Table 5-3. Example 5.1 Peak Accelerations				
1st Harmonic		2nd Harmonic		Combined Peak
$1f_{step}$, Hz	a_1/g, %g	$2f_{step}$, Hz	a_2/g, %g	a_p, %g
1.50	0.13	3.00	0.06	0.16
1.60	0.15	3.20	0.08	0.18
1.70	0.17	3.40	0.09	0.21
1.80	0.19	3.60	0.11	0.24
1.90	0.21	3.80	0.13	0.28
2.00	0.24	4.00	0.16	0.32
2.10	0.27	4.20	0.19	0.37
2.20	0.30	4.40	0.23	0.42
2.30	0.33	4.60	0.29	0.49
2.40	0.37	4.80	0.36	0.58
2.50	0.40	5.00	0.46	0.69
2.60	0.44	5.20	0.61	0.85
2.70	0.49	5.40	0.85	1.08

Evaluation

From Table 5-1, a tolerance acceleration limit of 2%g is selected; i.e., $a_o/g = 0.02$. The maximum predicted peak acceleration occurs at a step frequency of 2.70 Hz and is 1.08%g, which is less than this selected tolerance acceleration limit. Therefore, the framing is satisfactory for dining and dancing for the size of the dance floor shown in Figure 5-1.

Example 5.2—Second Floor of General Purpose Building Used for Aerobics

Given:

Aerobics is to be considered in a bay of an occupied second floor of a six-story office building. The structural plan shown in Figure 5-3 satisfies all strength requirements. The floor construction consists of a concrete slab on hot-rolled beams, supported on hot-rolled girders and steel columns. The floor slab is 5½ in. total depth, normal weight concrete ($f'_c = 4.0$ ksi) on 2-in.-deep deck. The weight of the slab and deck is 56.4 psf. There are ceiling and ductwork below with an estimated weight of 4 psf. From Table 5-2, the weight of the aerobic participants is estimated to be 4.2 psf. The effective composite moments of inertia of the beams and girder are 1,920 in.^{4} and 4,740 in.^{4}, respectively. (See Example 4.1 for calculation procedures.) The spandrel girder is supported by the exterior cladding for vibration analysis purposes.

Solution:

The uniform load supported by a beam is

$$\begin{aligned} w_b &= (4.00 \text{ psf} + 56.4 \text{ psf} + 4.20 \text{ psf})(7.50 \text{ ft}) + 35.0 \text{ plf} \\ &= 520 \text{ plf} \end{aligned}$$

The equivalent uniform load supported by the girder assuming 11 psf live load on the adjacent bay is

$$\begin{aligned} w_g &= \left(\frac{520 \text{ plf}}{7.50 \text{ ft}}\right)\left(\frac{36.0 \text{ ft}}{2}\right) + (4.00 \text{ psf} + 56.4 \text{ psf} + 11.0 \text{ psf})\left(\frac{15.0 \text{ ft}}{2}\right) + 55.0 \text{ plf} \\ &= 1{,}840 \text{ plf} \end{aligned}$$

Natural Frequency

The natural frequency of the system is estimated by use of Equation 3-5. The required deflections due to the weight supported by each element (beams, girders and columns) are determined in the following.

The deflection of the beams due to the supported weight is

$$\begin{aligned}\Delta_b &= \frac{5w_bL_b^4}{384E_sI_b}\\ &= \frac{5(520\text{ plf})(36.0\text{ ft})^4(1{,}728\text{ in.}^3/\text{ft}^3)}{384(29\times10^6\text{ psi})(1{,}920\text{ in.}^4)}\\ &= 0.353\text{ in.}\end{aligned}$$

The deflection of the girders due to the supported weight is

$$\begin{aligned}\Delta_g &= \frac{5w_gL_g^4}{384E_sI_g}\\ &= \frac{5(1{,}840\text{ plf})(30.0\text{ ft})^4(1{,}728\text{ in.}^3/\text{ft}^3)}{384(29\times10^6\text{ psi})(4{,}740\text{ in.}^4)}\\ &= 0.244\text{ in.}\end{aligned}$$

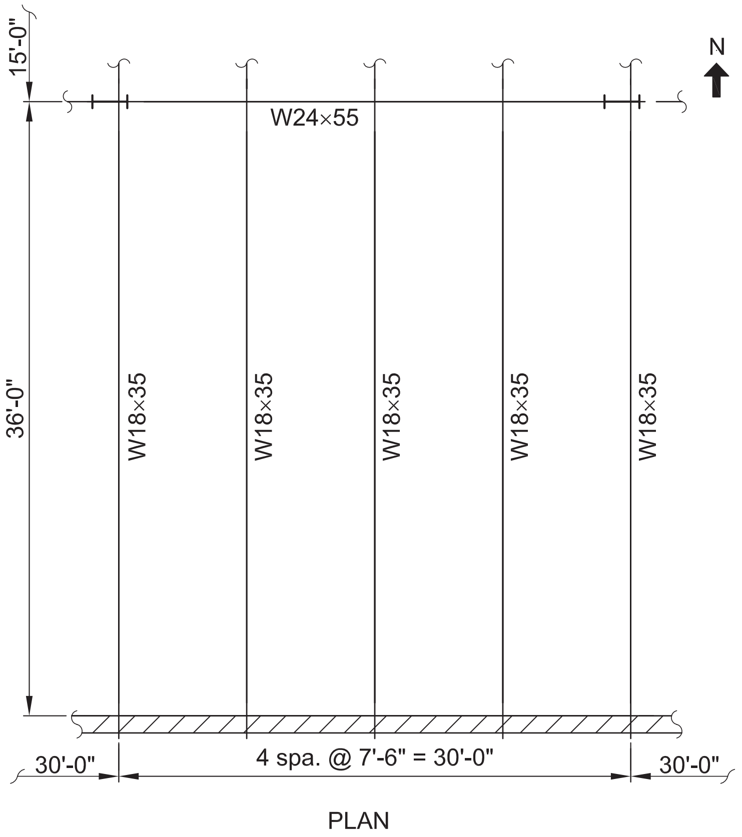

Fig 5-3. Aerobics floor structural layout for Example 5.2.

Table 5-4. Example 5.2 Peak Accelerations

1st Harmonic		2nd Harmonic		3rd Harmonic		Peak
$1f_{step}$, Hz	a_1/g, %g	$2f_{step}$, Hz	a_2/g, %g	$3f_{step}$, Hz	a_3/g, %g	a_p, %g
2.0	2.96	4.0	17.4	6.0	1.70	18.5
2.1	3.35	4.2	26.8	6.3	1.53	27.9
2.2	3.79	4.4	38.1	6.6	1.41	39.1
2.23	**3.93**	**4.44**	**39.0**	**6.69**	**1.38**	**40.1**
2.3	4.26	4.6	35.0	6.9	1.32	36.2
2.4	4.80	4.8	25.9	7.2	1.25	27.4
2.5	5.39	5.0	19.9	7.5	1.19	21.9
2.6	6.05	5.2	16.2	7.8	1.15	18.8
2.7	6.80	5.4	13.9	8.1	1.11	17.1
2.75	7.21	5.5	13.0	8.25	1.09	16.6

The axial shortening of the columns is calculated from the axial stress due to the weight supported. Assuming an axial stress, f_a, of 6 ksi and a first floor column length of 16 ft:

$$\begin{aligned}\Delta_c &= \frac{f_a L_c}{E} \\ &= \frac{(6.00\text{ ksi})(16.0\text{ ft})(12\text{ in./ft})}{29{,}000\text{ ksi}} \\ &= 0.0397\text{ in.}\end{aligned}$$

The natural frequency from Equation 3-5 is

$$f_n = 0.18\sqrt{\frac{g}{\Delta_b + \Delta_g + \Delta_c}} \tag{3-5}$$

$$\begin{aligned} &= 0.18\sqrt{\frac{386\text{ in./s}^2}{0.353\text{ in.} + 0.244\text{ in.} + 0.0397\text{ in.}}} \\ &= 4.43\text{ Hz}\end{aligned}$$

Predicted Peak Acceleration

The peak accelerations due to the 1st, 2nd and 3rd harmonics of the dynamic force, as required in Table 5-2, are determined from Equation 5-2 with w_t = (4.00 psf + 56.4 psf + 4.20 psf) + 5.40 psf for steel framing = 70.0 psf, w_p = 4.2 psf, β = 0.060, i = 1, 2, 3 and with α_1 = 1.50, α_2 = 0.60 and α_3 = 0.10 from Table 5-2. The predicted peak accelerations for step frequencies from 2.0 to 2.75 Hz using Equations 5-1 and 5-2 are shown in Table 5-4 and plotted in Figure 5-4. The maximum predicted acceleration, 40.1%g, occurs when the step frequency is 2.23 Hz, for which the second harmonic frequency approximately equals the natural frequency, resulting in resonance.

Evaluation

The maximum peak acceleration of 40.1%g, which far exceeds the recommended tolerance limit of 4 to 7%g, indicates that the vibrations will be unacceptable, not only for the aerobics floor, but also for adjacent areas on the second floor. Furthermore, other areas of the building supported by the aerobics floor columns will probably be subjected to vertical accelerations that are unacceptable for most occupancies.

The floor framing shown in Figure 5-3 should not be used for aerobic activities. For an acceptable structural system, the natural frequency of the structural system needs to be substantially increased. Significant increases in the stiffnesses of both the beams and the girders are required. A lightweight concrete masonry (CMU) wall over the girder may be a cost effective means for stiffening the girder.

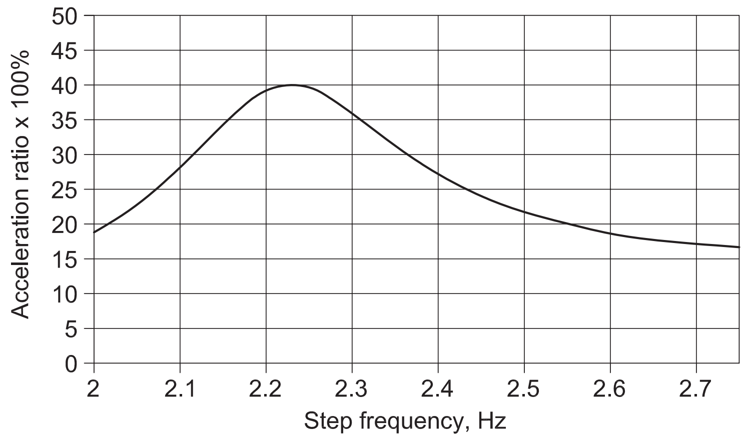

Fig. 5-4. Peak acceleration versus step frequency results for Example 5.2.

Chapter 6
Design for Sensitive Equipment and Sensitive Occupancies

This chapter provides guidance for evaluation of vibrations of floors supporting sensitive equipment, such as precision imaging, measurement, manufacturing instruments, and floors supporting sensitive occupancies such as hospital patient rooms and operating rooms. The vibration tolerance limits for sensitive equipment are usually much more stringent than the limits for human comfort. In relation to human comfort, often only vibrations at relatively low frequencies—up to about 9 Hz—need to be considered, but for sensitive equipment and occupancies, vibrations at higher frequencies also need to be considered.

6.1 EVALUATION OF VIBRATIONS IN AREAS WITH SENSITIVE EQUIPMENT

6.1.1 Tolerance Limits

The tolerance limits relating to human comfort typically are stated in terms of sinusoidal acceleration or velocity at a single frequency. In contrast, the suppliers of sensitive equipment often provide specific tolerance limits in terms of (1) peak (zero-to-peak) or peak-to-peak velocity or acceleration, (2) narrowband spectral velocity or acceleration, or (3) one-third octave spectral velocity or acceleration. Floor evaluations and designs should be based on specific limits for the equipment items of concern when possible. However, if the equipment items or their tolerances are not known, it has become typical practice to rely on generic tolerance limits for specifying the required vibration performance of floors.

Section 6.1.4 provides an evaluation procedure against generic velocity limits and Section 6.1.5 against specific tolerance limits.

6.1.2 Modal Parameters and Mode Shape Scaling

The response prediction methods in this chapter are based on the fundamental mode of the floor. Thus, for a given floor structure, one needs estimates of the natural frequency, modal mass or effective weight, mode shape, and damping of that mode.

The fundamental frequency used in the present chapter is somewhat different from that given in Chapter 3; here it is taken as the minimum of the beam mode and girder mode fundamental frequencies (Liu, 2015):

$$f_n = \min(f_b, f_g) \tag{6-1}$$

where the beam and girder natural frequencies, f_b and f_g, are computed using Equation 3-1 or 3-3.

The effective weight, W, may be obtained from Equation 4-5, or rational analysis and damping ratios may be estimated by use of Table 4-2.

The unity-normalized mode shape, which has the value of 1.0 at midbay, is estimated using the following, which describe single-curvature, two-way bending within the bay.

$$\phi = \sin\left(\frac{\pi x}{L_b}\right)\sin\left[\frac{\pi\left(y+L_g\right)}{3L_g}\right] \text{ if } f_b \le f_g \tag{6-2a}$$

$$\phi = \sin\left[\frac{\pi\left(x+L_b\right)}{3L_b}\right]\sin\left(\frac{\pi y}{L_g}\right) \text{ if } f_b > f_g \tag{6-2b}$$

where x, y, L_b and L_g are defined as shown in Figure 6-1. Note that the x-direction is always parallel to the beam spans. Alternatively, the mode shapes may be determined by finite element methods or other computational means.

When the equipment location and/or the walker location are not at midbay, the vibration response at the equipment location may be found by multiplying the midbay response by the mode shape value at the location of interest, ϕ_E, and the mode shape value at the walker location, ϕ_W, as determined from Equation 6-2a or 6-2b or from finite element analysis.

6.1.3 Conceptual Models of Floor Vibrations Due to Footfalls

Floor vibrations due to walking result from a series of footfall impulses; typical oscillatory motion is illustrated in Figures 1-4(a) and 1-8(a).

Severe vibrations can build up in a floor if it is subjected to a force at its natural frequency causing a resonant build-up such as the one shown in Figure 1-8(a). Because a walking person exerts significant dynamic forces only in the first four harmonics, as evident from the dynamic coefficients in Table 1-1, only low-frequency floors—floors with natural frequencies below the fourth harmonic maximum frequency—can experience such resonant build-ups due to walking.

In high-frequency floors—floors with all natural frequencies above the fourth harmonic maximum frequency— several cycles of vibration at the floor's natural frequency occur between successive impacts. A typical high-frequency floor

Walking Speed	f_{step}, Hz	f_{4max}, Hz	Intermediate Zone Boundaries		γ
			f_L, Hz	f_U, Hz	
Very slow	1.25	—	—	—	—
Slow	1.60	6.8	6	8	0.10
Moderate	1.85	8.0	7	9	0.09
Fast	2.10	8.8	8	10	0.08

Table 6-1. Walking Parameters

f_L = intermediate zone lower boundary, Hz
f_U = intermediate zone upper boundary, Hz
f_{4max} = fourth harmonic maximum frequency, Hz

response to walking is shown in Figure 1-4(a). In this case, it is appropriate to consider each footstep as exerting an impulse, resulting in a vibration that rapidly reaches a peak and then decays until arrival of the next footstep.

Floors with natural frequencies in an intermediate zone around the fourth harmonic maximum frequency probably exhibit behavior between that of low- and high-frequency floors.

Table 6-1 includes the four walking speeds used in this chapter. Very slow walking applies to areas with one or two walkers and limited walking paths; examples are laboratories with fewer than three workers and medical imaging rooms. Slow walking applies to areas with three or four potential walkers and limited walking paths. Moderate walking applies to busy areas with fairly clear walking paths. Fast walking applies to areas with clear walking paths, such as corridors.

Table 6-1 includes the average step frequency, f_{step}, the fourth harmonic maximum frequency, f_{4max}, and the intermediate zone lower and upper boundaries for each speed. A dynamic load parameter, γ, based on the Willford et al. (2007) second through fourth dynamic coefficients in Table 1-1, is also included. Note that the response to very slow walking is computed using the impulse response equations in the following sections, thus f_{4max}, f_L, f_U and γ are not required.

6.1.4 Evaluation Against Generic Velocity Limits

The generic tolerance limits are given in terms of root-mean-square (RMS) spectral velocities in one-third octave bands of frequency. These limits, typically labeled as vibration criteria (VC) curves, are shown in Figure 6-2; Table 6-2 presents a list of equipment and activities to which the generic

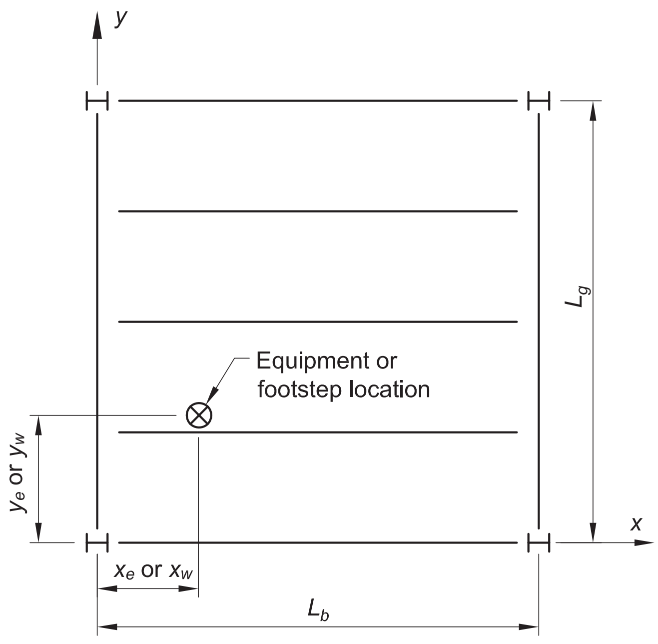

Fig. 6-1. Bay showing equipment and footstep locations.

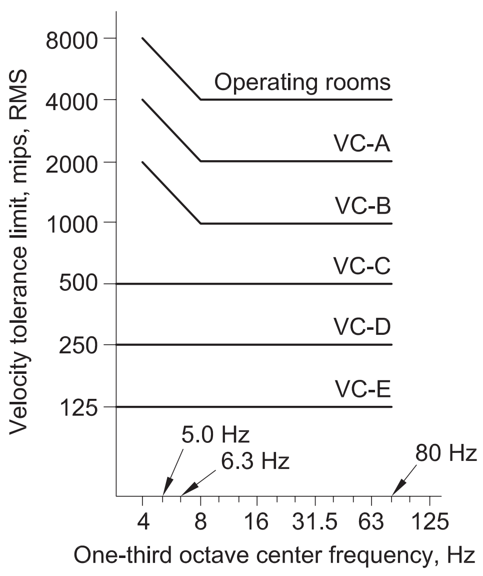

Fig. 6-2. Generic vibration criteria (VC) curves.

Table 6-2. Generic Vibration Criteria Tolerance Limits		
Designation	Tolerance Limit[1], mips	Applicability
—	32,000	Ordinary workshops[2]
—	16,000	Offices[2]
—	8,000	Computer equipment Residences[2,3]
—	6,000	Hospital patient rooms[4]
—	4,000	Surgery facilities, laboratory robots Bench microscopes up to 100×, operating rooms[5]
VC-A	2,000	Microbalances, optical comparators, mass spectrometers Industrial metrology laboratories, spectrophotometers Bench microscopes up to 400×
VC-B	1,000	Microsurgery, microtomes and cryotomes for 5 to10 μm slices Tissue and cell cultures, optical equipment on isolation tables Bench microscopes at greater than 400×, atomic force microscopes
VC-C	500	High-precision balances, spectrophotometers, magnetic resonance imagers Microtomes and cryotomes for <5 μm slices, chemotaxis Electron microscopes at up to 30,000×
VC-D	250	Cell implant equipment, micromanipulation Confocal microscopes, high-resolution mass spectrometers Electron microscopes (SEMs, TEMs) at greater than 30,000×
VC-E	125	Unisolated optical research systems, extraordinarily sensitive systems

[1] As measured in one-third octave bands over the frequency range 8 to 80 Hz (VC-A and VC-B) or 1 to 80 Hz (VC-C through VC-E); see Figure 6-2.
[2] Provided for reference only. Evaluate using Chapter 4 or Chapter 7.
[3] Corresponds to approximate average threshold of perception (ASA, 1983).
[4] When required by FGI (2014). Evaluate using Section 6.2.
[5] Corresponds to approximate threshold of perception of most sensitive humans (ASA, 1983). Evaluate using Section 6.2.

limits apply (Ungar et al., 2006, adapted from Amick et al., 2005). The velocity tolerance unit is micro-in./s (mips). The curves are valid below 80 Hz, but the prediction methods in this chapter have only been verified to approximately 15 Hz.

In terms of these velocities, the root-mean-square (RMS) floor vibrations at midbay due to walking at midbay may be estimated using Equation 6-3a for very slow walking and Equation 6-3b for slow, moderate or fast walking. Equation 6-3a predicts the impulse response. The first expression in Equation 6-3b predicts the resonant response of low-frequency floors, and the second expression predicts the impulse response of high-frequency floors. In the intermediate zone, the response is predicted using linear interpolation between the resonant response at f_L and impulse response at f_U.

$$V_{1/3} = \frac{250\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-3a}$$

$$V_{1/3} = \begin{vmatrix} \dfrac{175\times10^6}{\beta W\sqrt{f_n}}e^{-\gamma f_n} \text{ if } f_n \le f_L \\ \dfrac{250\times10^6}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{1.8}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right)\text{if } f_n \ge f_U \end{vmatrix} \tag{6-3b}$$

where γ, f_L, f_U and f_{step} are from Table 6-1 and W is the effective weight in lb.

The resonant response expression is based on Liu (2015) and pertains to a six-footstep resonant build-up. The impulse response expression is based on Liu and Davis (2015) and the effective impulse approach from Chapter 1; it considers a four-second series of footsteps. The expressions are for a 168-lb walker and a total walking event duration of 8 seconds and include an empirical adjustment based on measured data selected so that the calculated results are exceeded by only 10% of the measured data.

Figure 6-3 presents design aid plots of $V_{1/3}W$ versus f_n obtained from calculations based on Equation 6-3 for two values of damping and the four walking speeds of Table 6-1.

6.1.5 Evaluation Against Specific Tolerance Limits

As mentioned in Section 6.1.1, limits for sensitive equipment are often expressed in other than generic terms; evaluation against these limits is addressed in this section. The prediction equations presented here are based on the references and assumptions cited after Equation 6-3. All of these equations pertain to midbay vibrations due to walking at midbay; mode shape scaling can be used to obtain estimates for other walking and vibration observation locations.

The design aid plots in this section are for the four walking speeds of Table 6-1 and, where applicable, for damping ratios of 0.03 and 0.05.

Waveform Peak Velocity or Acceleration Specific Limit

Figure 6-4 shows a typical acceleration waveform associated with walking, identifies the record's peak value, a_p, and illustrates a peak acceleration tolerance limit, $a_{p,Lim}$.

The peak velocity, v_p, (sometimes called the zero-to-peak velocity) in mips may be estimated from Equation 6-4, and the peak acceleration, a_p, may be estimated from Equation 6-5. Equations 6-4a and 6-5a apply for very slow walking, and Equations 6-4b and 6-5b apply for slow, moderate and fast walking. When the natural frequency is lower than the fourth harmonic maximum frequency, f_{4max}, from Table 6-1, the maximum of the resonant response (the first expression) and the impulse response (the second expression) is used in Equations 6-4b and 6-5b. When the natural frequency is f_{4max} or higher, the impulse response is used. The peak-to-peak values may be taken as twice the peak zero-to-peak values. Figures 6-5 and 6-6 are design aids for the evaluation of Equations 6-4 and 6-5.

$$v_p = \frac{19\times10^9}{W}\frac{f_{step}^{1.43}}{f_n^{1.3}} \tag{6-4a}$$

$$v_p = \max\left|\begin{array}{l} \dfrac{1.3\times10^9}{\beta W f_n}e^{-\gamma f_n} \text{ if } f_n \le f_{4max} \\ \dfrac{19\times10^9}{W}\dfrac{f_{step}^{1.43}}{f_n^{1.3}} \end{array}\right. \tag{6-4b}$$

$$\frac{a_p}{g} = \frac{310}{W}\frac{f_{step}^{1.43}}{f_n^{0.3}} \tag{6-5a}$$

$$\frac{a_p}{g} = \max\left|\begin{array}{l} \dfrac{22}{\beta W}e^{-\gamma f_n} \text{ if } f_n \le f_{4max} \\ \dfrac{310}{W}\dfrac{f_{step}^{1.43}}{f_n^{0.3}} \end{array}\right. \tag{6-5b}$$

where γ, f_{4max} and f_{step} are from Table 6-1 and W is the effective weight in lb.

Narrowband Spectral Velocity or Acceleration Specific Limit

Figure 6-7 shows the narrowband acceleration spectrum corresponding to the acceleration waveform in Figure 6-4. It identifies the maximum value, A_{NB}, and includes a curve corresponding to a specific illustrative limit, $A_{NB,Lim}$.

The greatest narrowband spectral velocity, V_{NB}, and the greatest narrowband spectral acceleration, A_{NB}, occur at the natural frequency of the floor. The former may be estimated (in mips) from Equation 6-6 and the latter from Equation 6-7. Equations 6-6a and 6-7a apply for very slow walking. Equations 6-6b and 6-7b apply for slow, moderate

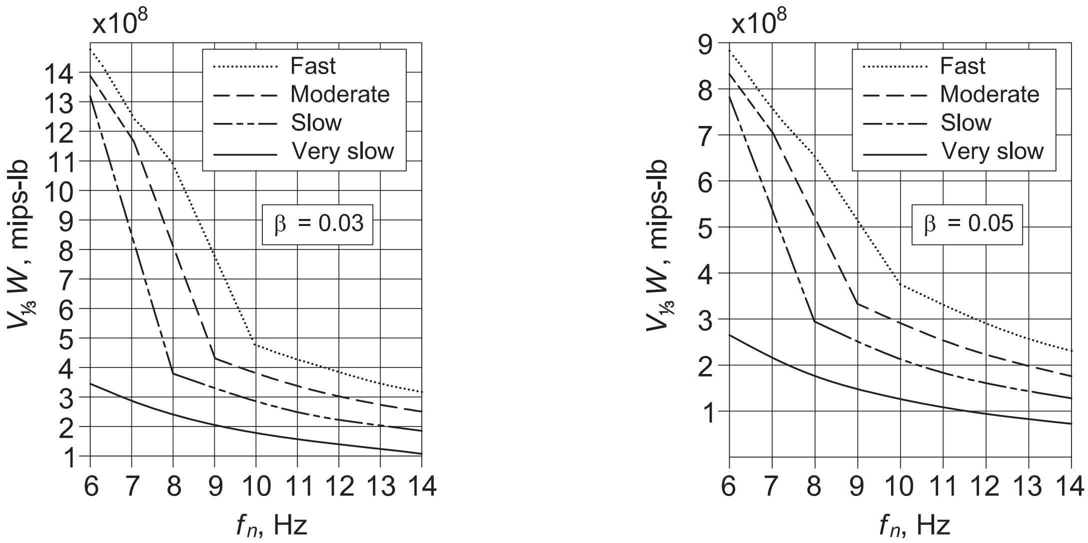

Fig. 6-3. One-third octave spectral velocity design aid (Equation 6-3).

and fast walking; the first expression predicts the resonant response of low-frequency floors, and the second expression predicts the impulse response of high-frequency floors. The response of a floor with fundamental natural frequency between f_L and f_U (Table 6-1) is predicted using linear interpolation between the resonant response at f_L and the impulse response at f_U. Figures 6-8 and 6-9 are design aids for evaluating Equations 6-6 and 6-7.

$$V_{NB} = \frac{490\times10^6}{\beta W}\frac{f_{step}^{2.43}}{f_n^{2.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-6a}$$

$$V_{NB} = \begin{cases} \dfrac{440\times10^6}{\beta W f_n}e^{-\gamma f_n} & \text{if } f_n \le f_L \\ \dfrac{490\times10^6}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{2.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) & \text{if } f_n \ge f_U \end{cases} \tag{6-6b}$$

$$\frac{A_{NB}}{g} = \frac{8.0}{\beta W}\frac{f_{step}^{2.43}}{f_n^{1.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) \tag{6-7a}$$

$$\frac{A_{NB}}{g} = \begin{cases} \dfrac{7.2}{\beta W}e^{-\gamma f_n} & \text{if } f_n \le f_L \\ \dfrac{8.0}{\beta W}\dfrac{f_{step}^{2.43}}{f_n^{1.3}}\left(1-e^{-2\pi\beta f_n/f_{step}}\right) & \text{if } f_n \ge f_U \end{cases} \tag{6-7b}$$

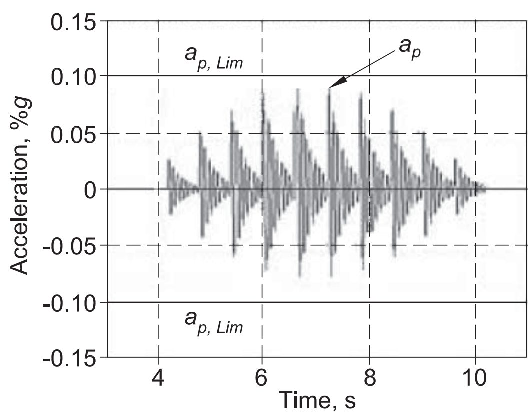

Fig. 6-4. Example acceleration waveform and peak acceleration tolerance limit.

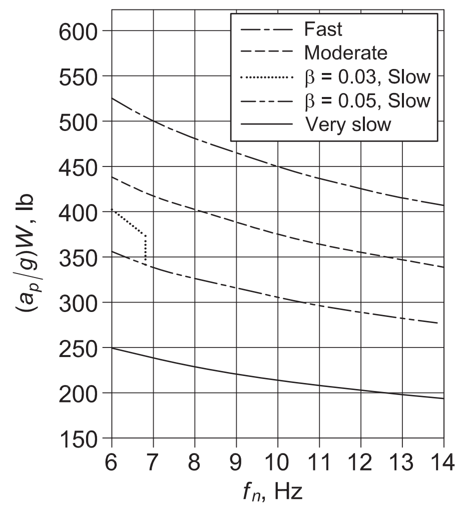

Fig. 6-6. Waveform peak acceleration design aid (Equation 6-5).

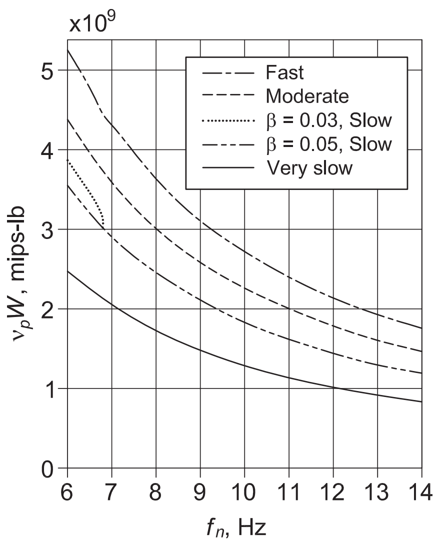

Fig. 6-5. Waveform peak velocity design aid (Equation 6-4).

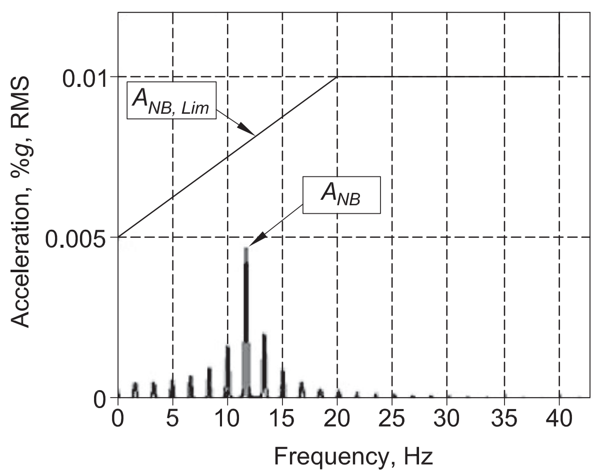

Fig. 6-7. Example narrowband acceleration spectrum and tolerance limit.

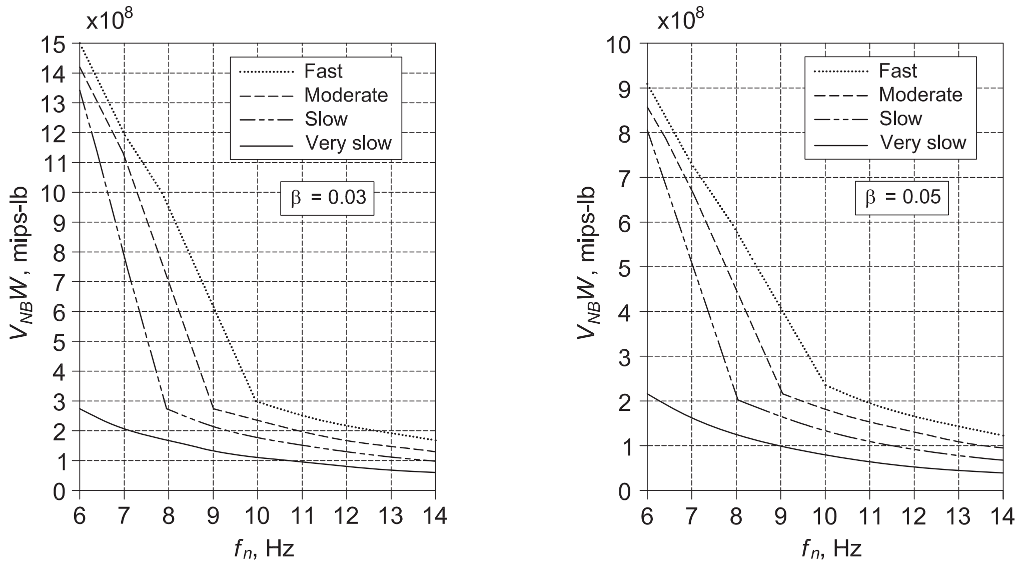

Fig. 6-8. Narrowband spectral velocity design aid (Equation 6-6).

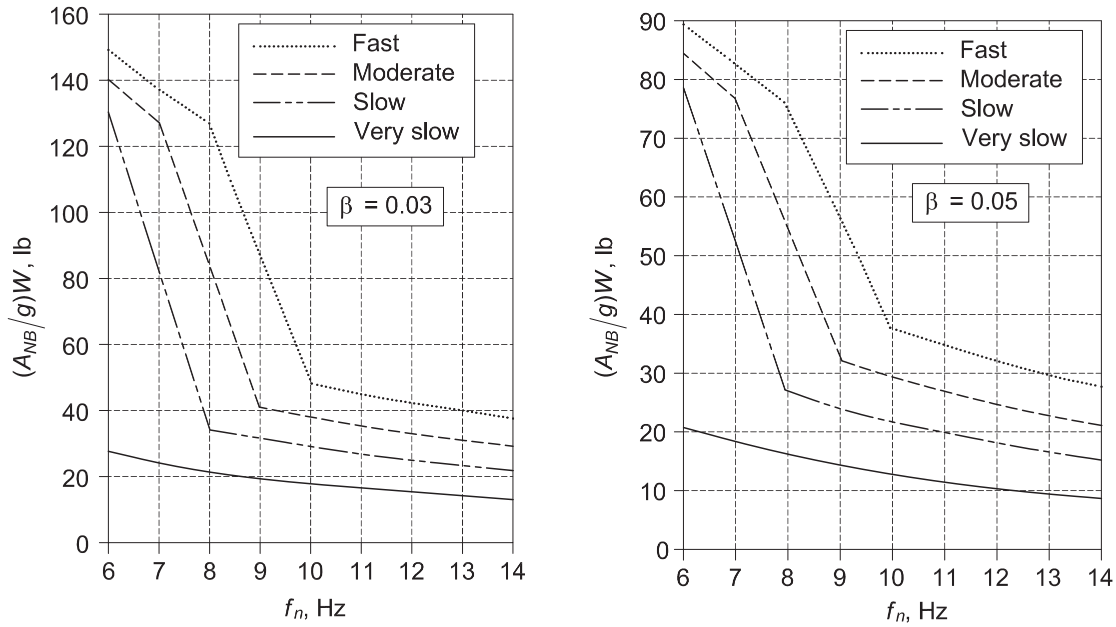

Fig. 6-9. Narrowband spectral acceleration design aid (Equation 6-7).