amount of reinforcement on both faces it was found that the effect of shrinkage was insignificant and consequently it has not been considered in this study (5, 12).

The increase in the compressive strength of the concrete due to maturing was also included in the analysis. For the design studies carried out later in this paper the strength at $t = \infty$ was conservatively assumed to be 1.10 times the 28 day strength. The decrease in concrete strength in the presence of very high sustained stresses was assumed to be 20 percent of the short time strength (14).

Method of Analysis for Short Time Loads

In the first stage of the analysis a family of moment curvature relationships is computed. In the second stage the deflections and load capacity of the given column are computed by an iterative numerical integration procedure making use of the stored moment curvature relationships. With the exception of the method used to account for tension stiffening, the analysis resembles the procedures described by Pfrang (15).

The cross section of a column subjected to uniaxial bending and compression is divided into a number of concrete and steel fibres perpendicular to the axis of symmetry and parallel to the neutral axis. The ultimate compression strain $\varepsilon_u$ is also divided into NE segments of strain. These segments are selected as strain increments at the compressed edge of the cross section as shown in Fig. 2. By assuming an arbitrary strain distribution over the depth of the cross section for any value of $\varepsilon_j$ the concrete fibre forces C and the steel layer forces S may be computed from the basic stress-strain relationships. Eqns. (9) and (10) give the forces P and M associated with the assumed strain distributions.

$$\Sigma C + \Sigma S = P \tag{9}$$

$$\Sigma(Cy_c) + \Sigma(Sy_s) = M \tag{10}$$

By varying the strain distribution for a given value $\varepsilon_j$ at the compressed edge, by a trial and error procedure as shown in Fig. 2 the sum of the internal forces P is made equal to the extenal force $P_i$. By repeating this procedure for all given values of $P_i$ (i = 1,2,...NP) and all given values of $\varepsilon_j$ (j = 1,2,...NE) the moment curvature relationship $M_{i,j}$, $\Phi_{i,j}$, results.

In the procedure just described, the influence of the concrete tensile zone has not been considered. This influence will be included by subtracting a correction $\Delta\Phi$ from the computed curvature values, $\Phi$ (Fig. 3). No correction is required if the whole cross section is subjected to compression strains only. The influence of the tensile zone and as a consequence the correction $\Delta\Phi$ reaches a maximum at the cracking moment, $M_{cc}$. To compute the correction $\Delta\Phi_{cc}$ the following method has been applied. For any value of the axial force $P_i$ the cracking moment is computed by a trial and error procedure similar to that shown in Fig. 2 except that the strain distributions are selected with the ultimate tensile strain $\varepsilon_2 = f_t'/E_c$ at the bottom edge of the cross section. Because the cross section is considered as uncracked in this particular analysis, Eqns. (9) and (10) also include concrete tensile forces. The value $\Delta\Phi_{cc}$ can now be found as the difference between the curvature, $\bar{\Phi}_{cc}$, computed for the uncracked cross section and the curvature, $\Phi_{cc}$, computed for the cracked cross section where both $\bar{\Phi}_{cc}$ and $\Phi_{cc}$ are evaluated for the moment $M = M_{cc}$. For other values of M the following corrections are then applied:

$$M \leq M_o \qquad : \Delta\Phi = 0 \tag{11}$$

$$M_o < M < M_{cc} \qquad : \Delta\Phi = \Delta\Phi_{cc} \frac{M - M_o}{M_{cc} - M_o} \tag{12}$$

$$M_{cc} \leq M \qquad : \Delta\Phi = \Delta\Phi_{cc} \frac{M_{cc} - M_o}{M - M_o} \tag{13}$$

where: $M_o$ is the moment for zero strain at the tensile edge.

These equations are compared to analyses and tests in Ref. 11 and good agreement is found.

The resulting moment curvature relationships can be used in an iterative numerical intergration procedure (15) to determine the deflections, w, of the column subjected to $P_i$. By repeating the computation of deflections for different values of $P_i$ a relationship $P_i$, M representing the column behavior, can be found where M is the maximum moment at midheight given by $M = P_i(e + w)$. Failure is assumed to occur at the intersection of this $P_i$, M curve with the P, M interaction curve as shown in Fig. 4. The interaction curve is given by $P_i$, $M_{i,j=NE}$ based on $J = NE$ ($\varepsilon_1 = \varepsilon_u$) from the moment curvature relationships. The result of the analysis is the ultimate load capacity $P_u$ and the ultimate moment $M_u$. In the case of stability failures the ultimate load, $P_u$, was that of instability and the ultimate moment was taken as the point on the P, M interaction diagram corresponding to the computed ultimate load, $P_u$.

## Method of Analysis for Sustained Loads

To include the effect of creep, the moment-curvature relationship is expanded to $M_{i,j}$, $\Phi_{i,j,k}$ where index k stands for time and k = 1 indicates values for the short term loading situation so far referred to as $\Phi_{i,j}$. The values $\Phi_{i,j,k}$ for k = 2, 3...NT are computed by a Rate-of-Creep Method using time as a discrete variable. Fig. 5 shows the strain distribution and curvature $\Phi_{i,j,k}$ at the beginning of creep stage k. First the unrestrained creep strain $\Delta\varepsilon_{cr}$ is computed for each compressed concrete fibre using Eqn. (3) and (4). Next, residual forces $\Delta C$ and $\Delta S$ are applied to maintain equilibrium and restore the linear strain distribution. The values of $\Delta C$ and $\Delta S$ and the strain increments $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ follow from the equilibrium conditions:

$$\Sigma\Delta C + \Sigma\Delta S = 0 \tag{14}$$

$$\Sigma(\Delta C\ y_c) + (\Delta S\ y_s) = 0 \tag{15}$$

The curvature $\Phi_{i,j,k+1}$ at the beginning of creep stage k+1 can now be computed from:

$$\Phi_{i,j,k+1} = \Phi_{i,j,k} + \frac{\Delta\varepsilon_1 - \Delta\varepsilon_2}{h} \tag{16}$$

This procedure must be repeated for all creep stages.

The influence of the concrete tensile zone is included as described earlier giving rise to $\bar{\Phi}$ as shown in Fig. 3. It is assumed that the cracking moment $M_{cc}$ does not vary with time.

## Computation of Column Deflections and Capacities

The time dependent deflections of a column subjected to a sustained load $P_s$ are computed by a Rate-of-Creep Method making use of the $M_{i,j}$, $\bar{\Phi}_{i,j,k}$ relationship. The length of the column is divided into a number of intervals for the numerical integration of the curvature. The instantaneous column deflection, $w_o$, is computed from the $M_{i,j}$, $\bar{\Phi}_{i,j,1}$ relationship. To compute the creep deflections, $\Delta w_1$, occurring in the first creep stage the incremental increase in curvature due to creep is determined from:

$$\Delta\bar{\Phi}_{i,j,1} = \bar{\Phi}_{i,j,2} - \bar{\Phi}_{i,j,1} \tag{17}$$

$\Delta\bar{\Phi}_{i,j,k}$ has to be evaluated for each interval of column length. The creep deflections, $\Delta w_1$, due to $\Delta\bar{\Phi}_{i,j,1}$ will cause an increase of the external moment thereby causing an additional instantaneous deflection, $\Delta w_{o,1}$. It is assumed that this increase in the short term deflections $\Delta w_{o,1}$ takes place at the end of the creep stage. The total deflection at the beginning of creep stage 2 is then given by:

$$w_1 = w_o + \Delta w_1 + \Delta w_{o,1} \tag{18}$$

or generally, at the beginning of creep stage k + 1:

$$w_k = w_o + \sum_{m=1}^{k} (\Delta w_m + \Delta w_{o,m}) \tag{19}$$

A more detailed description of this procedure is presented in Ref. 11. In the case of a homogeneous linear visco-elastic material this procedure converges to the solution of the appropriate differential equation. Because the quality of the convergence decreases with increasing second order deformations, eight creep stages have been used in the present analysis.

For the purpose of computing the final short term load capacity of a column after a period of sustained load, an additional set of moment curvature relationships is required to account for the increase in concrete strength with increasing age. The computational procedure for these short term relationships is the same as for $M_{i,j}$, $\bar{\Phi}_{i,j,1}$. The analysis after a period of sustained load is carried out in the same way as the computation of the behavior due to an initial short term loading situation but the residual creep deflections (the deflections remaining after unloading the column quickly) must be added to the initial geometrical eccentricity of the load.

The analysis described here is essentially the same as that described in Ref. 11. That analysis is compared in Ref. 11 to the results of columns tested under short time and sustained loads by Ramu et.al. (5) and gave good predictions of both the load capacity and the load deflection response of these columns.

## SUSTAINED LOAD BEHAVIOR OF SLENDER COLUMNS

The typical behavior of a slender hinged column bent in single curvature is shown in Fig. 6. Curve (A) shows the instantaneous short term behavior expressed in terms of load and moment at midheight. Failure is assumed to occur at the point of intersection of Curve (A) with the P, M - interaction curve ($\bar{A}$) computed with the initial 28 day concrete strength. Curve (B) shows the behavior of a column loaded up to failure after being subjected to a sustained load $P_s$ for the period of time indicated by the index k. Failure is now assumed to occur at the intersection with an interaction curve ($\bar{B}$) computed with a slightly increased concrete strength. Curves (C) and ($\bar{C}$) shows a similar behavior for a column loaded to failure after the total amount of creep possible under the sustained load $P_i$ has occurred.

The interaction curves are given by the values $P_i$, $M_{i,j=NE,k}$. The fact that the interaction curves have been found from the condition $\varepsilon_1 = \varepsilon_u = 0.0035$ does not mean that the maximum total strain in the column at failure is equal to $\varepsilon_u$. Instead, the total strain minus the residual creep strains at the edge of the failure section is assumed to reach $\varepsilon_1 = \varepsilon_u$. Thus it is assumed that the creep process does not affect the ability of the concrete to develop its full ultimate short term strain for a final short term load (10). This assumption is in fairly good agreement with test results.

For all eccentrically loaded columns there is a "critical sustained load", $P_{scr}$. For sustained loads less than $P_{scr}$ the column load can be increased to failure at time $t = \infty$ as shown by curve (C) in Fig. 6. For sustained loads equal to or greater than $P_{scr}$ the column will fail under the sustained load (9, 11).

The strength of concrete tends to decrease under sustained high stresses (14). This decrease has been assumed to be 20 percent of the short-time strength of concrete of the same age. Since the short-time strength at time infinity has been conservatively assumed equal to $1.10\ f'_{c\ 28}$, in this investigation, the critical sustained stress was assumed to be $0.8 \times 1.10\ f'_{c28}$. The interaction curve ($\bar{D}$) in Fig. 6 then results from the $M_{i,j}$, $\Phi_{i,j,k=NT}$ relationship where j indicates the strain level, $\varepsilon_1$, in the extreme compression fibre which is set equal to the critical sustained load strain $\varepsilon_{us}$ computed from Eqn. (8) for $f_c/f'_{c28} = 0.88$ and $t = \infty$ in computing the interaction curve. Curve (D) in Fig. 6 represents the maximum possible increase in the moment at failure section due to creep. The intersection of this limiting curve and the interaction curve ($\bar{D}$) yields the critical sustained load which is the lowest load which can cause a creep failure for any given initial eccentricity, e.

Generally, significant differences between the interaction curve

$(\bar{A})$ and the curves $(\bar{B})$, $(\bar{C})$ and $(\bar{D})$ are only noticeable for load levels above the balanced load.

## DERIVATION OF DESIGN EQUATIONS

### Computer Experiment

Earlier studies of the behavior of slender columns (2) suggested that the following five variables would have the most effect on the strength and behavior of such columns: slenderness ratio, $\ell/h$; shape of cross section and reinforcement position; reinforcement ratio, $\rho_t$; ratio of the distance between the outer reinforcement layers and the overall thickness, $\gamma$; and eccentricity, e/h.

The principal groups of columns studied were a fully factorial block (16) including:

1. Eighty-one tied columns with bars in 2 faces (Shape A in Fig. 7) and every combination of the following variables:

   $\gamma$ = 0.6, 0.75 and 0.9

   e/h = 0.1, 0.25 and 0.40

   $\ell/h$ = 10, 20 and 40

   $\rho_t$ = 0.008, 0.040 and 0.064

   $f'_c$ = 4 ksi

   $f_y$ = 60 ksi

   (creep coefficient) = 5.0

2. Eighty-one tied columns with bars in 4 faces (Shape B in Fig. 7) and every combination of the variables given above.

3. Sixty-four other columns with 8 cross section shapes, 8 slenderness ratios, 8 values of $\rho_t$, 8 ratios of e/h, 8 values of $f'_c$, 8 values of $\gamma$ and 8 values of the creep coefficient.

The columns in the third group were selected according to a 8 x 8 Graeco-Latin square (17) to extend the data in a more or less random way to sample the effects of other variables. The combinations of the variables have been chosen according to the pattern of a 8 x 8 Graeco-Latin square as shown in Table 2. Each of the 64 cells in Table 2 represents one "computer experiment" in the third group of columns and each of the seven numbers in each cell represents the level of one variable listed in Table 1. For example, the combination 4356273 for cross section shape C and slenderness ratio $\ell/h$ = 20 in Table 2 means that the properties of this column were:

First Number - slenderness ratio, $\ell/h$ = 20 (the 4th level of $\ell/h$ in Table 1)

Second Number - cross sectional shape - C (the 3rd level in Table 1 - Fig. 7 shows this to be a spiral column)

Third Number - $\rho_t$ = 0.04 (the 5th value in Table 1)

Fourth Number - $d_c/h$ = 0.175 (the 6th value in Table 1)

Fifth Number - $f'_c$ = 3,000 psi

Sixth Number - e/h = 0.35

Seventh Number - $\phi$ = 2.0 (creep coefficient)

It is a feature of the Graeco-Latin square variable arrangement that each value of each variable appears once with each value of every other variable. Latin squares and Graeco-Latin squares of different sizes are described in Ref. 17. An estimation of the total interaction of the variables is possible from the Analysis of Variance using the residual sum of squares. A residual sum of squares exists in the present case because the 8 x 8 Graeco-Latin square could contain nine instead of the six or seven variables used.

The ranges of the variables e/h and $\ell/h$ in this study were chosen on the basis of a study of the range of variables encountered in over 22,000 actual columns in buildings (18).

Thus, a total of 226 columns have been analyzed to determine their capacity under short time loads. In addition, these columns have been studied under varying sustained loads yielding a total of 972 different loading cases.

The study was limited to hinged columns bent in symmetrical single curvature without transverse loads between their ends. The moment-magnifier procedure used in the ACI Code was originally derived for this particular type of column (2). Other loading patterns and other end restraint conditions are accounted for in the ACI Code using correction factors ($C_m$,k). By considering only hinged columns bent in single curvature, errors in $C_m$ or k will not affect the accuracy of the EI expressions which are derived.

EVALUATION OF EFFECTIVE STIFFNESS EI FOR SHORT TIME LOADS

The maximum moment in an elastic beam-column bent in uniform single curvature can be calculated with sufficient accuracy for design purposes using Eqn. (20) (2):

$$M_u = \frac{P_u e}{1 - \frac{P_u}{P_c}} \qquad (20)$$