From Figure 5.1 for $\frac{\rho f_y}{f'_c} = 0.60$ no point on the curves for $\frac{e}{t} = 0.872$ and $\frac{Pu'}{f'_c bt} = 0.52$ - hence section too small.

Increase column size to 15" x 15"

$$\frac{Pu'}{f'_c bt} = 0.33$$

$$\frac{e}{t} = \frac{10.48}{15.00} = 0.698$$

$$\theta = 26.6^o$$

At the ends of the column estimating between $\theta = 0$ and $\theta = 45^o$ and using Figure 5.2

Max. strain = 0.0018

$\frac{z}{t} = 0.66$ .. z = 9.88" (z is the perpendicular distance from the neutral axis to the point of maximum strain)

$$\text{curvature} = \frac{0.0022}{9.88}$$

At mid-column height - load is the same but the eccentricity and its angle are unknown.

Assume max. strain = 0.0030
From Fig. 5.1

$\frac{e}{t} = 0.83$ e = 12.45"

$\frac{z}{t} = 0.65$ z = 9.73"

$$\text{curvature at midheight} = \frac{0.0030}{9.73}$$

Hence from moment area (noting that $\varepsilon/z \equiv \frac{M}{EI}$) the lateral deflection about the neutral axis is Δ = 1.17"

e actual = (e top + Δ) = 10.48 = 11.65

e possible = 12.45"

e possible > e actual .. section OK.

Obviously the maximum strain at the mid height to satisfy compatibility of deflections is less than 0.0030

Try ε = 0.0024 by interpolation

$\frac{z}{t} = 0.66$ z = 9.88"

$\frac{e}{t} = 0.78$ $\qquad\qquad$ $e = 11.70"$

$\Delta = 0.93"$

e actual = e top + Δ = 10.48 + 0.93 = 11.41"

e possible = 11.7"

section OK

Comparison with ACI 318.71

Comparison with the current ACI method can best be made by comparing the moment magnifiers of the two methods.

Exact analysis $\delta_{exact} = \frac{11.41}{10.48} = 1.09$

Calculations as per ACI 318.71

$C_m = 1$

$\beta_d = 0.0$ $\rightarrow$ to correspond with previous

$I_g = 4218$ ins.^{4}

$E_c = 3.5 \times 10^6$

$EI = 5900 \times 10^6$ lbs. ins.^{2}

$\delta_{ACI} = 1.20$

Comparison shows the two moment magnification factors $\delta_{exact} = 1.09$ and $\delta_{ACI} = 1.20$ to be in close agreement. Similar agreement has been found for all the trial columns checked by the authors.

Discussion of Method

A possible criticism of the method is that the neutral axis is not necessarily perpendicular to the line of eccentricity; giving rise to deflection of the column lateral to the line of eccentricity. For square section columns this effect is not large and can be ignored.

Lateral buckling is not generally considered critical for reinforced concrete columns but circumstances can be envisaged where the lateral stiffness of the partially 'plastic' section is not adequate to prevent buckling lateral to the line of action of the load. Hence the lateral flexural stiffness of the 'plastic' section is required. However as reinforced concrete columns fail by material failure the buckling problem can be avoided by resolving the curvatures onto the symmetric axes and calculating separately the lateral deflections about each axis.

CONCLUSIONS

Design charts are presented which simplify the design of slender square section symmetrically reinforced concrete columns under biaxially eccentric loads. The method satisfies equilibrium and compatibility across the section and along the length of the column.

Comparison with the ACI moment magnifier method shows the ACI method to be approximately 5% conservative.

ACKNOWLEDGEMENT

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REFERENCES

1. Abdel-Sayed, I. "Behaviour of slender reinforced concrete columns under biaxially eccentric loading." Ph.D. thesis 1974, University of Ottawa.

2. Au, T. "Ultimate strength design of rectangular concrete members subjected unsymmetrical bending." ACI Journal V.29, No. 8, Feb. 8, 1958, pp.657-674.

3. Bresler, B. "Design criteria for reinforced columns under axial load and biaxial bending." ACI Journal, V. 57, No. 5, Nov. 1960, pp.481-490.

4. Building Code Requirements, ACI 318-63, American Concrete Institute, 1963.

5. Building Code Requirements, ACI 318-71, American Concrete Institute, 1971.

6. Chu, K.-H. and Pabarcius, A. "Biaxially loaded reinforced concrete columns." Journal of the Structural Division ST8, December 1958.

7. Drysdale, R.G. and Huggins, M. "Sustained biaxial load on slender concrete columns." Journal of the Structural Division ST5, Vol. 97, May 1971, pp.1423-1443.

8. Furlong, R. "Ultimate strength of square columns under biaxially eccentric loads." ACI Journal, V. 32, No. 9, March 1961, pp.1129-1140.

9. Hognestad, E. "A study of combined and axial load in reinforced concrete members." University of Illinois Engineering Experiment Station Bulletin #399.

10. Pannell, F.N. "Failure surfaces for members in compression and biaxial bending, ACI Journal, V. 60, No. 1, Jan. 1963, pp.129-140.

11 Pannell, F.N. and Robinson, J.L. "Slender reinforced concrete columns with biaxial eccentricity of loading." Magazine of Concrete Research, Vol. 20, No. 65, Dec. 1968.

NOTATION

b	smaller dimension of column cross section
C_m	factor relating the actual moment diagram to an equivalent moment diagram as per ACI 318-71
e	eccentricity of load
E_c	modulus of elasticity of concrete
F	factor to correct for cover ratio - defined in text
f'_c	specified cylinder strength of concrete
fy	specified yield strength of reinforcement
g	cover ratio
Ig	second moment of area of gross concrete area
I	second moment of area
Mfx	ultimate moment about y axis
Mfy	ultimate moment about x axis
My	design ultimate moment about y axis
Mx	design ultimate moment about x axis
N	defined in Figure 2
Pu'	ultimate load at actual eccentricity $e = \sqrt{ex^2 + ey^2}$
Px'	ultimate load at ex with ey = 0
Py'	ultimate load at ex with ex = 0
Po'	axial ultimate load
t	larger dimension of column cross-section
z	perpendicular distance from neutral axis to point of maximum strain
β_d	ratio of maximum design dead load to maximum design total load
Δ	lateral displacement of column

δ moment magnifier - as per ACI 318-71

ε strain

ρ steel ratio

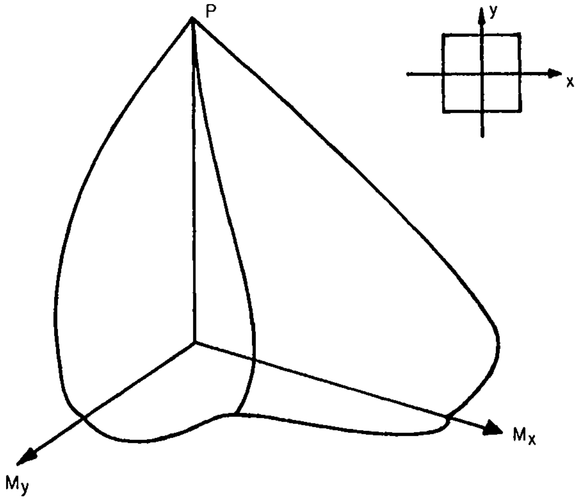

Fig. 1--Load biaxial moment interaction surface

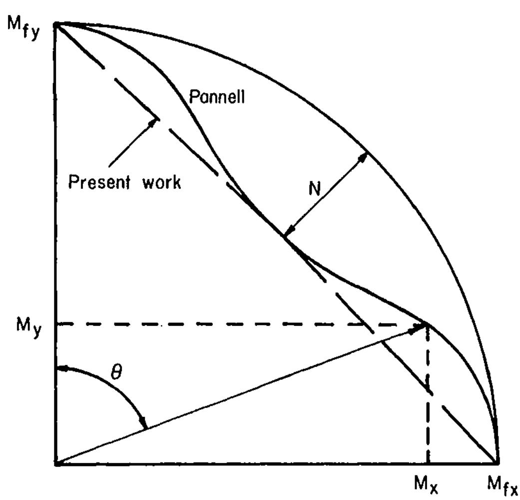

Fig. 2--Horizontal section through interaction diagram

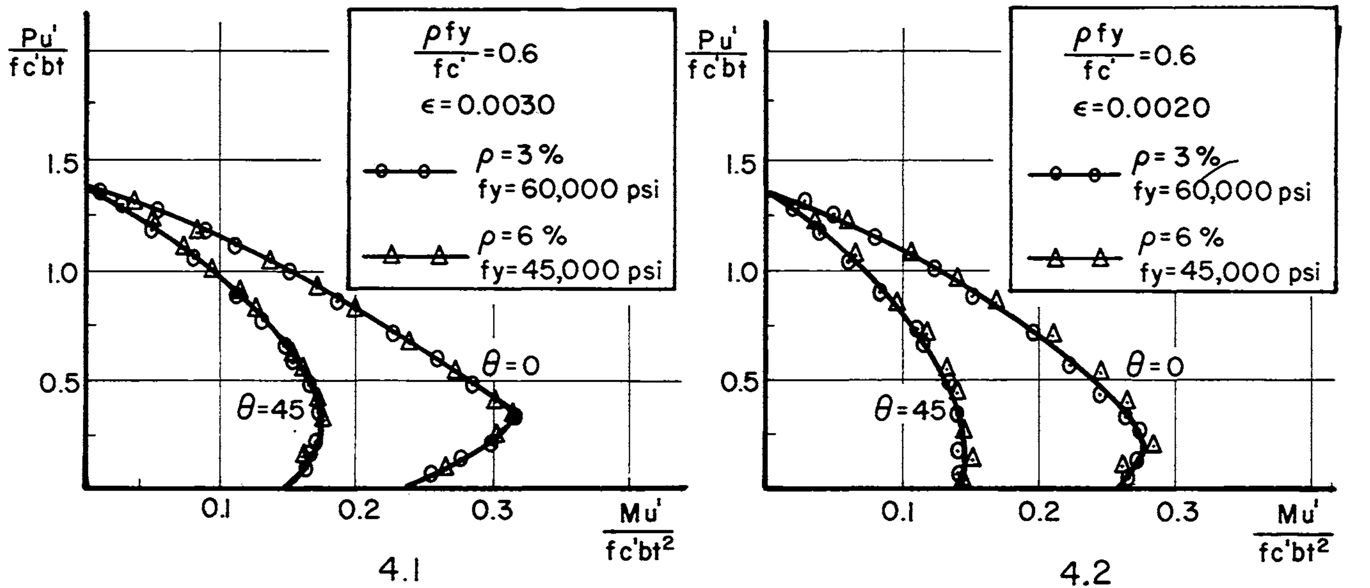

Fig. 3--Justification of $\frac{\rho f_y}{f_c'}$ as an independent parameter

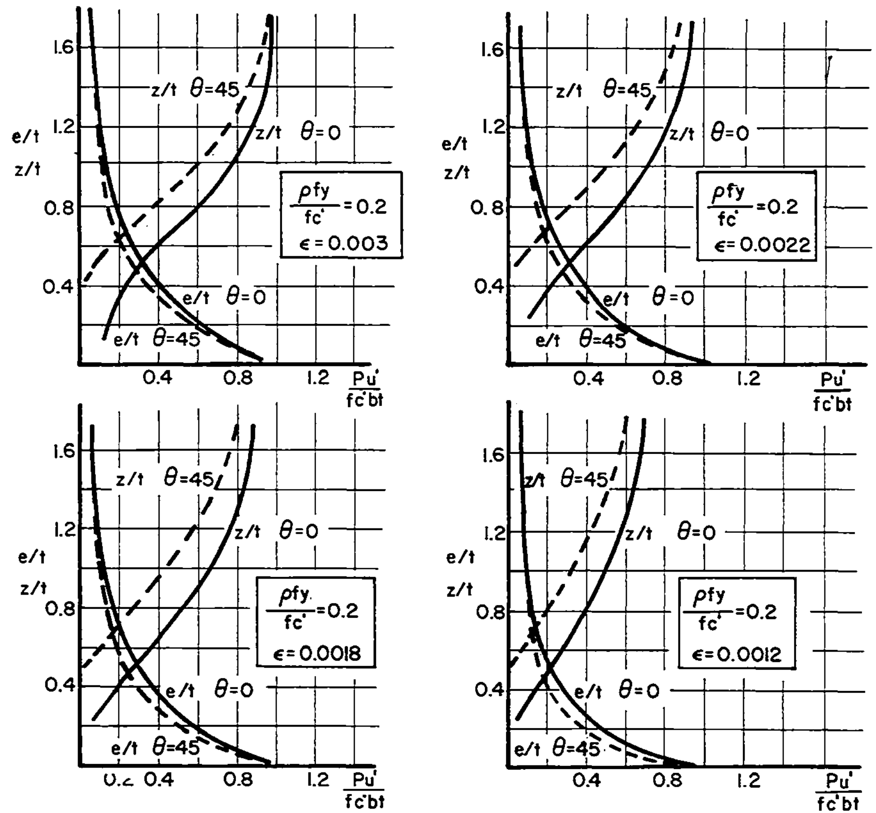

Fig. 4 Interaction curves for ... 20

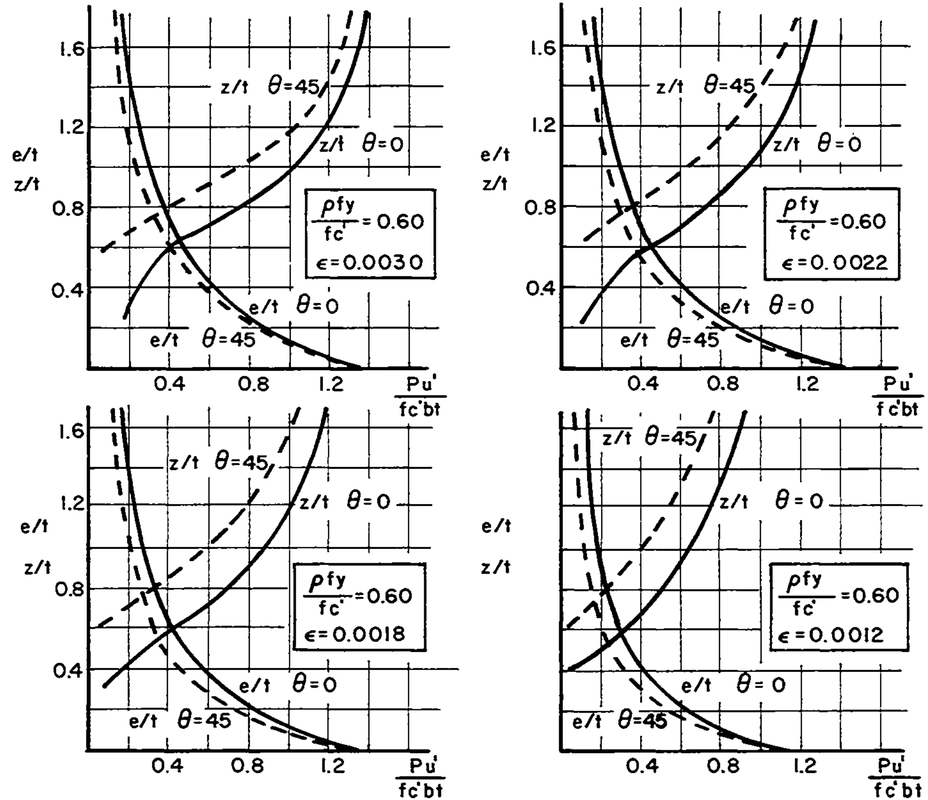

Fig. 5--Interaction curves for $\frac{\rho f_y}{f'_c} = 0.60$

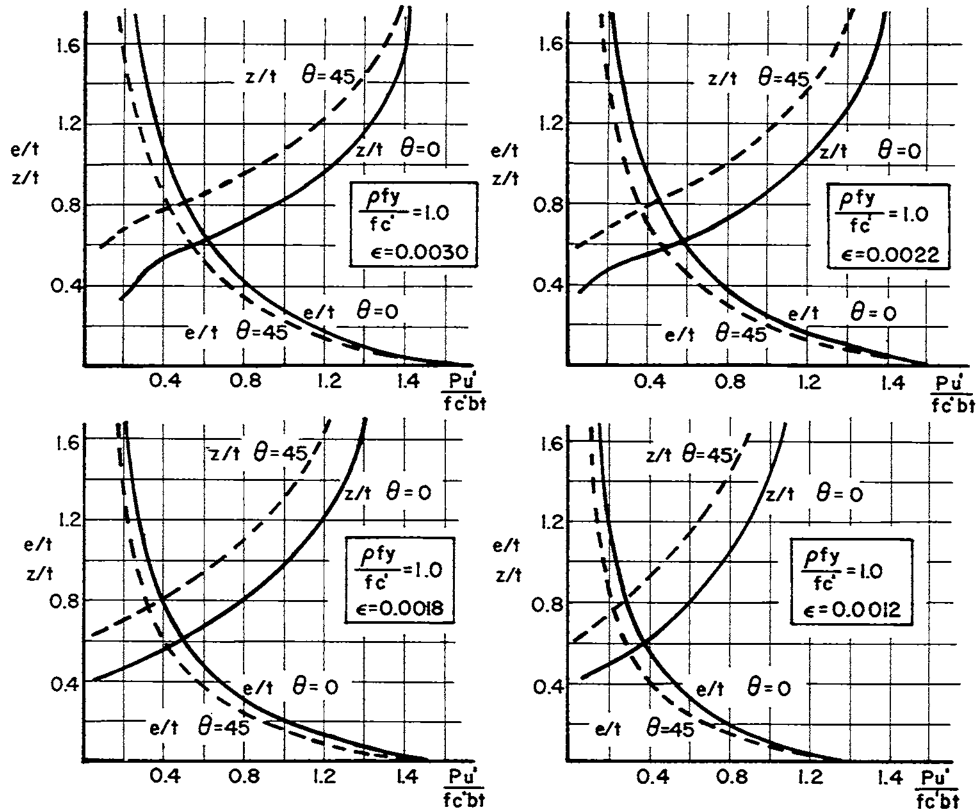

Fig. 6--Interaction curves for $\frac{\rho f_y}{\;} = 1.0$

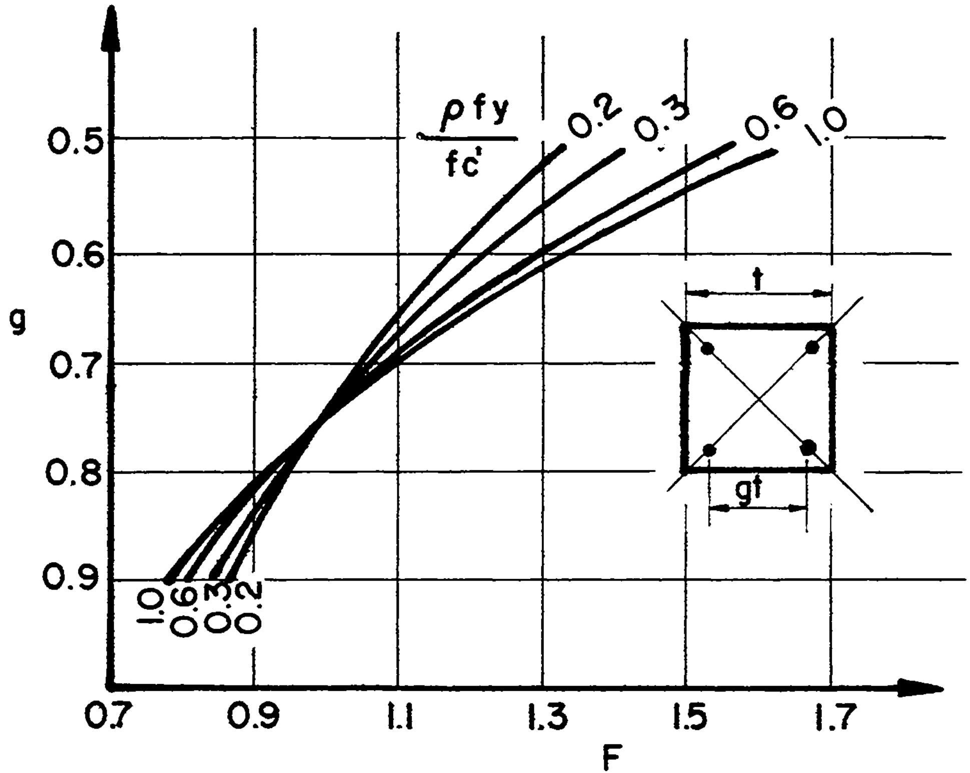

Fig. 7--Factor F to modify results with a cover ratio g = 0.75 to any other cover ratio