$$\theta = \tan^{-1}\left(\frac{H}{L}\right)$$
$$= \tan^{-1}\left(\frac{15.0 \text{ ft}}{35.0 \text{ ft}}\right)$$
$$= 23.2°$$

Then, the elongation, assuming the top of the column moves horizontally is:

$$\Delta L = \Delta \cos\theta$$
$$= (4.24 \text{ in.})(\cos 23.2°)$$
$$= 3.90 \text{ in.}$$

Thus, the elastic displacement of the 38-ft-long rod is:

$$\Delta_{el} = \frac{f_{ds} L}{E}$$
$$= \frac{(65.0 \text{ ksi})(38.0 \text{ ft})(12 \text{ in./ft})}{29{,}000 \text{ ksi}}$$
$$= 1.02 \text{ in.}$$

Therefore, the ductility demand in the rod is:

$$\mu = \frac{3.86 \text{ in.}}{1.02 \text{ in.}}$$
$$= 3.78$$

This is the same value as the one obtained for the whole lateral system in Example 5.1. From Table 6-5, the allowable deformation is $9\Delta_T$. This is analogous to a response ratio of $\mu = 9$. With a ductility of 3.78, the demand is less than the capacity and therefore acceptable.

(b) Diagonal Brace of Example 5.2

The diagonal brace used in Example 5.2 is checked and the tension-compression behavior is defined in the following. The brace is a 17-ft-long HSS6×6×¼.

UFC 3-340-02 defines a maximum slenderness for compression elements as:

$$C_c = \sqrt{\frac{2\pi^2 E}{f_{ds}}}$$
$$= \sqrt{\frac{2\pi^2 (29{,}000 \text{ ksi})}{65.0 \text{ ksi}}}$$
$$= 93.8$$

This can be compared to the AISC *Specification* Section E3 limit for inelastic behavior of:

$$\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$$

For blast loading, $F_y = f_{ds} = 65.0$ ksi and $\phi = 1.00$, and the limit is:

$$4.71\sqrt{\frac{29{,}000\text{ ksi}}{65.0\text{ ksi}}} = 99.5$$

Note that this limit is significantly less than the suggested maximum slenderness of 200 given in the User Note in Section E2 of the AISC *Specification*. Assuming $K = 1.0$, the slenderness of this element is:

$$\begin{aligned}\frac{KL}{r} &= \frac{1.0(17.0\text{ ft})(12\text{ in./ft})}{2.34\text{ in.}}\\ &= 87.2 < 93.8\end{aligned}$$

The buckling stress is determined from AISC *Specification* Section E3 as:

$$\begin{aligned}F_e &= \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}\\ &= \frac{\pi^2(29{,}000\text{ ksi})}{(87.2)^2}\\ &= 37.6\text{ ksi}\end{aligned}$$

The critical stress is:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(}\textit{Spec}\text{. Eq. E3-2)}$$

For blast loading, $F_y = f_{ds} = 65.0$ ksi, and the critical stress is:

$$\begin{aligned}F_{cr} &= \left(0.658^{\frac{f_{ds}}{F_e}}\right)f_{ds}\\ &= \left(0.658^{\frac{65.0\text{ ksi}}{37.6\text{ ksi}}}\right)(65.0\text{ ksi})\\ &= 31.5\text{ ksi}\end{aligned}$$

Note that the buckling stress used here corresponds to the AISC *Specification*. This differs from UFC 3-340-02 which uses a buckling stress corresponding to the 1989 *Specification* (AISC, 1989).

Hence, for blast loading, with $\phi = 1.00$, the available compressive strength of the diagonal brace is:

$$\begin{aligned}\phi P_n &= \phi F_{cr} A_g \qquad \text{(6-10)}\\ &= 1.00(31.5\text{ ksi})(5.24\text{ in.}^2)\\ &= 165\text{ kips}\end{aligned}$$

The available tensile yielding strength of the diagonal brace, from AISC *Specification* Section D2, is:

$$\phi P_n = \phi F_y A_g \qquad \text{(6-9)}$$

Table 6-7. Tension-Compression Hinge Parameters			
Loading	**a**	**b**	**c**
T	$11\Delta_T$	$14\Delta_T$	$0.8P_n$
C	$0.5\Delta_c$	$4.1\Delta_c$	$0.3F_{cr}$

For blast loading, $F_y = f_{ds} = 65.0$ ksi and $\phi = 1.00$, and the available tensile strength is:

$$\begin{aligned}\phi P_n &= \phi f_{ds} A_g \\ &= 1.00(65.0 \text{ ksi})(5.24 \text{ in.}^2) \\ &= 341 \text{ kips}\end{aligned}$$

With these tensile and compressive capacities, the tension-compression hinge properties used in Example 5.2 are derived as put forth in Figure 6-7, Table 5-6 and Table 5-7 of FEMA 356 (FEMA, 2000b). These hinge properties are used to model plastic hinges in the braces to allow for a nonlinear plastic analysis of the structure, as shown in Chapter 5.

From Table 5-7 of FEMA 356, the HSS section has the modeling parameters shown in Table 6-7 for tension and compression. Δ_T refers to the axial deformation at expected tensile yielding load, and Δ_c is the axial deformation at expected buckling load. Based on $E = 29{,}000$ ksi, its relationship to stress and strain, i.e., $\varepsilon = \sigma/E$ and the definition of strain, i.e., $\varepsilon = \Delta_L/L$, the values of Δ_T and Δ_c can be determined. Setting these two equations for ε equal to each other and solving for Δ_L results in:

$$\Delta_L = \frac{\sigma L}{E}$$

For tension:

$$\begin{aligned}\Delta_T &= \frac{\sigma L}{E} \\ &= \frac{(65.0 \text{ ksi})(17.0 \text{ ft})(12 \text{ in./ft})}{29{,}000 \text{ ksi}} \\ &= 0.457 \text{ in.}\end{aligned}$$

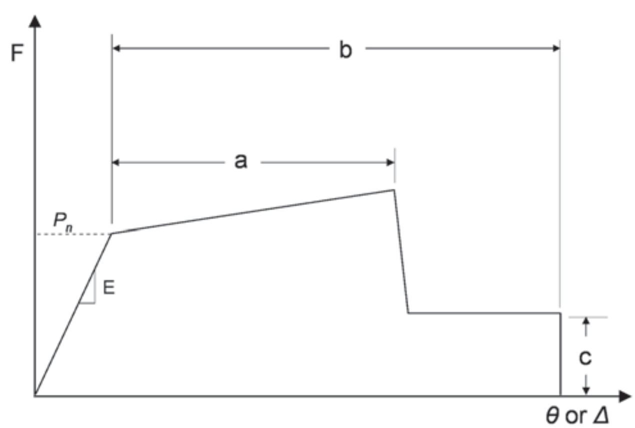


Fig. 6-7. FEMA 356 hinge parameters (FEMA, 2000b).

For compression:

$$
\begin{aligned}
\Delta_c &= \frac{\sigma L}{E} \\
&= \frac{(31.5\text{ ksi})(17.0\text{ ft})(12\text{ in./ft})}{29{,}000\text{ ksi}} \\
&= 0.222
\end{aligned}
$$

Using the allowable strain-hardening slope of 1.5% of the elastic slope for tension produces a maximum stress of approximately 1.15 of the yield stress (Fell et al., 2006). For compression, to avoid computational instabilities, use a 0.1% hardening slope. With this information, the hinge is compiled as Figure 6-8.

Results from Section 5.3, Example 5.2, indicate that only one diagonal at the first floor fails in compression while the others only started yielding. The structure remains stable despite the failure of one brace in compression due to its redundancy.

(c) Columns of Example 5.1

Columns 1 and 2 used in Example 5.1 are checked and designed to remain elastic. The column section used is a W12×53 with an effective length, $KL = 15$ ft.

For compression elements, the maximum slenderness and buckling load are checked. The UFC 3-340-02 maximum slenderness described previously is calculated to be:

$$
\begin{aligned}
C_c &= \sqrt{\frac{2\pi^2 E}{f_{ds}}} \\
&= \sqrt{\frac{2\pi^2 (29{,}000\text{ ksi})}{65.0\text{ ksi}}} \\
&= 93.8
\end{aligned}
$$

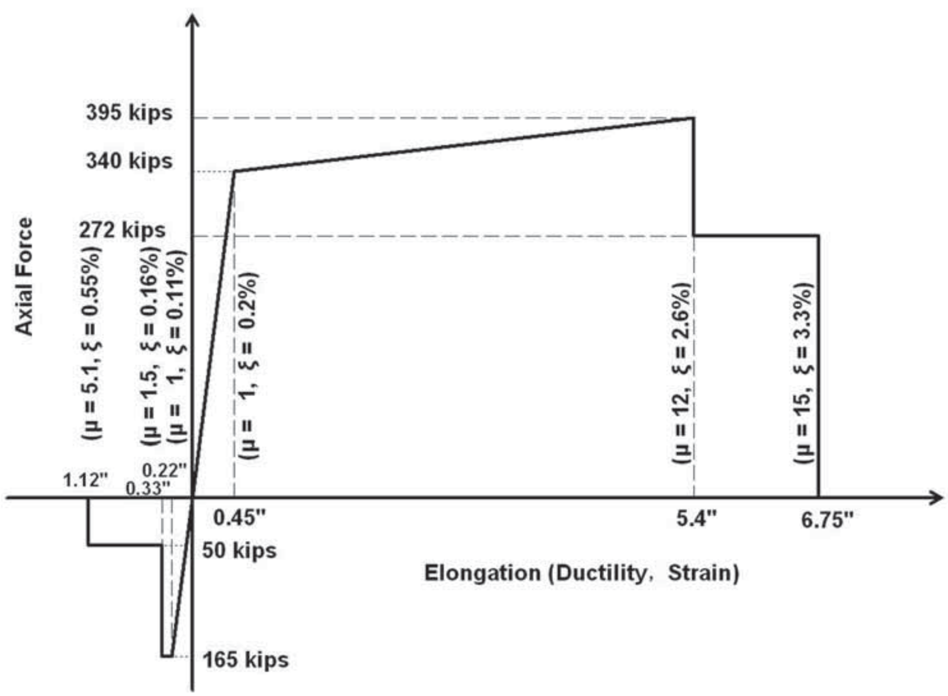


Fig. 6-8. Tension-compression hinge properties.

This can be compared to the AISC *Specification* Section E3 limit for inelastic behavior:

$$\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$$

As determined previously, for blast loading, $F_y = f_{ds} = 65.0$ ksi, and the limit becomes:

$$\begin{aligned} 4.71\sqrt{\frac{E}{f_{ds}}} &= 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{65.0 \text{ ksi}}} \\ &= 99.5 \end{aligned}$$

Assuming $K = 1.0$, the slenderness of the W12×53 is:

$$\begin{aligned} \frac{KL}{r_y} &= \frac{1.0(15.0 \text{ ft})(12 \text{ in./ft})}{2.48 \text{ in.}} \\ &= 72.6 < 93.8 \end{aligned}$$

The buckling stress is defined in Equation 6-14 and for this case is:

$$\begin{aligned} F_e &= \frac{\pi^2 E}{(KL/r)^2} \\ &= \frac{\pi^2(29{,}000 \text{ ksi})}{(72.6 \text{ in.})^2} \\ &= 54.3 \text{ ksi} \end{aligned} \qquad \text{(\textit{Spec}. Eq. E3-4)}$$

The critical stress is:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$

For blast loading, $F_y = f_{ds} = 65.0$ ksi, and the critical stress is:

$$\begin{aligned} F_{cr} &= \left(0.658^{\frac{f_{ds}}{F_e}}\right)f_{ds} \\ &= \left(0.658^{\frac{65.0 \text{ ksi}}{54.3 \text{ ksi}}}\right)65.0 \text{ ksi} \\ &= 39.4 \text{ ksi} \end{aligned}$$

For blast loading, $\phi = 1.00$, and the available compressive strength is:

$$\begin{aligned} \phi P_n &= \phi A_g F_{cr} \\ &= 1.00\left(15.6 \text{ in.}^2\right)(39.4 \text{ ksi}) \\ &= 615 \text{ kips} \end{aligned}$$

The available tensile yielding strength of the W12×53 columns from AISC *Specification* Section D2 is:

$$\phi P_n = \phi F_y A_g$$

For blast loading, $F_y = f_{ds} = 65.0$ ksi and $\phi = 1.00$, and the available tensile strength is:

$$\begin{aligned}\phi P_n &= \phi f_{ds} A_g \\ &= 1.00(65.0 \text{ ksi})(15.6 \text{ in.}^2) \\ &= 1{,}010 \text{ kips}\end{aligned}$$

The maximum axial compressive load in Column 1 due to the blast load is 11.5 kips, as shown in Figure 5-10. Considering a 17.5 ft by 12.5 ft tributary area, a dead load of 50 psf, a live load of 30 psf, and the load combination given by Equation 6-7, the total column load is:

$$\begin{aligned}P_u &= 1.0D + 0.25L + 1.0B \\ &= 1.0(17.5 \text{ ft})(12.5 \text{ ft})(0.050 \text{ ksf}) + 0.25(17.5 \text{ ft})(12.5 \text{ ft})(0.030 \text{ ksf}) + 1.0(11.5 \text{ kips}) \\ &= 24.1 \text{ kips} < \phi P_n\end{aligned}$$

The maximum axial compressive load in Column 2 due to the blast load is 11.5 kips, also shown in Figure 5-10. Considering a 35 ft × 12.5 ft tributary area, the total axial compressive load for this column is:

$$\begin{aligned}P_u &= 1.0D + 0.25L + 1.0B \\ &= 1.0(35 \text{ ft})(12.5 \text{ ft})(0.050 \text{ ksf}) + 0.25(35 \text{ ft})(12.5 \text{ ft})(0.030 \text{ ksf}) + 1.0(11.5 \text{ kips}) \\ &= 36.7 \text{ kips} < \phi P_n\end{aligned}$$

The maximum compression is 24.1 kips in Column 1 and 36.7 kips in Column 2. These are significantly below the buckling load of the column ($\phi P_n = 615$ kips). The maximum stresses in the columns are 1.5 ksi and 2.4 ksi, respectively, therefore, the columns remain elastic.

(d) Second Floor Beam of Example 5.2

The second floor beam in Example 5.2, which is part of the braced frame, is checked next and its behavior is determined. The W12×35 beam has a length, $L = 24$ ft. It is modeled as a simply supported beam with a concentrated load at midspan where the braces meet.

For a starting point, it is initially assumed that the element has a ductility ratio smaller than 3. From Section 6.3.5 it is seen that for ductility ratios smaller than 3, the elastic-plastic flexural strength is given by:

$$M'_p = f_{ds}\left(\frac{S+Z}{2}\right) \tag{6-12}$$

$$\begin{aligned}&= \left(\frac{65.0 \text{ ksi}}{12 \text{ in./ft}}\right)\left(\frac{45.6 \text{ in.}^3 + 51.2 \text{ in.}^3}{2}\right) \\ &= 262 \text{ kip-ft}\end{aligned}$$

where f_{ds} was determined previously as 65.0 ksi for blast loading using the simplified Equation 6-5.

From AISC *Manual* Table 3-23, the elastic deflection for this moment due to the concentrated load at midspan is:

$$\begin{aligned}
\Delta &= \frac{M'_p L^2}{12EI} \\
&= \frac{(262\text{ kip-ft})(12\text{ in./ft})(24\text{ ft})^2\,(12\text{ in./ft})^2}{12(29{,}000\text{ ksi})(285\text{ in.}^4)} \\
&= 2.63\text{ in.}
\end{aligned}$$

This deflection gives an elastic rotation of:

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\Delta}{L/2}\right) \\
&= \tan^{-1}\left[\frac{2.63\text{ in.}}{(24\text{ ft})(12\text{ in./ft})/2}\right] \\
&= 1.05^{\circ}
\end{aligned}$$

Note that the hinge rotation is double this support rotation (see Figure 6-2). These parameters define an elastic-perfectly plastic moment-rotation curve. Since many programs have convergence problems with a perfectly plastic zone, a sloped line should be introduced in this plastic region. This slope shown in Figure 6-9 is based on f_{ds} determined from Figure 6-1 and Equation 6-2 for ductility greater than 10. Therefore, the dynamic design stress is:

$$f_{ds} = f_{dy} + \frac{f_{du} - f_{dy}}{4} \tag{6-2}$$

where

$$\begin{aligned}
f_{dy} &= SIF(DIF)F_y \qquad (6\text{-}1) \\
&= 1.10(1.19)(50\text{ ksi}) \\
&= 65.5\text{ ksi} \\
f_{du} &= DIF(F_u) \qquad (6\text{-}3) \\
&= 1.19(65\text{ ksi}) \\
&= 77.4\text{ ksi}
\end{aligned}$$

and

$$\begin{aligned}
f_{ds} &= 65.5\text{ ksi} + \frac{77.4\text{ ksi} - 65.5\text{ ksi}}{4} \\
&= 68.5\text{ ksi}
\end{aligned}$$

The ultimate bending moment is:

$$\begin{aligned}
M_{ult} &= f_{ds} Z_x \\
&= \frac{(68.5\text{ ksi})(51.2\text{ in.}^3)}{12} \\
&= 292\text{ kip-ft}
\end{aligned}$$

The maximum rotation, determined from Table 6-2 is:

$$\theta_{ult} = \min\left(10°,\ 20\,\theta\right)$$
$$= \min\left(10°,\ 21°\right)$$
$$= 10°$$

Again, note that hinge rotation is double this support rotation. The moment-rotation diagram for the beam hinge is given in Figure 6-9.

Figure 5-20 shows the time-history deflection at midspan for the second-floor beam. The maximum deflection results in the first cycle with a value of 3 in. For this deflection the ductility is smaller than the maximum value defined in Table 6-2 for beams:

$$\mu = \frac{3.00 \text{ in.}}{2.63 \text{ in.}}$$
$$= 1.14 < 20$$

Note, also, that the ductility ratio is smaller than 3, indicating that the initial assumption was valid. If, conversely, the ratio was larger than 3 this process would have to be repeated with the correct M_p.

Example 6.2—Design of Structural Elements Subject to Direct Blast Loading: Façade Girt and Column

Given:

In the previous examples, the element behaviors were defined and included in the structural models used in Examples 5.1 and 5.2. The results from these examples were used in this chapter to check the adequacy of the members to support the loads defined in Chapter 5. These particular elements were not directly exposed to blast loads. In this example, the elements are designed and analyzed based on the blast load applied directly to them. Elements designed in Example 5.1 are simplified into an SDOF model and will be redesigned according to the requirements defined in this chapter. All steel is ASTM A992 material. Specifically, the following elements are designed:

(a) Façade Girt Design: Design an 8-in.-deep section. This element has been designed for wind as a MC8×20 of ASTM A992 material, with a deflection limitation of L/260.

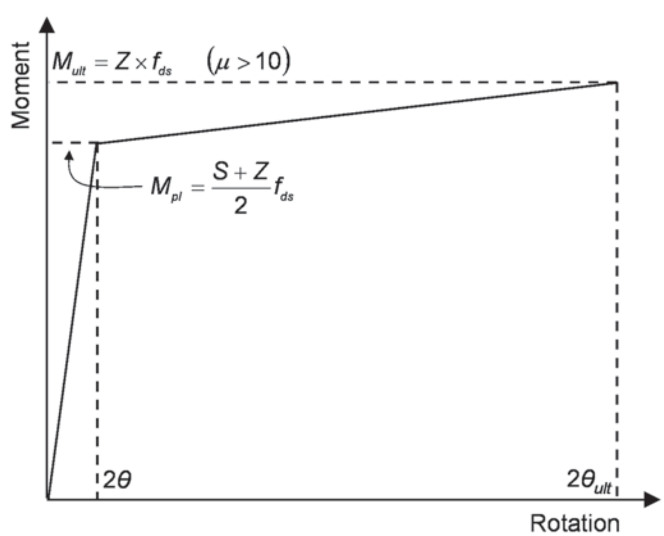


Fig. 6-9. Beam hinge moment-rotation curve.

(b) Façade Column Design: This element has been designed for wind as a W12×53 of ASTM A992 material, with a deflection limitation of $L/240$.

An introduction to the MDOF-to-SDOF simplification method was presented in Chapter 4. Figure 6-10 provides an overview of this method.

Note that the sample procedure equations given in Figure 6-10 are based on a simply supported beam with load and mass uniformly distributed. Equations for other boundary conditions were defined in Chapter 4. The same transformation as shown in Figure 6-10 can be performed by multiplying only the mass by the load-mass factor (Biggs, 1964). Figure 6-11 summarizes the SDOF solution.

$$M_{SDOF} = MK_{LM} = M\frac{K_M}{K_L} \tag{6-14}$$

This can be seen by starting from the simple force equilibrium equation, and applying the transformation shown in Figure 6-10. The simple force equilibrium equations are:

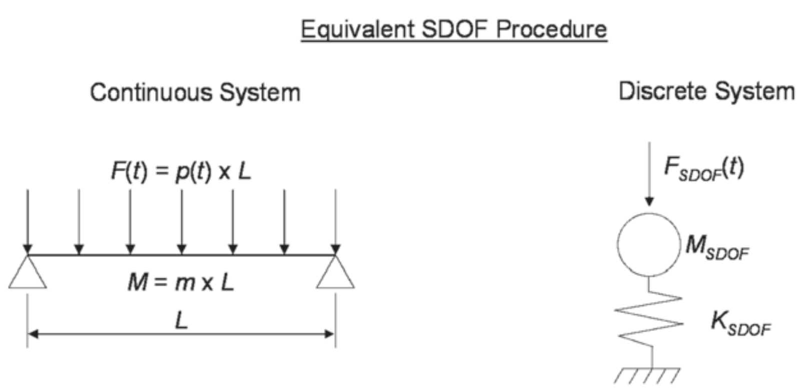
$$F = ku + M\ddot{u} \tag{6-15}$$

$$K_L F = kK_L u + MK_M \ddot{u} \tag{6-16}$$

This can then be simplified to show:

$$F = ku + M\frac{K_M}{K_L}\ddot{u} \tag{6-17}$$

The K_{LM} approach is simpler because it only uses one transformation factor and is standard practice in blast analysis/design. Here, the load factor, K_L, and the mass factor, K_M, are used because they have a more physical interpretation.



Mass = $M = m \times L$ ⟶ Equivalent Mass = $M_{SDOF} = M \times K_M$

Load = $F(t) = p(t) \times L$ ⟶ Equivalent Load = $F_{SDOF}(t) = F(t) \times K_L$

Elastic Stiffness = $K = \dfrac{384 \times E \times I}{5 \times L^3}$ ⟶ Equivalent Stiffness = $K_{SDOF} = K \times K_L$

Plastic Stiffness = $K_{pl} = 0$

Yield Force = $R_{yield} = \dfrac{8 \times M_{pl}}{L}$ ⟶ Equivalent Yield Force = $R_{yield,SDOF} = R_{yield} \times K_L$

Fig. 6-10. MDOF-to-SDOF simplification.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

(a) Façade Girt Design

The girt shown in Figure 6-12 is designed to support the blast load calculated in Chapter 2. The blast deflection criteria given in Table 6-2 shows that the ductility should be less than 20 and the support rotation should be less than 10°. As a preliminary design, the support rotation criterion is used because it does not assume the knowledge of the actual section used. Rigid-perfectly plastic behavior is assumed and the element is assumed sufficiently braced against lateral-torsional buckling.

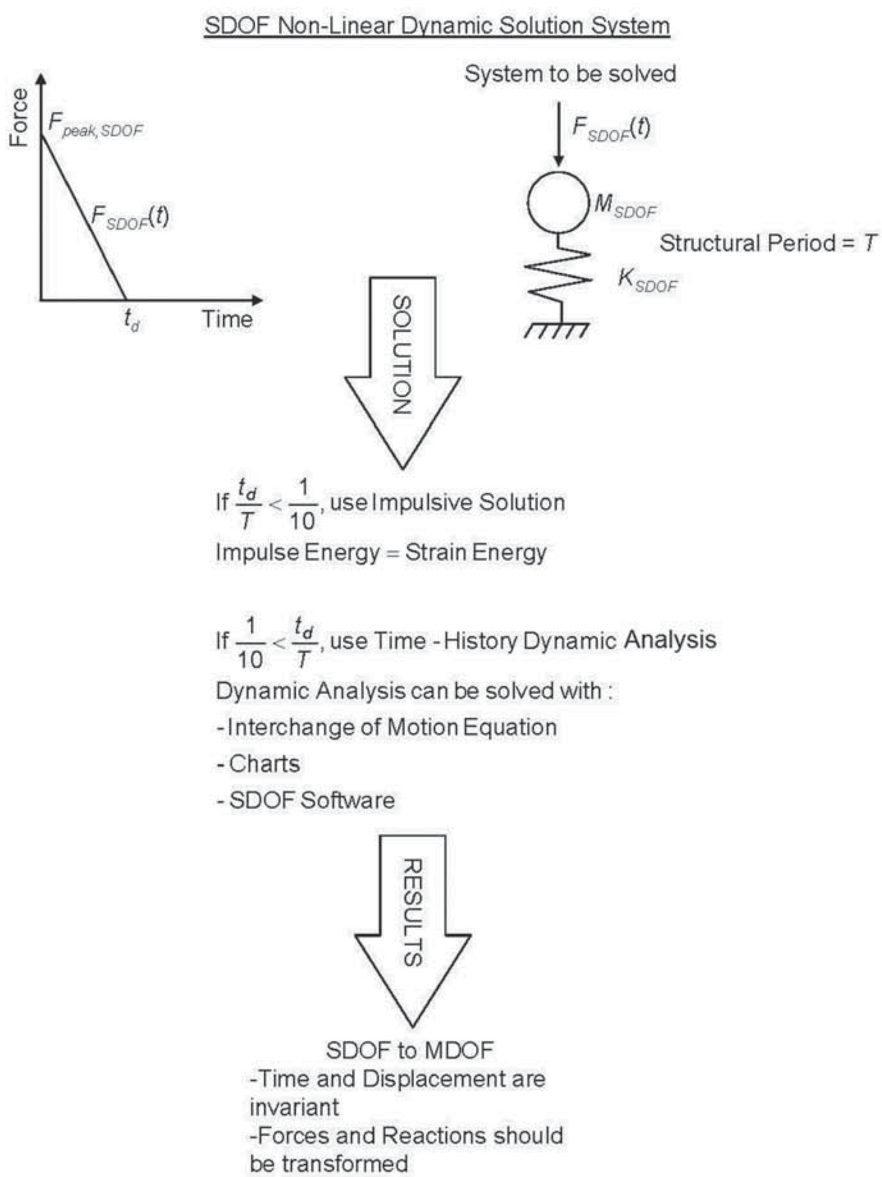


Fig. 6-11. SDOF solution.