Chapter 7
Horizontally Curved Members

7.1 INTRODUCTION

This chapter discusses the strength and behavior of beams loaded perpendicular to the plane of curvature as shown in Figure 7-1. Horizontally curved beams must resist both flexural and torsional moments and are subject to the same limit states as straight beams. In most cases, torsional rotations lead to a design controlled by serviceability considerations.

7.2 BEHAVIOR

The deflected shape of a horizontally curved beam is characterized by vertical and horizontal translation, and torsional rotation of the cross section. Second-order effects and potential yielding of the beam cause nonlinear deformations until failure occurs by excessive deformations and/or yielding of the member. Due to their high torsional stiffness, closed sections provide efficient resistance to these deformations.

The behavior of curved beams is dependent on the span angle, θ_s, in Figure 7-1. Beams with span angles less than 1° are dominated by flexure, acting as a nominally straight beam with an initial geometric imperfection. For beams with span angles between 1° and 20°, both bending and torsion have a significant influence on the behavior. When the span angle is greater than 20°, the behavior is affected primarily by torsion (Pi et al., 2000).

Because torsional deformations dominate the behavior of beams with span angles greater than 20°, efficient framing systems typically utilize infill members to provide torsional restraint. Where the curved member is continuous across torsional supports, as shown in Figure 7-2, warping restraint increases the torsional efficiency. Analogous to the flexural behavior of a continuous beam, warping restraint is provided by equal and opposite warping moments in the adjacent span. The total resisting moment at the end of the infill beam is the sum of the torsional loads at the end of each span, M_e, shown in Figure 7-2(c). Connections between the curved member and the infill beams are discussed in Section 7.9.

7.3 STRUCTURAL ANALYSIS

Several methods are available to calculate the required loads in a curved beam. The finite element method is generally used for final design. Both the M/R method and the eccentric-load method are accurate enough for use in final design; however, they may be more appropriate for preliminary design in cases where complicated geometry and loadings are required. Also, these methods can provide valuable insight into the fundamental behavior of horizontally curved beams.

The required loads can also be calculated using equations published by Lebet and Hirt (2013), Young and Budynas (2002) and Nakai and Yoo (1988); however, the equations are cumbersome for design office use, and they are available only for a limited number of idealized cases. For the simplest case shown in Figure 7-2(a), where the beam is subjected to equal and opposite flexural moments, M_x, at the ends of the unbraced segment, the flexural moment is (Pi et al., 2000):

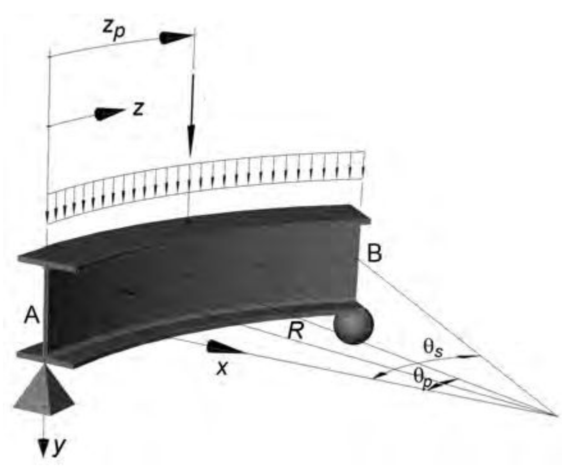


Fig. 7-1. Horizontally curved beam.

$$M_{x\theta} = \frac{M_x \cos\left(\frac{\theta_b}{2} - \theta_z\right)}{\cos\left(\frac{\theta_b}{2}\right)} \tag{7-1}$$

where

θ_b = angle between torsional restraints, rad

θ_z = angle from the end of the segment to the location of interest, rad

The torsional moment is:

$$M_{z\theta} = \frac{M_x \sin\left(\frac{\theta_b}{2} - \theta_z\right)}{\cos\left(\frac{\theta_b}{2}\right)} \tag{7-2}$$

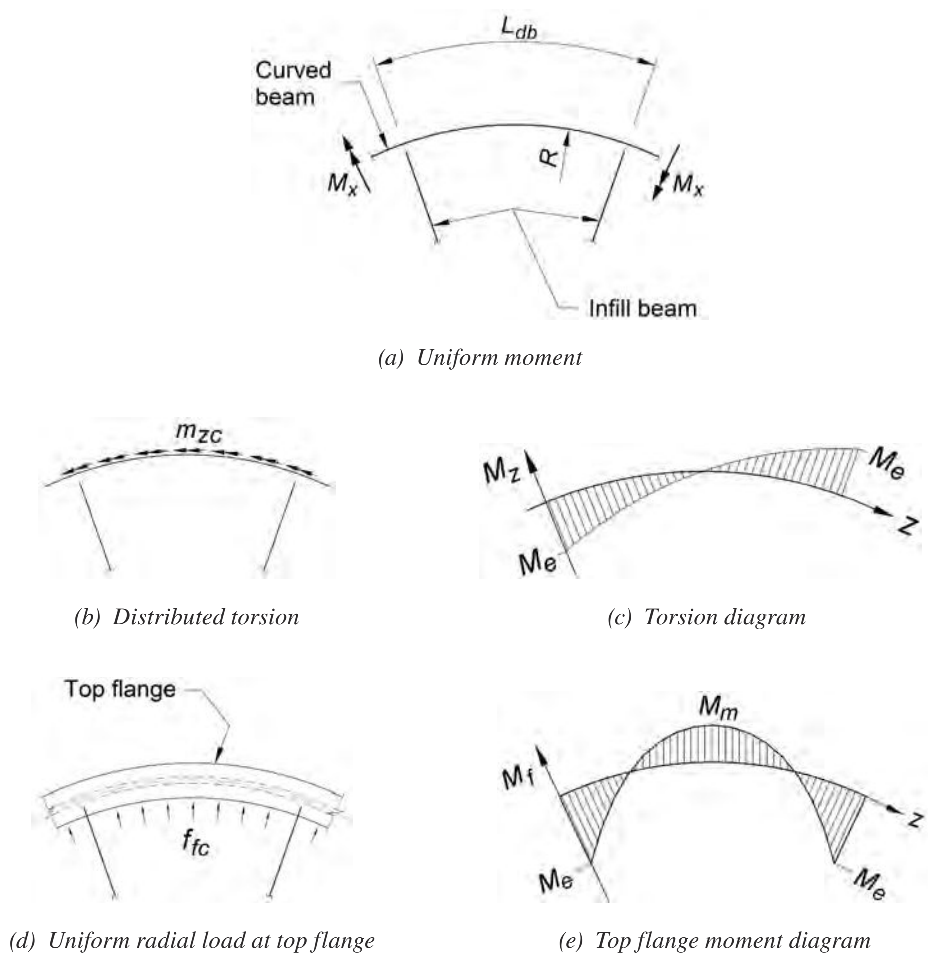
The maximum flexural moment, which occurs at the mid-span, is:

$$M_{xm} = \frac{M_x}{\cos\left(\frac{\theta_b}{2}\right)} \tag{7-3}$$

The maximum torsional moment, which occurs at the ends, is:

$$M_{ze} = M_x \tan\left(\frac{\theta_b}{2}\right) \tag{7-4}$$

In addition to shear and axial loads, helical members are subjected to biaxial flexure and torsion. Several solutions are available for calculating the loads in spiral stairs of various geometries (Bangash and Bangash, 1999; Abdul-Baki and Bartel, 1969; Bergman, 1956). However, the equations are cumbersome for design office use and they are available only for a limited number of idealized cases. Because the solutions were derived by modeling the treads and stringers as a



(a) Uniform moment

(b) Distributed torsion *(c) Torsion diagram*

(d) Uniform radial load at top flange *(e) Top flange moment diagram*

Fig. 7-2. Curved beam continuous across torsional supports.

single member, they are applicable to steel stairs only in special cases. A finite element model may be the best method to determine the required member loads. A conservative model can be obtained by neglecting the treads and modeling the stringers as independent spiral members. In many cases, this will be extremely conservative because the treads can provide significant torsional restraint to the stringer. The level of torsional restraint provided by the treads is dependent on the tread type, arrangement and connection details.

7.3.1 Finite Element Models

Either a two- or three-dimensional finite element model can be used to model the structural behavior of horizontally curved beams. As discussed in Section 6.6, curved members are usually modeled with a series of straight elements. Although a three-dimensional model requires a greater engineering effort, the accuracy may only be slightly better than a two-dimensional analysis with similar element sizes (Nevling et al., 2006).

Two-Dimensional Finite Element Model

A two-dimensional segmented finite element model, with several straight beam elements representing the curved member will usually provide the accuracy required for design purposes. The accuracy increases with the number of elements. Between 10 and 20 elements is adequate for modeling most semi-circular members (King and Brown, 2001). For models with highly nonlinear behavior, a convergence study may be required to determine the appropriate number of elements.

Most commercial finite element programs use the basic beam finite element formulation, which does not have the capability to model the warping stiffness. In this case, only the St. Venant stiffness is utilized in the analysis, which causes an overestimate of the torsional deformations for most open cross sections. The accuracy can be improved by using equivalent torsion constants (Ahmed and Weisgerber, 1996; White and Coletti, 2013). For members with warping fixed at both ends of the span, the equivalent torsion constant is:

$$J_e = \frac{J}{1 - \dfrac{\sinh\gamma}{\gamma} + \dfrac{(\cosh\gamma - 1)^2}{\gamma\sinh\gamma}} \tag{7-5}$$

where

J = torsional constant, in.^{4}

$$\gamma = L_{db}\sqrt{\frac{GJ}{EC_w}} \tag{7-6}$$

and where

C_w = warping constant, in.^{6}

E = modulus of elasticity, ksi

G = shear modulus, ksi

L_{db} = developed length (arc length) along the curved member between torsional restraints, in.

For members with warping fixed at one end of the span and warping free at the other end, the equivalent torsion constant is:

$$J_e = \frac{J}{1 - \dfrac{\sinh\gamma}{\gamma\cosh\gamma}} \tag{7-7}$$

Because torsion in closed cross sections is resisted primarily by St. Venant torsion, accurate results can be expected for closed sections when the warping stiffness is neglected.

Three-Dimensional Finite Element Model

A three-dimensional finite element model uses several elements to make up the cross section. The webs are typically modeled with plate elements, but can also be modeled with shell or solid elements. The flanges of I-shaped members are typically modeled with beam elements, but can also be modeled with plate, shell or solid elements (FHWA, 2015; AASHTO/NSBA, 2014; King and Brown, 2001). The warping stiffness is addressed properly in these models without the need for modified torsion constants.

Infill members and cross frames that are rigidly connected to restrain torsion can be connected to nodes at the top and bottom flanges of the curved member. They can be modeled with beam elements or with plate/shell/solid elements (FHWA, 2015; AASHTO/NSBA, 2014).

Any deck or slab can be modeled with plate, shell or solid elements. If plate or shell elements are used, the elements should be offset vertically above the top flange of the curved member using linking elements. For example, a composite slab can be modeled with eight-node solid elements attached to the curved member top flange with beam elements representing the shear headed stud anchors (FHWA, 2015).

If the flanges of the curved member are modeled with beam elements, the required stresses from the model can be compared with the available stresses in the AISC *Specification* (AISC, 2016c) and AISC Design Guide 9 *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997). However, the available strengths in the AISC *Specification* were not developed to be compared to the results from finite element models built with plate, shell or solid elements. The ASD safety factors and LRFD resistance factors in the AISC *Specification* were calibrated to provide a specific target reliability when compared with required loads calculated using truss and beam elements in the structural analysis model, not plate, shell or solid elements. If the results of a model with plate, shell or solid elements are used with the AISC *Specification* provisions, the required member loads should be determined by summing the element stresses over the entire cross section. This will complicate the calculations and will likely produce similar results compared to a model with beam elements used for the flanges.

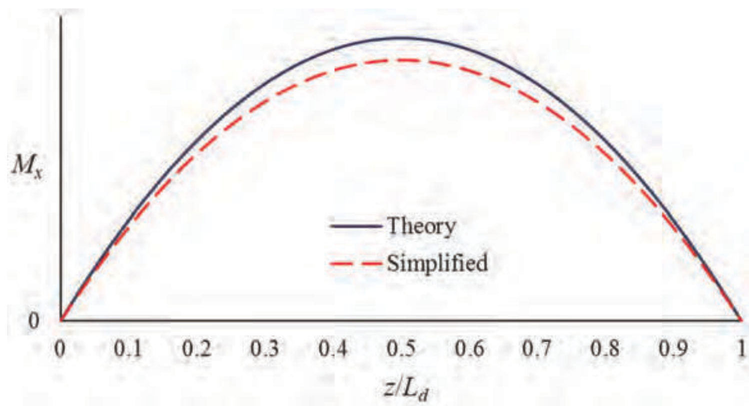
7.3.2 *M/R* Method

The *M/R* method (Tung and Fountain, 1970) has been used extensively in design where the curved beam is modeled as a straight member with a length equal to the developed span length, $L_{ds} = R\theta_s$, where R is the radius and θ_s is the span angle in radians. The shear force, V, and the out-of-plane flexural moment, M_x, are calculated as for a straight beam. Figure 7-3(a) shows the bending moment diagram for a horizontally curved, simply supported, uniformly loaded beam. The solid line shows the moment for the exact solution (Lebet and Hirt, 2013), and the dashed line shows the moment calculated using the straight beam approximation. The bending moment diagrams for a horizontally curved, simply supported beam with a midspan concentrated load is shown in Figure 7-4(a). In both cases, the developed member length, L_d, is equal to the developed span length, L_{ds}.

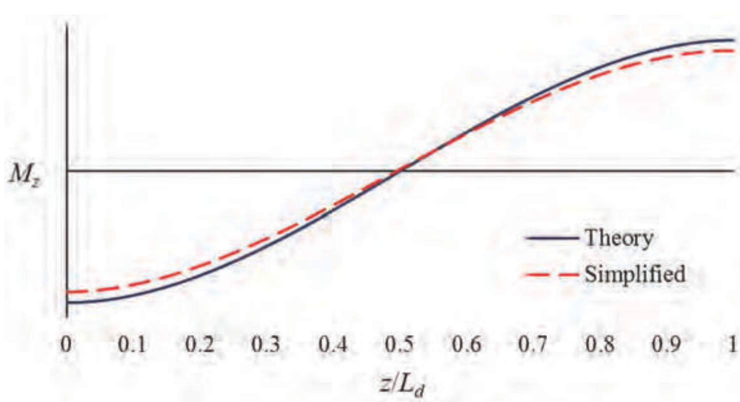
The torsional moment per unit length resulting from the beam curvature can be estimated with Equation 7-8. Torsion diagrams can be constructed in a manner similar to the method used for shear and moment diagrams, where the change in torsional moment, M_z, between two points along the developed beam length is equal to the summation of m_{zc} over the segment of interest. Because m_{zc} accounts only for shear-center loads caused by the beam curvature, any additional torsional moments should be added algebraically to m_{zc}.

$$m_{zc} = \frac{M_x}{R} \tag{7-8}$$

For the curved beam in Figure 7-2(a), m_{zc} is shown in Figure 7-2(b) and the torsion diagram is shown in Figure 7-2(c). Because the torsion diagram is shown on a curved axis, the diagram is curved; however, the variation in torsional moment is linear along the member arc. The torsion diagrams for the bending moment diagrams in Figures 7-3(a)



(a) Moment diagram



(b) Torsion diagram

Fig. 7-3. Moment and torsion diagrams for a horizontally curved, simply supported, uniformly loaded beam.

and 7-4(a) are shown in Figures 7-3(b) and 7-4(b), respectively. In both cases, torsional restraints are located only at the supports ($L_d = L_{ds} = L_{db}$).

The required shear calculated using the straight beam model is equal to the theoretical value; however, the flexural and torsional moments are under-predicted. When $\theta_s \le \pi/6$ (30°), the error for the simplified method is less than 3%. For $\theta_s > \pi/6$ (30°), the flexural and torsional moments can be calculated using correction factors according to Equations 7-9 and 7-10, respectively.

$$M_{xc} = CM_x \tag{7-9}$$

$$M_{zc} = CM_z \tag{7-10}$$

where

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2} \tag{7-11}$$

Idealized Cases

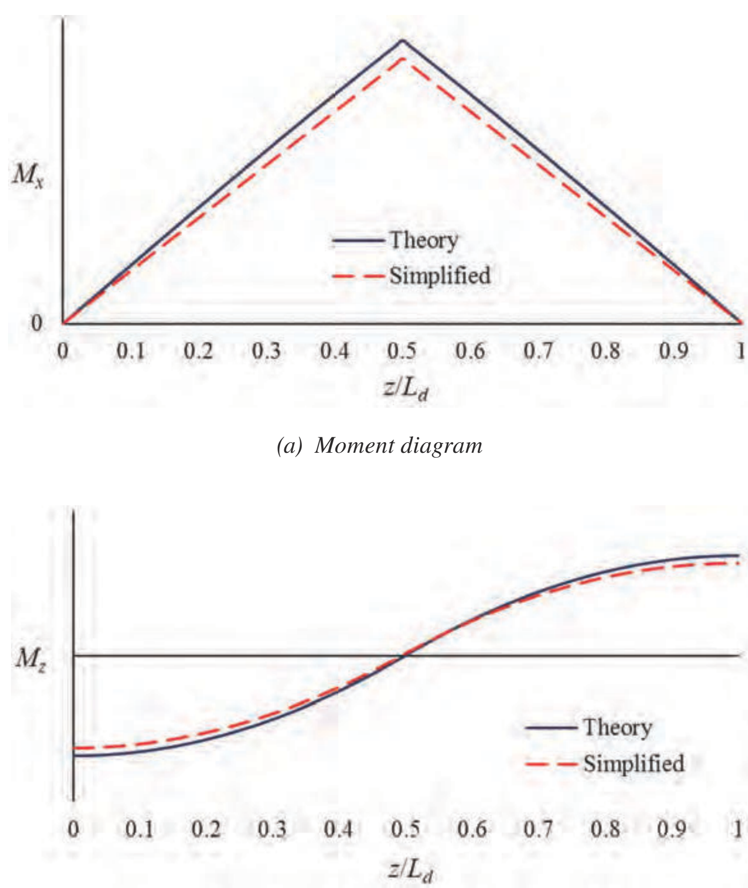
For several idealized cases, the M/R method can be used to develop equations for the torsional moment at any location along the member. For all cases discussed, torsional restraints are located only at the supports ($L_d = L_{ds} = L_{db}$). For a horizontally curved, simply supported, uniformly loaded beam, the torsional moment represented by the diagram in Figure 7-3(b) is:

$$M_z = \frac{w}{24R}\left[L_d^3 - z^2\left(6L_d - 4z\right)\right] \tag{7-12}$$

where

L_d = developed beam length, in.
w = uniform load, kip/in.
z = distance along the developed beam length, in.

The torsional moment is zero at the midspan and the maximum/minimum values at the ends are:



(a) Moment diagram

(b) Torsion diagram

Fig. 7-4. Moment and torsion diagrams for a horizontally curved, simply supported beam with a midspan concentrated load.

$$M_z = \pm \frac{wL_d^3}{24R} \tag{7-13}$$

For a horizontally curved, simply supported beam with a midspan concentrated load, the torsional moment represented by the diagram in Figure 7-4(b) is:

$$M_z = \frac{P}{16R}\left(L_d^2 - 4z^2\right) \quad \text{for } z \le \frac{L_d}{2} \tag{7-14}$$

where

P = concentrated load, kips

The torsional moment is zero at the midspan and the maximum/minimum values at the ends are:

$$M_z = \pm \frac{PL_d^2}{16R} \tag{7-15}$$

For a horizontally curved, fixed-end, uniformly loaded beam, the torsional moment is:

$$M_z = \frac{wz}{12R}\left(3L_d z - L_d^2 - 2z^2\right) \tag{7-16}$$

The torsional moment is zero at the ends and midspan. The maximum/minimum values at z=0.211L_d and z=0.789L_d are:

$$M_z = \pm \frac{wL_d^3}{125R} \tag{7-17}$$

For a horizontally curved, fixed-end beam with a midspan concentrated load, the torsional moment is:

$$M_z = \frac{Pz}{8R}\left(2z - L_d\right) \quad \text{for } z \le \frac{L_d}{2} \tag{7-18}$$

The torsional moment is zero at the ends and midspan. The maximum/minimum values at z=L_d/4 and z=3L_d/4 are:

$$M_z = \pm \frac{PL_d^2}{64R} \tag{7-19}$$

7.3.3 Eccentric Load Method

A simple method to approximate the torsional loads on a horizontally curved beam is based on the horizontal eccentricity from the load to a chord drawn between the supports (Heins and Firmage, 1979). For members that are loaded along their curved shear center axis, the equivalent eccentricity is the distance perpendicular to the chord, from the chord to the centroid of the load. For uniformly distributed loads, the equivalent eccentricity is:

$$e_w = R\left[\cos\left(\frac{\theta_s}{4}\right) - \cos\left(\frac{\theta_s}{2}\right)\right] \tag{7-20}$$

And the uniformly distributed torsion is:

$$m_z = we_w \tag{7-21}$$

For midspan concentrated loads, the equivalent eccentricity is:

$$e_p = R\left[1 - \cos\left(\frac{\theta_s}{2}\right)\right] \tag{7-22}$$

And the concentrated midspan torsion is:

$$M_z = Pe_p \tag{7-23}$$

The support moments are accurately predicted with the eccentric load method; however, the span moments are only approximate. Use of these approximate span moments results in a linear torsion diagram, which is advantageous in design because the required cases for these loading conditions are available in AISC Design Guide 9, Appendix B.

7.4 FLEXURAL STRENGTH

The local buckling provisions in AISC *Specification* Chapter B are applicable to horizontally curved beams without modification. As discussed in Section 7.2, the behavior at ultimate strength is characterized by excessive vertical, horizontal and torsional deformations rather than a classical lateral-torsional buckling failure. However, as with straight beams, the flexural strength of curved beams is reduced for members that are susceptible to lateral-torsional buckling (Yoo et al., 1996; Nishida et al., 1978). Because closed sections have a high torsional rigidity, they are typically not subject to lateral-torsional buckling.

The effect of curvature on the lateral-torsional buckling strength is negligible when the angle between torsional restraints, θ_b, is equal to or less than $\pi/8$ (22.5°). In this case, AISC *Specification* Chapter F is applicable. For doubly symmetric I-shaped members with $\theta_b > \pi/8$ (22.5°), the provisions of Chapter F can be used with a revised lateral-torsional buckling modification factor according to Equation 7-24 (adapted from Yoo et al., 1996). In the AISC *Specification* equations, the developed length along the beam between torsional restraints, L_{db}=$R\theta_b$, must be used in lieu of the straight-member unbraced length, L_b.

$$C_{bo} = C_{bs}\left[1 - \left(\frac{\theta_b}{\pi}\right)^2\right]^2 \tag{7-24}$$

where

C_{bs} = lateral-torsional buckling modification factor for an equivalent straight member

θ_b = angle between torsional restraints, rad

7.5 TORSIONAL STRENGTH

After the required torsional loading diagrams have been constructed using one of the structural analysis methods in Section 7.3, the torsional strength can be determined with one of the methods in this section. Because both torsion and flexure are present in curved beams, the second-order effects and interaction equations in Section 7.6 are required to verify the member strength. In all cases, the torsional strength is calculated for an equivalent straight member, based on the developed length between torsional restraints, L_{db}, and properly accounting for any warping restraints.

7.5.1 Elastic Method

AISC Design Guide 9 can be used to calculate the elastic torsional strength of an equivalent straight member. Because the torsion diagrams for curved beams are typically nonlinear, conservative assumptions are usually required to accommodate the design charts in Appendix B of the Design Guide. The simplest loading case with a uniform moment over the unbraced length, as shown in Figure 7-2(a), results in a uniformly distributed torsion, as shown in Figure 7-2(b). For this loading condition, the torsional functions can be determined with Case 4 or Case 7, depending on the warping boundary conditions at the supports. For simply supported beams subjected to uniformly distributed loads, the maximum value of m_{zc} within the span can be used as a conservative estimate of the uniform torsion per unit length. Using this simplification with Case 4 for a beam with free warping at the boundaries results in an overestimate of torsional rotations by 23%.

For composite I-shaped beams, as shown in Figure 7-5(a), the torsional properties are based on the idealized transformed section shown in Figure 7-5(b) (Heins and Kuo, 1972). The normalized warping functions and warping statical moments are shown in Figures 7-5(c) and 7-5(d), respectively. The torsional constant is:

$$J = \frac{1}{3}\left(b_f t_f^3 + d_e t_w^3 + \frac{b_e t_c^3}{m}\right) \tag{7-25}$$

The warping constant is:

$$C_w = \frac{1}{12}\left[y^2 b_e^3 t_e + (d_e - y)^2 b_f^3 t_f\right] \tag{7-26}$$

The normalized warping function for the slab is:

$$W_{nc} = \frac{y b_e}{2} \tag{7-27}$$

The normalized warping function for the steel section is:

$$W_{ns} = \frac{(d_e - y) b_f}{2} \tag{7-28}$$

The warping statical moment for the slab is:

$$S_{wc} = \frac{y b_e^2 t_e}{8} \tag{7-29}$$

The warping statical moment for the steel section is:

$$S_{ws} = \frac{(d_e - y) b_f^2 t_f}{8} \tag{7-30}$$

The shear center location is:

$$y = \frac{b_f^3 t_f d_e}{b_e^3 t_e + b_f^3 t_f} \tag{7-31}$$

where

E = modulus of elasticity of the steel section, ksi

E_c = modulus of elasticity of the slab, ksi

G = shear modulus of the steel section, ksi

G_c = shear modulus of the slab, ksi

b_e = effective slab width, in.

b_f = flange width, in.

d_e = $d + (t_e - t_f)/2$ = distance between flange centroids of the idealized section, in.

m = G/G_c = shear modular ratio

n = E/E_c = modular ratio

t_c = slab thickness, in.

t_e = t_c/n = transformed slab thickness, in.

t_f = flange thickness, in.

7.5.2 Isolated Flange Method

If the St. Venant torsion is neglected, the torsional loads are resisted exclusively by warping. For I-shaped members, the warping strength can be approximated by isolating the flanges and treating them as independent rectangular beams loaded in the horizontal plane by a distributed radial force per unit length calculated with Equation 7-32. The radial force is applied toward the center of curvature at the tension flange and away from the center of curvature at the compression flange as shown in Figure 7-6.

$$\begin{aligned} f_{fc} &= \frac{m_{zc}}{h_o} \\ &= \frac{M_x}{R h_o} \end{aligned} \tag{7-32}$$

where

h_o = distance between flange centroids, in.

The flexural boundary conditions of the isolated flange are based on the warping boundary conditions of the curved member. If warping is restrained at the support, the isolated flange will be modeled with a flexurally fixed end. For free warping, the isolated flange will be modeled with a flexurally pinned end. For sections with compact flanges, the nominal flexural strength of the isolated flange is:

$$M_{nw} = F_y Z_f \tag{7-33}$$

The plastic modulus about the strong axis of the flange is:

$$Z_f = \frac{t_f b_f^2}{4} \tag{7-34}$$

where

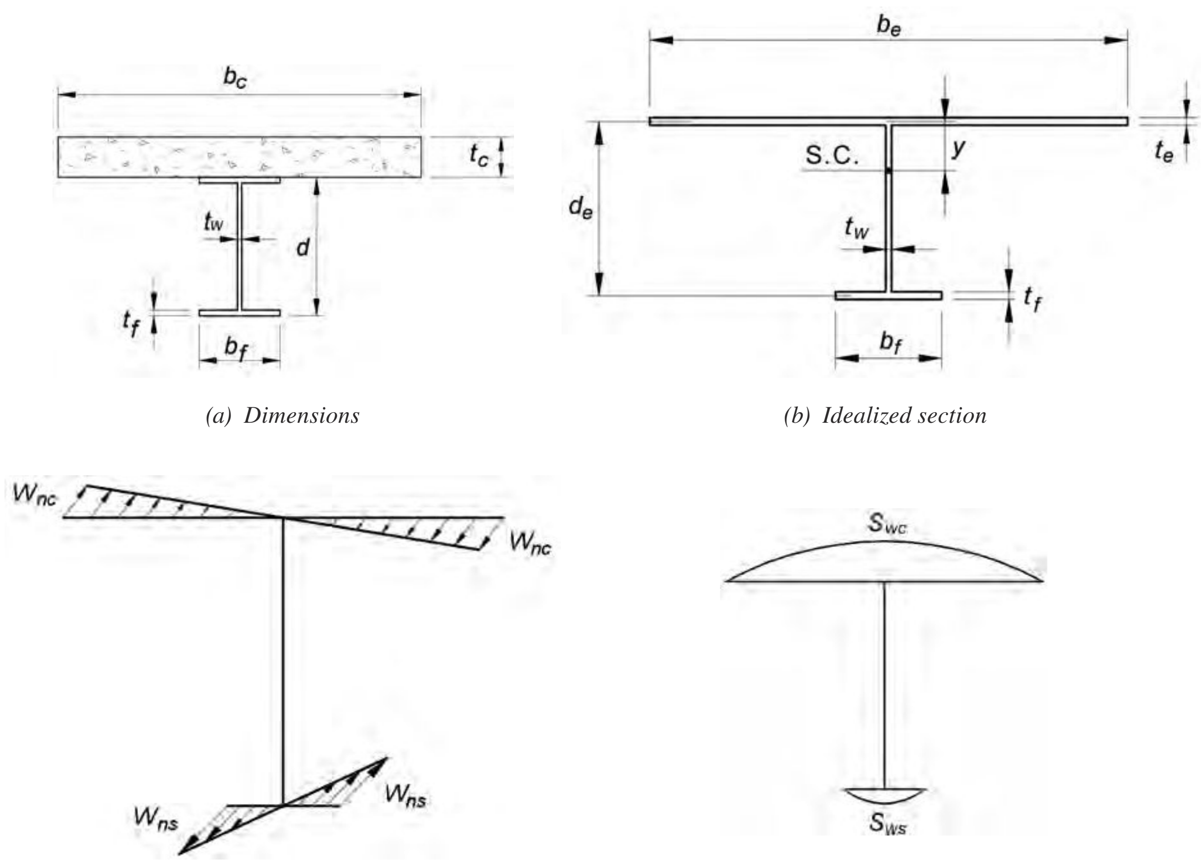
b_f = flange width, in.

t_f = flange thickness, in.

For the beam in Figure 7-2(a), the moment diagram for the isolated top flange, shown in Figure 7-2(e), is based on the compression flange radial load shown in Figure 7-2(d). Because the member is continuous across the infill beams, warping is restrained at the ends of the unbraced segment. For this condition, the moment diagram for the equivalent straight beam isolated flange is based on a fixed-fixed uniformly loaded beam.

The basic steps for the isolated flange method are:

1. Construct the primary moment diagram for the equivalent straight beam segment between points of torsional restraint, L_{db}.
2. Convert the warping boundary conditions to the appropriate flexural boundary conditions for the isolated flange.
3. Using the primary moment diagram and Equation 7-32, calculate the distributed radial force per unit length, f_{fc}, to be applied to the isolated flange.
4. Construct the moment diagram for the isolated flange.



(a) Dimensions *(b) Idealized section*

(c) Normalized warping function *(d) Warping statical moment*

Fig. 7-5. Torsional properties of a composite beam.

5. Evaluate both flanges under the combined actions, including second-order effects, as discussed in Section 7.6.

7.6 COMBINED FLEXURE AND TORSION

As with straight members, curved members subjected to both flexure and torsion must consider the effects of load interaction. After the required flexural and torsional moments are determined, the available member strength is calculated either by combining the stresses or by combining the load ratios in an interaction equation. In either case, second-order effects must be considered.

7.6.1 Second-Order Effects

Second-order torsional moments and rotations can be calculated either by using a rigorous second-order analysis or by amplifying the results of a first-order analysis. Amplification factors similar to those in AISC *Specification* Appendix 8 for straight members can be used for curved members (Rettie, 2015; AASHTO, 2014; Ashkinadze, 2008; Lindner and Glitsch, 2005; Boissonnade et al., 2002; Trahair and Teh, 2000; Pi and Trahair, 1994). For open sections subjected to both torsion and strong-axis flexure, the second-order torsional rotation is:

$$\theta_2 = B_o\theta_1 \tag{7-35}$$

The second-order torsional moment is:

$$M_{rz} = B_o M_z \tag{7-36}$$

The amplification factor is:

$$B_o = \frac{0.85}{1-\alpha\dfrac{M_{ro}}{M_{eo}}} \geq 1.0 \tag{7-37}$$

where

M_{eo} = elastic critical lateral-torsional buckling moment for out-of-plane flexure, kip-in.
M_{ro} = required out-of-plane flexural moment, kip-in.
M_z = first-order torsional moment, kip-in.
θ_1 = first-order torsional rotation, rad
α = 1.0 (LRFD); 1.6 (ASD)

Where compression flange bracing is spaced close enough to satisfy $L_b \leq L_p$ according to AISC *Specification* Chapter F, torsional moments can be based on a first-order analysis. Because closed sections are typically not subject to lateral-torsional buckling, the second-order contribution to the torsional moment is negligible for these members.

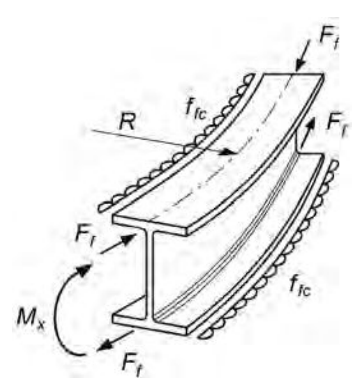


Fig. 7-6. The isolated flange method (adapted from King and Brown, 2001).

7.6.2 Noncomposite I-Shaped Members

Noncomposite I-shaped members can be evaluated using either the elastic method or the isolated flange method, which are discussed in Sections 7.5.1 and 7.5.2, respectively. When the isolated flange method is used to determine the flange warping moment, the out-of-plane moment can be combined with the flange warping moment by adapting AISC *Specification* Equation H1-1a:

$$\frac{M_{ro}}{M_{co}} + \frac{8}{9}\frac{M_{rw}}{M_{cw}} \leq 1.0 \tag{7-38}$$

where

M_{co} = available out-of-plane flexural strength of the member, kip-in.
M_{cw} = available flexural strength of the isolated flange, kip-in.
= ϕM_{nw} (LRFD)
= M_{nw}/Ω (ASD)
M_{nw} = nominal flexural strength of the isolated flange, kip-in.
M_{ro} = required out-of-plane flexural strength of the member, kip-in.
M_{rw} = required second-order flexural strength of the isolated flange, kip-in.
Ω = 1.67
ϕ = 0.90

When the elastic method is used, the warping stresses calculated with AISC Design Guide 9 can be combined with the member flexural stresses by adapting AISC *Specification* Equation H1-1a for elastic stresses according to Equation 7-39. The 16/27 value for the constant is the result of dividing 8/9, which is the constant from Equation H1-1a, by 3/2, which is the shape factor of the flange.

$$\frac{\sigma_{ro}}{\sigma_{co}} + \frac{16}{27}\frac{\sigma_{rw}}{\sigma_{cw}} \leq 1.0 \tag{7-39}$$

where

σ_{co} = available out-of-plane flexural stress, ksi
σ_{cw} = available warping stress, ksi
= ϕF_y (LRFD)
= F_y/Ω (ASD)
σ_{ro} = required out-of-plane flexural stress, ksi
σ_{rw} = required second-order warping stress, ksi
Ω = 1.67
ϕ = 0.90

Analysis by Finite Element model

For finite element models with the flanges modeled as rectangular beam elements, the strong axis of the element will be oriented vertically with the warping stresses varying across the element depth. The analysis will result in both axial and flexural loads on the element due to out-of-plane flexure and warping of the curved beam, respectively. The elements can be evaluated using the equations in *Specification* Section H1. For finite element models with the flanges modeled as plate, shell or solid elements, the required member loads for use with the equations in AISC *Specification* Section H1 should be determined by summing the stresses over the element as discussed in Section 7.3.1.

7.6.3 HSS and Box-Shaped Members

Round, square and rectangular HSS members and box-shaped members can be designed according to AISC *Specification* Section H3.2. For combined flexure, shear and torsion, Equation H3-6 reduces to:

$$\frac{M_{ro}}{M_{co}} + \left(\frac{V_r}{V_c} + \frac{M_{rz}}{M_{cz}}\right)^2 \le 1.0 \tag{7-40}$$

where

M_{cz} = available torsional strength, kip-in.
M_{rz} = required torsional strength, kip-in.
V_c = available shear strength, kips
V_r = required shear strength, kips

For evaluation using stresses, Equation 7-40 can be expressed using the stress ratios:

$$\frac{\sigma_{ro}}{\sigma_{co}} + \left(\frac{\tau_{rv}}{\tau_{cv}} + \frac{\tau_{rt}}{\tau_{ct}}\right)^2 \le 1.0 \tag{7-41}$$

where

τ_{ct} = available shear stress for torsional loads, ksi
τ_{cv} = available shear stress for shear loads, ksi
τ_{rt} = required shear stress due to torsional loads, ksi
τ_{rv} = required shear stress due to shear loads, ksi

7.6.4 Composite I-Shaped Members

For partially and fully composite straight and curved beams subjected to torsion, the concrete slab provides most of the torsional resistance. The steel member enhances the slab strength by restraining the longitudinal deformation. Torsional strength increases when the member is subjected to flexural loading because flexural compression in the slab partially opposes the torsional tensile stresses, which decreases the concrete cracking (Tan and Uy, 2011; Tan and Uy, 2009; Nie et al., 2009). Therefore, the interaction between torsion and flexure can be neglected for partially and fully composite beams, and the torsional and flexural strengths can be verified independently.

7.7 SERVICEABILITY

As discussed in Sections 7.1 and 7.2, large vertical, horizontal and torsional deformations at ultimate strength often result in designs based on serviceability rather than strength. A reasonable limit on the maximum angle of rotation will ensure nonstructural elements are not damaged by excessive rotations. There are no formal limits in building codes; therefore, judgment should be used to define the appropriate deflection and rotation limits based on the type of building elements supported by the beam. When Equation 7-37 is used to calculate the second-order amplification factor for serviceability conditions, α = 1.00 can be used for both LRFD and ASD. Additional considerations, such as floor vibrations, may result in other serviceability performance criteria.

In the serviceability evaluation, the maximum normal stress in the member should be limited to the first-yield stress (Bremault et al., 2008; Driver and Kennedy, 1989). Alternatively, for I-shaped members in the inelastic range, the torsional rotation, θ_{2i}, can be estimated with Equation 7-42a (Pi and Trahair, 1994). For closed shapes, a strength evaluation according to Equation 7-40 or 7-41 will ensure nominally elastic behavior and the elastic deformations can be used to evaluate serviceability limits.

$$\theta_{2i} = \frac{\theta_2}{1 - \dfrac{\alpha_t}{2}} \tag{7-42a}$$

where

α_t = ratio of required torsional moment to plastic torsional strength

The value of α_t can be estimated with Equations 7-42b and 7-42c for the isolated flange method and the elastic method, respectively:

$$\alpha_t = \frac{M_{rw}}{M_{cw}} \tag{7-42b}$$

$$\alpha_t = \frac{2}{3}\frac{\sigma_{rw}}{\sigma_{cw}} \tag{7-42c}$$