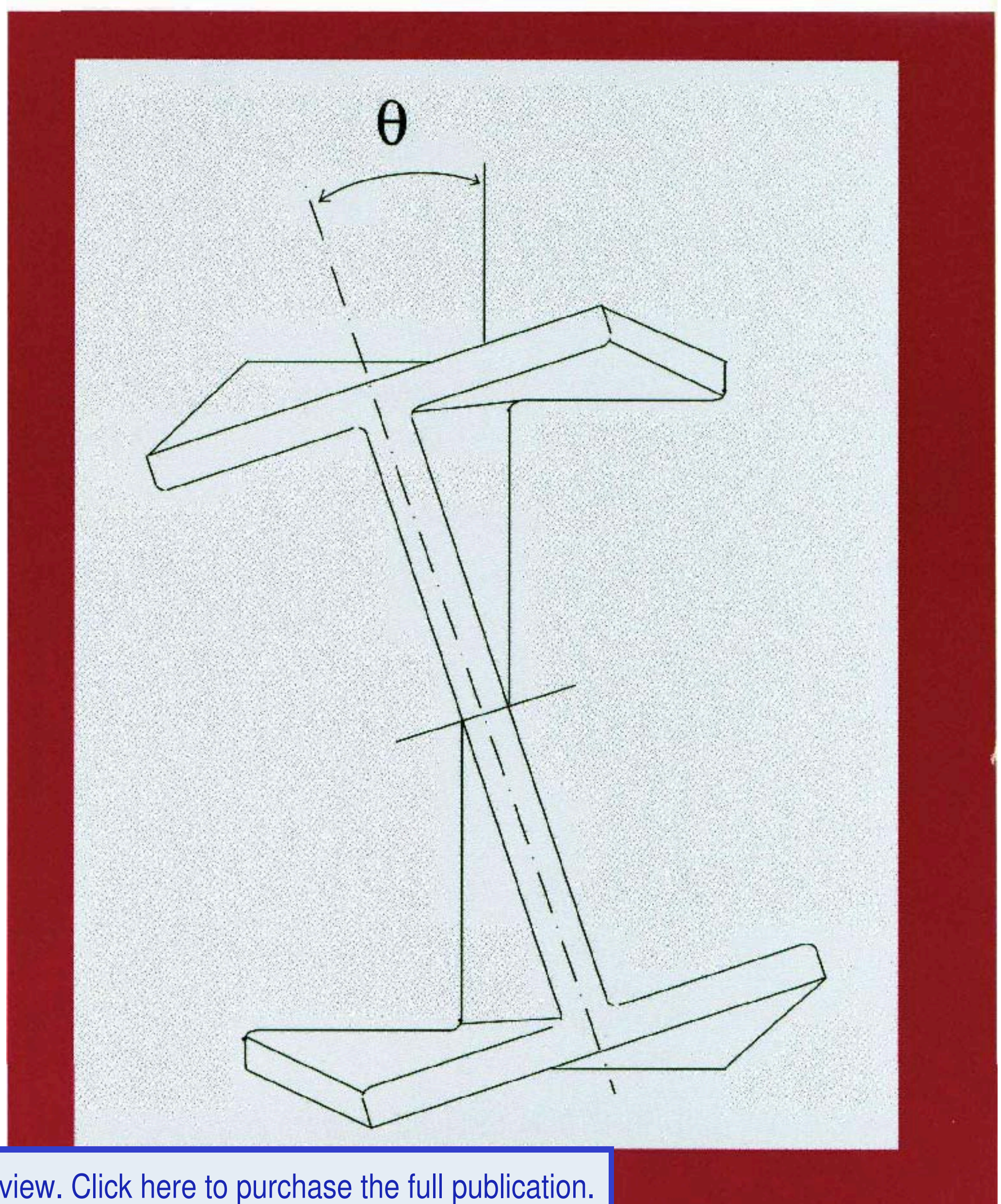
9
Steel Design Guide Series

Torsional Analysis of Structural Steel Members



Steel Design Guide Series 9

Torsional Analysis of Structural Steel Members

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Chapter 1
INTRODUCTION

This design guide is an update to the AISC publication *Torsional Analysis of Steel Members* and advances further the work upon which that publication was based: Bethlehem Steel Company's *Torsion Analysis of Rolled Steel Sections* (Heins and Seaburg, 1963). Coverage of shapes has been expanded and includes W-, M-, S-, and HP-Shapes, channels (C and MC), structural tees (WT, MT, and ST), angles (L), Z-shapes, square, rectangular and round hollow structural sections (HSS), and steel pipe (P). Torsional formulas for these and other non-standard cross sections can also be found in Chapter 9 of Young (1989).

Chapters 2 and 3 provide an overview of the fundamentals and basic theory of torsional loading for structural steel members. Chapter 4 covers the determination of torsional stresses, their combination with other stresses, Specification provisions relating to torsion, and serviceability issues. The design examples in Chapter 5 illustrate the design process as well as the use of the design aids for torsional properties and functions found in Appendices A and B, respectively. Finally, Appendix C provides supporting information that illustrates the background of much of the information in this design guide.

The design examples are generally based upon the provisions of the 1993 AISC LRFD *Specification for Structural Steel Buildings* (referred to herein as the LRFD Specification). Accordingly, forces and moments are indicated with the subscript u to denote factored loads. Nonetheless, the information contained in this guide can be used for design according to the 1989 AISC ASD *Specification for Structural Steel Buildings* (referred to herein as the ASD Specification) if service loads are used in place of factored loads. Where this is not the case, it has been so noted in the text. For single-angle members, the provisions of the AISC *Specification for LRFD of Single-Angle Members* and *Specification for ASD of Single-Angle Members* are appropriate. The design of curved members is beyond the scope of this publication; refer to AISC (1986), Liew et al. (1995), Nakai and Heins (1977), Tung and Fountain (1970), Chapter 8 of Young (1989), Galambos (1988), AASHTO (1993), and Nakai and Yoo (1988).

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Chapter 2
TORSION FUNDAMENTALS

2.1 Shear Center

The shear center is the point through which the applied loads must pass to produce bending without twisting. If a shape has a line of symmetry, the shear center will always lie on that line; for cross-sections with two lines of symmetry, the shear center is at the intersection of those lines (as is the centroid). Thus, as shown in Figure 2.la, the centroid and shear center coincide for doubly symmetric cross-sections such as W-, M-, S-, and HP-shapes, square, rectangular and round hollow structural sections (HSS), and steel pipe (P).

Singly symmetric cross-sections such as channels (C and MC) and tees (WT, MT, and ST) have their shear centers on the axis of symmetry, but not necessarily at the centroid. As illustrated in Figure 2. lb, the shear center for channels is at a distance e_o from the face of the channel; the location of the shear center for channels is tabulated in Appendix A as well as Part 1 of AISC (1994) and may be calculated as shown in Appendix C. The shear center for a tee is at the intersection of the centerlines of the flange and stem. The shear center location for unsymmetric cross-sections such as angles (L) and Z-shapes is illustrated in Figure 2.1c.

2.2 Resistance of a Cross-section to a Torsional Moment

At any point along the length of a member subjected to a torsional moment, the cross-section will rotate through an angle θ as shown in Figure 2.2. For non-circular cross-sections this rotation is accompanied by warping; that is, transverse sections do not remain plane.[1] If this warping is completely unrestrained, the torsional moment resisted by the cross-section is:

$$T_t = GJ\theta' \qquad (2.1)$$

where

- T_t = resisting moment of unrestrained cross-section, kip-in.
- G = shear modulus of elasticity of steel, 11,200 ksi
- J = torsional constant for the cross-section, in.^{4}
- θ' = angle of rotation per unit length, first derivative of 0 with respect to z measured along the length of the member from the left support

When the tendency for a cross-section to warp freely is prevented or restrained, longitudinal bending results. This bending is accompanied by shear stresses in the plane of the cross-section that resist the externally applied torsional moment according to the following relationship:

$$T_w = -EC_w\theta''' \qquad (2.2)$$

where

- T_w = resisting moment due to restrained warping of the cross-section, kip-in,
- E = modulus of elasticity of steel, 29,000 ksi
- C_w = warping constant for the cross-section, in.^{4}
- θ''' = third derivative of 6 with respect to z

The total torsional moment resisted by the cross-section is the sum of $T,$ and T_w. The first of these is always present; the second depends upon the resistance to warping. Denoting the total torsional resisting moment by T, the following expression is obtained:

$$T = GJ\theta' - EC_w\theta''' \qquad (2.3)$$

Rearranging, this may also be written as:

$$\frac{T}{EC_w} = \frac{\theta'}{a^2} - \theta''' \qquad (2.4)$$

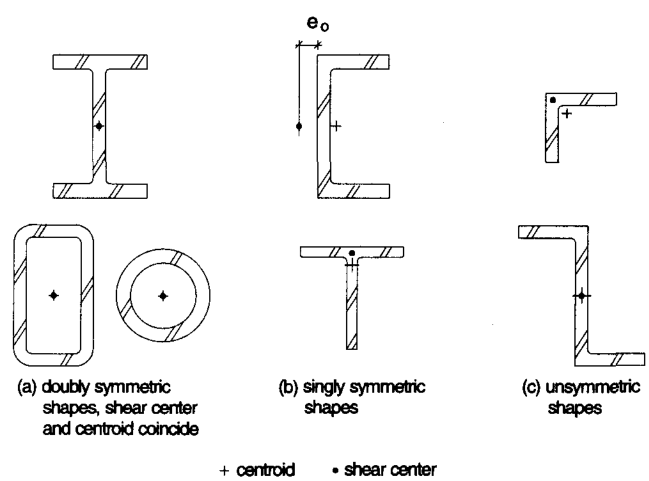

Figure 2.1.

[1] An exception to this occurs in cross-sections composed of plate elements having centerlines that intersect at a common point such as a structural tee. For such cross-sections, $W_{ns} = S_{ws} = C_w = a = 0$.

where

$$a^2 = \frac{EC_w}{GJ} \tag{2.5}$$

2.3 Avoiding and Minimizing Torsion

The commonly used structural shapes offer relatively poor resistance to torsion. Hence, it is best to avoid torsion by detailing the loads and reactions to act through the shear center of the member. However, in some instances, this may not always be possible. AISC (1994) offers several suggestions for eliminating torsion; see pages 2-40 through 2-42. For example, rigid facade elements spanning between floors (the weight of which would otherwise induce torsional loading of the spandrel girder) may be designed to transfer lateral forces into the floor diaphragms and resist the eccentric effect as illustrated in Figure 2.3. Note that many systems may be too flexible for this assumption. Partial facade panels that do not extend from floor diaphragm to floor diaphragm may be designed with diagonal steel "kickers," as shown in Figure 2.4, to provide the lateral forces. In either case, this eliminates torsional loading of the spandrel beam or girder. Also, torsional bracing may be provided at eccentric load points to reduce or eliminate the torsional effect; refer to Salmon and Johnson (1990).

When torsion must be resisted by the member directly, its effect may be reduced through consideration of intermediate torsional support provided by secondary framing. For example, the rotation of the spandrel girder cannot exceed the total end rotation of the beam and connection being supported. Therefore, a reduced torque may be calculated by evaluating the torsional stiffness of the member subjected to torsion relative to the rotational stiffness of the loading system. The bending stiffness of the restraining member depends upon its end conditions; the torsional stiffness k of the member under consideration (illustrated in Figure 2.5) is:

$$k = \frac{T}{\theta} \tag{2.6}$$

where

T = torque
θ = the angle of rotation, measured in radians.

A fully restrained (FR) moment connection between the framing beam and spandrel girder maximizes the torsional restraint. Alternatively, additional intermediate torsional supports may be provided to reduce the span over which the torsion acts and thereby reduce the torsional effect.

As another example, consider the beam supporting a wall and slab illustrated in Figure 2.6; calculations for a similar case may be found in Johnston (1982). Assume that the beam

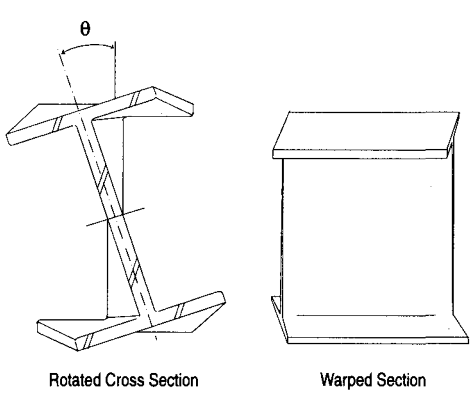

Figure 2.2.

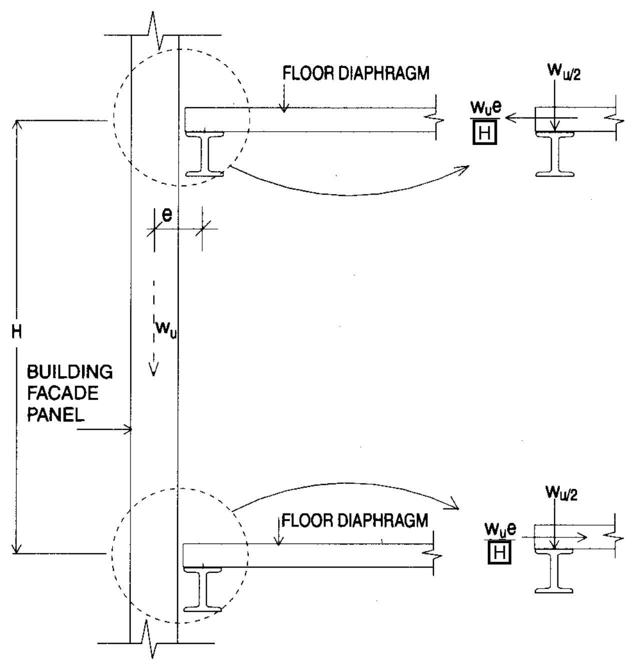

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Figure 2.3.

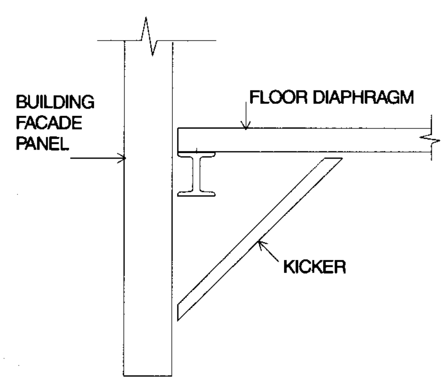


Figure 2.4.

alone resists the torsional moment and the maximum rotation of the beam due to the weight of the wall is 0.01 radians. Without temporary shoring, the top of the wall would deflect laterally by nearly $^3/_4$-in. (72 in. x 0.01 rad.). The additional load due to the slab would significantly increase this lateral deflection. One solution to this problem is to make the beam and wall integral with reinforcing steel welded to the top flange of the beam. In addition to appreciably increasing the torsional rigidity of the system, the wall, because of its bending stiffness, would absorb nearly all of the torsional load. To prevent twist during construction, the steel beam would have to be shored until the floor slab is in place.

2.4 Selection of Shapes for Torsional Loading

In general, the torsional performance of closed cross-sections is superior to that for open cross-sections. Circular closed shapes, such as round HSS and steel pipe, are most efficient for resisting torsional loading. Other closed shapes, such as square and rectangular HSS, also provide considerably better resistance to torsion than open shapes, such as W-shapes and channels. When open shapes must be used, their torsional resistance may be increased by creating a box shape, e.g., by welding one or two side plates between the flanges of a W-shape for a portion of its length.

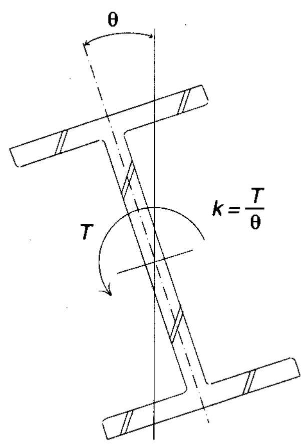


Figure 2.5.

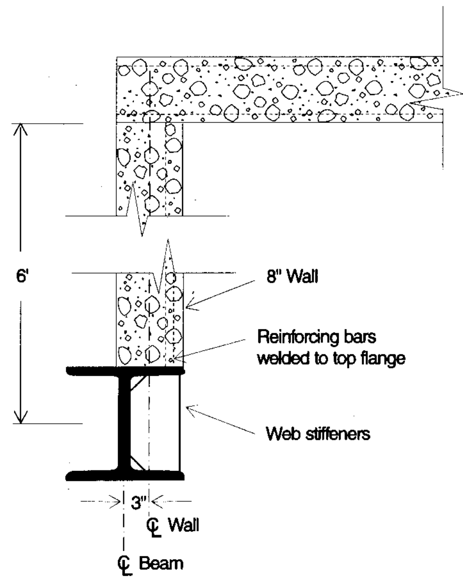


Figure 2.6.

Chapter 3
GENERAL TORSIONAL THEORY

A complete discussion of torsional theory is beyond the scope of this publication. The brief discussion that follows is intended primarily to define the method of analysis used in this book. More detailed coverage of torsional theory and other topics is available in the references given.

3.1 Torsional Response

From Section 2.2, the total torsional resistance provided by a structural shape is the sum of that due to pure torsion and that due to restrained warping. Thus, for a constant torque T along the length of the member:

$$T = GJ\theta' - EC_w\theta''' \tag{3.1}$$

where

G = shear modulus of elasticity of steel, 11,200 ksi
J = torsional constant of cross-section, in.^{4}
E = modulus of elasticity of steel, 29,000 ksi
C_w = warping constant of cross-section, in.^{6}

For a uniformly distributed torque t:

$$t = EC_w\theta'''' - GJ\theta'' \tag{3.2}$$

For a linearly varying torque $t \times (z/l)$:

$$\frac{tz}{l} = EC_w\theta'''' - GJ\theta'' \tag{3.3}$$

where

t = maximum applied torque at right support, kip-in./ft
z = distance from left support, in.
l = span length, in.

In the above equations, θ', θ'', θ''', and θ'''' are the first, second, third, and fourth derivatives of 9 with respect to z and θ is the total angle of rotation about the Z-axis (longitudinal axis of member). For the derivation of these equations, see Appendix C.1.

3.2 Torsional Properties

Torsional properties J, a, C_w, W_{ns}, and S_{ws} are necessary for the solution of the above equations and the equations for torsional stress presented in Chapter 4. Since these values are dependent only upon the geometry of the cross-section, they have been tabulated for common structural shapes in Appendix A as well as Part 1 of AISC (1994). For the derivation of torsional properties for various cross-sections, see Appendix C and Heins (1975). Values for Q_f and Q_w, which are used to compute plane bending shear stresses in the flange and edge of the web, are also included in the tables for all relevant shapes except Z-shapes.

The terms J, a, and C_w are properties of the entire cross-section. The terms W_n and S_w vary at different points on the cross-section as illustrated in Appendix A. The tables give all values of these terms necessary to determine the maximum values of the combined stress.

3.2.1 *Torsional Constant J*

The torsional constant J for solid round and flat bars, square, rectangular and round HSS, and steel pipe is summarized in Table 3.1. For open cross-sections, the following equation may be used (more accurate equations are given for selected shapes in Appendix C.3):

$$J \approx \Sigma\left(\frac{bt^3}{3}\right) \tag{3.4}$$

where

b = length of each cross-sectional element, in.
t = thickness of each cross-sectional element, in.

3.2.2 *Other Torsional Properties for Open Cross-Sections* [2]

For rolled and built-up I-shapes, the following equations may be used (fillets are generally neglected):

$$C_w = \frac{I_y h^2}{4} \tag{3.5}$$

$$a = \frac{h}{2}\sqrt{\frac{EI_y}{GJ}} = \sqrt{\frac{EC_w}{GJ}} \tag{3.6}$$

$$W_{no} = \frac{hb_f}{4} \tag{3.7}$$

$$S_w = \frac{W_{no} b_f t_f}{4} = \frac{hb_f^2 t_f}{16} \tag{3.8}$$

$$Q_f = \frac{ht_f(b_f - t_w)}{4} \tag{3.9}$$

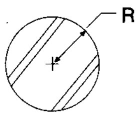
$$Q_w = \frac{hb_f t_f}{2} + \frac{(h - t_f)^2 t_w}{8} \tag{3.10}$$

where

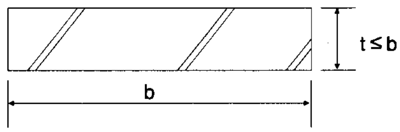
[2] For shapes with sloping-sided flanges, sloping flange elements are simplified into rectangular elements of thickness equal to the average thickness of the flange.

Table 3.1
Torsional Constants J

Solid Cross-Sections



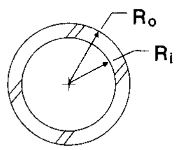
$$J = \frac{\pi R^4}{2}$$



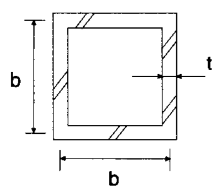
$$J \approx \frac{bt^3}{3};\ \frac{b}{t} \geq 10$$

$$J = \left(\tfrac{1}{3} - 0.2\frac{t}{b}\right)bt^3;\ \frac{b}{t} < 10$$

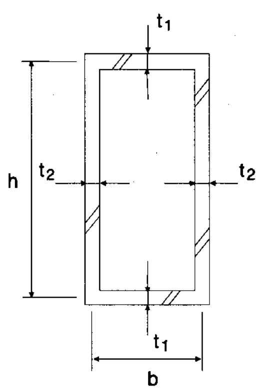
Closed Cross-Sections



$$J = \frac{\pi(R_o^4 - R_i^4)}{2}$$



$$J = tb^3;\ \frac{b}{t} \geq 10$$



$$J = \frac{2t_1 t_2 b^2 h^2}{bt_2 + ht_1};\ \frac{b}{t} \geq 10$$

Note: tabulated values for HSS in Appendix A differ slightly because the effect of corner radii has been considered.

$$h = d - t_f \tag{3.11}$$

For channels, the following equations may be used:

$$a = \sqrt{\frac{EC_w}{GJ}} \tag{3.12}$$

$$W_{no} = \frac{uh}{2} \tag{3.13}$$

$$W_{n2} = \frac{E_o h}{2} \tag{3.14}$$

$$S_{w1} = \frac{u^2 h t_f}{4} \tag{3.15}$$

$$S_{w2} = \frac{hb' t_f (b' - 2E_o)}{4} \tag{3.16}$$

$$S_{w3} = S_{w2} - \frac{E_o h^2 t_w}{8} \tag{3.17}$$

$$C_w = \frac{h^2 b'^2 t_f (b' - 3E_o)}{6} + E_o^2 I_x \tag{3.18}$$

where, as illustrated in Figure 3.1:

$$E_o = \frac{t_f b'^2}{2b' t_f + \frac{ht_w}{3}} = e_o + \frac{t_w}{2} \tag{3.19}$$

$$h = d - t_f \tag{3.20}$$

$$b' = b_f - \frac{t_w}{2} \tag{3.21}$$

$$u = b' - E_o \tag{3.22}$$

For Z-shapes:

$$a = \sqrt{\frac{EC_w}{GJ}} \tag{3.23}$$

$$W_{no} = \frac{uh}{2} \tag{3.24}$$

$$W_{n2} = \frac{u'h}{2} \tag{3.25}$$

$$S_{w1} = \frac{hb'^2 t_f (ht_w + b' t_f)^2}{4(ht_w + 2b' t_f)^2} \tag{3.26}$$

$$S_{w2} = \frac{t_w t_f h^2 b'^2}{4(ht_w + 2b' t_f)} \tag{3.27}$$

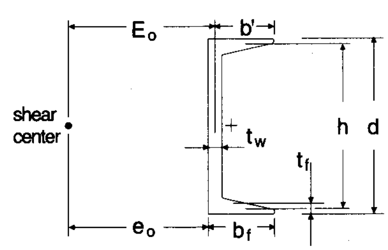


Figure 3.1.