From AISC Manual Table 3-2, for a W40×149:

 $L_p = 8.09 \text{ ft}$   $L_r = 23.5 \text{ ft}$   $L_b = 6.67 \text{ ft (purlin spacing)} < L_p = 8.09 \text{ ft}$   $\frac{M_{px}}{\Omega_L} = 1,490 \text{ kip-ft}$ 

Because  $L_b < L_p$ ,  $\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b} = 1,490$  kip-ft, according to AISC Specification Section F2.

 $\frac{M_n}{\Omega_b} = (1,490 \text{ kip-ft})(12 \text{ in./ft})$ = 17,900 kip-in. > 4,930 kip-in. **o.k**.

East-West Braced Frame, Tension Only Member, T1

The required tensile strength of the 1<sup>1</sup>/<sub>4</sub>-in.-diameter rod is:

 $P_r = 22.1$  kips for Comb9 from Table 4-14

For tension-only rod ( $F_y$  =36 ksi), the allowable tensile yielding strength is:

$$\frac{P_n}{\Omega_t} = \frac{(36 \text{ ksi})(\pi)(1\frac{1}{4} \text{ in.})^2/4}{1.67}$$
  
= 26.5 kips > 22.1 kips **o.k**.

The allowable tensile rupture strength is (assume the effective net tension area is  $0.75A_g$ ; this must be confirmed once connections are completed):

$$\frac{P_n}{\Omega_t} = \frac{(58 \text{ ksi})(0.75)(\pi)(1\frac{1}{4} \text{ in.})^2/4}{2.00}$$
$$= 26.7 \text{ kips} > 22.1 \text{ kips} \text{ o.k.}$$

### Observations

- 1. The FOM provides a conservative design for this problem compared to the DM because (1) it was done using the more conservative 1.6 overall load factor used by ASD; (2) the  $\Delta_{2nd}/\Delta_{1st} \le 1.5$  stiffness limitation applies; and (3) other conservative approximations are invoked in its development (see Chapter 4 and Appendix B).
- 2. There were no seismic requirements for this problem, but the lateral load system in the north-south moment frame direction may not be satisfactory in higher seismic zones because of the magnitude of the second-order effects. This would need to be checked according to ASCE/SEI 7 Section 12.8.7.
- 3. The moment frame design is strongly influenced by the large leaning column load and the fact that there are relatively few moment connections.
- 4. The braced frame design is controlled by strength while the moment frame design is controlled by stiffness (necessary to satisfy the  $B_2 \le 1.5$  requirement). This is common in many buildings of this type.

# Chapter 5 Related Topics

## 5.1 APPLICATION TO SEISMIC DESIGN

### 5.1.1 Determination of Seismic Load Effect, E

The current approach to seismic design in the U.S. is based on provisions defined in ASCE/SEI 7 Chapter 12: Seismic Design Requirements for Building Structures. A design response spectrum is created (ASCE/SEI 7 Figure 11.4-1) from which a seismic base shear, V, is determined (ASCE/ SEI 7 Equation 12.8-1) based on the occupancy importance factor of the building, I, the response modification factor, R, dependent on the type of structural system used (e.g., braced frame, moment frame, and the level of detailing applied to it), and the fundamental building period, T. In the equivalent lateral force procedure (ELF) defined in Section 12.8, the base shear, V, is distributed up the building and applied as a seismic lateral force at each floor,  $F_x$ , according to the mass of the structure at each level and the building period based on ASCE/SEI 7 Equations 12.8-11 and 12.8-12. In the modal response spectrum analysis procedure (MRSA) defined in Section 12.9, an analysis to determine the natural modes of vibration of the structure is made and the internal member forces are determined from the modes of vibration along with a defined response spectrum divided by the quantity R/I. The member forces calculated for each mode are combined using either the square root of the sum of the squares (SRSS) method or the complete quadratic combination (CQC) method. The internal member forces are scaled using the base shear, V, from the ELF according to Section 12.9.4. Refer to the section in the following, "Member Properties Used in Structural Analysis Modeling," for a discussion of member properties to use for these different types of structural analysis.

The designer should understand that the seismic load effect, E, from either the ELF or MRSA involves a reduction in the elastic forces by the R factor to a "first significant yield" level of response for ease in design. It is fully expected that the building will perform inelastically during the design earthquake. This is in contrast to the ordinary design for wind loads, where the level of response is essentially elastic even under ultimate wind loads. This philosophy of design is depicted in Figure 5-1 which shows a monotonic pushover curve for a well proportioned structural steel moment frame building. Its use is based on ductile detailing in the steel frame as required to ensure good energy dissipation and life safety during the design level earthquake. This design philosophy is important for economy and is based on a long history of acceptable performance of steel buildings during earthquakes.

Typical code designed structures are expected to undergo large inelastic reversed cyclic deformations during strong earthquake motion. Systems with good detailing of members and connections as defined in the 2005 AISC *Seismic Provisions for Structural Steel Buildings* are expected to have ductile behavior and are rewarded with smaller seismic design forces. Good inelastic behavior is characterized by full and stable hysteretic force-deformation loops which provide the structure with the necessary energy dissipation capability to survive the severe shaking and ensure life safety of the occupants. The expectation is that all forms of member buckling and overall system buckling are reasonably controlled so that adequate ductility and inelastic deformation are achieved before buckling influences the overall performance of the structure.

Despite the fact that considerable research has been conducted on the problem of overall stability of steel structures under seismic loading (Ziemian, 2010, Chapter 20), very little has found its way into the building code. The code treatment of the problem is very simplistic and stems from the current practice of elastic analysis at code specified reduced force levels. Consideration of an overall P- $\Delta$  effect under the specified seismic load effect, E, is required but only under nominal (not ultimate) gravity load levels. While this requirement (discussed further below) is admirable, it certainly does not address in any meaningful way the real stability problem during major ground shaking, where excursions into the inelastic range may induce negative post-yield stiffness and possible collapse. The "true" seismic response can only be addressed by running second-order inelastic dynamic timehistory analyses under a suite of probable ground motions, which is permitted but not required by the current code. This is not done for the vast majority of steel structures. The Federal Emergency Management Agency (FEMA) is sponsoring long term research, directed by the Applied Technology Council (ATC) and currently known as the ATC-58 project, to develop a performance-based design approach based on assessing frame deformations under various levels of earthquake shaking using an appropriate method of analysis. These new design guidelines, when completed, are expected to be the foundation for the next generation of seismic design standards in the U.S.

Given this brief background, one can ask how design for stability using the methods described in this design guide fits into the seismic design process as defined by the building code. The answer lies in the fact that the building code has sanctioned the use of the traditional effective length method (ELM) by virtue of its widespread use and acceptance in all

building codes, including the IBC 2006 code. Therefore, by virtue of the fact that comparable design results are obtained using the new DM, this method can also be used along with the ASCE/SEI 7 load combinations, including *E*, the seismic load effect, defined in Section 1605 of the 2006 IBC Code and Chapter 2 of ASCE/SEI 7. The successful use of either of these stability design methods is predicated upon satisfaction of all the seismic detailing requirements of the code, and conformance with the prescribed drift limits defined in Section 12.12 and the *P*- $\Delta$  limitations specified in ASCE/SEI 7 Section 12.8.7, both of which are discussed in the following.

## 5.1.2 Member Properties to Use in Structural Analysis Modeling

It is recommended in this Design Guide that the seismic forces, whether from the ELF or MRSA, be determined from an analysis model based on the traditional member properties (nominal member properties for steel structures and cracked member properties for concrete structures) of the frame as traditionally done prior to the introduction of the DM. The DM was not developed with any intention to modify the determination of the seismic load effect, *E*, required by the building code or the ASCE/SEI 7 load standard. Any



Fig. 5-1. Building seismic design philosophy.

subsequent static second-order analysis may be conducted by the DM using the reduced properties of the members and considering this load effect.

#### 5.1.3 Drift Control Under Code Seismic Forces

Drift requirements for steel structures designed by the 2006 IBC are defined in ASCE/SEI 7 Section 12.8.6, Figure 12.8-2 (Story Drift Determination) and Section 12.12.1 (Story Drift Limit). In determining the story drift, an elastic analysis of the steel frame is conducted under the code prescribed seismic load effect, E, (the elastic load effect reduced by R) to determine elastic deflections,  $\delta_e$ . These deflections are then magnified by the deflection amplification factor,  $C_d$ , which is determined from Table 12.2-1 of ASCE/SEI 7 based on the type of structural system. This yields a deflection estimate under the true inelastic excursion of the structure. The story drift,  $\Delta$ , is then determined from the inelastic frame deflections,  $\delta = \delta_e (C_d/I)$ , and its value is compared against the code drift limits or allowable story drift,  $\Delta_a$ , defined in Table 12.12-1 of ASCE/SEI 7. Note that the importance factor, I, is included in the deflection equation to cancel it out of the drift determination since its effect is already included in the seismic load effect, E. For most steel frames,  $\Delta_a = 0.02 h_{sx}$ , where  $h_{sx}$  is the story height as defined in ASCE/SEI 7. For many moment frames, the member sizes are controlled by either wind or seismic drift limits rather than by the strength under the reduced code seismic load effect, E. These types of frames should be proportioned for drift first and then checked for strength. Conversely, most braced frame member sizes are controlled by strength rather than drift because of their inherent stiffness.

## 5.1.4 P- $\Delta$ Control Under Seismic Forces

In the current codes, the primary control for ensuring overall frame stability during earthquake ground shaking (and in the post-earthquake period from aftershocks) lies in drift control and in limiting the secondary effects by controlling the P- $\Delta$  moments. The control of P- $\Delta$  effects is covered in Section 12.8.7 of ASCE/SEI 7. The requirements of this section apply to all structures in a location assigned to seismic design categories B through F. The stability coefficient,  $\theta$ , is determined from the following:

$$\theta = \frac{P_x \Delta}{V_x h_{sx} C_d}$$
(ASCE/SEI 7 Eq. 12.8-16)

where

 $P_x$  = total vertical design load at and above level *x*, computed with a load factor of not greater than 1.0 on all gravity loads, kips

- $\Delta$  = design story drift under earthquake loads (including  $C_d$  and inelastic effects) occurring simultaneously with  $V_x$ , in.
- $V_x$  = seismic story shear below level x, kips
- $h_{sx}$  = story height below level x, in.
- $C_d$  = deflection amplification factor from Table 12.2-1

The maximum limit on  $\theta$  is defined by ASCE/SEI 7 as:

$$\theta_{MAX} = \frac{0.5}{\beta C_d} \le 0.25$$
(ASCE/SEI 7 Eq. 12.8-17)

where  $\beta$  is the ratio of shear demand to shear capacity for the story below level x. The shear demand is simply the story shear obtained from the code seismic forces. The story shear capacity can be defined as the shear in the story that occurs simultaneously with the attainment of the first significant vield level of the overall structure, computed by amplifying the code applied seismic forces until first yield occurs in any one member of the frame. In addition, the story shear capacity can be determined conservatively by using the code seismic force level and determining the largest  $\beta$  value in the particular story under consideration. It is always conservative to assume  $\beta = 1.0$ . However, it can be important to consider the actual value of  $\beta$  (< 1.0) because many moment frames have extra reserve strength because their sizes are larger than required for strength in order to control the drift. This is reflected as a reduction factor on  $C_d$ , the code prescribed deflection amplification factor.

The technical justification for ASCE/SEI 7 Equations 12.8-16 and 12.8-17 is controversial. Further discussion of  $\theta$  and  $\beta$ , along with additional references, can be found in the FEMA 450-2 Commentary (FEMA, 2003).

As discussed in ASCE/SEI 7 Section 12.8.7, the *P*- $\Delta$  effect (i.e., the *P*- $\Delta$  amplification) can be determined using computer software as part of the frame analysis or it can be calculated as follows:

$$P-\Delta \text{ effect} = \frac{1}{1-\theta}$$
(5-1)

ASCE/SEI 7 allows this effect to be ignored when  $\theta \le 0.10$  (a *P*- $\Delta$  amplification from the above equation equal to 1.11). Note that Equation 5-1 is equivalent to the AISC *B*<sub>2</sub> factor (Equations C2-3 and C2-6a in Chapter C) [in a different form in the 2010 AISC *Specification* Appendix 8, Equations A-8-6 and A-8-7], except that it uses  $R_M = 1.0$  rather than 0.85 for moment frames and combined systems and it uses a load factor on the gravity load no greater than 1.0.

Table 5-1 demonstrates the overall influence of the 2006 IBC Code on second-order effects using the common lateral load combination 1.2D + 0.5L + 1.6W (from IBC

Table 5-1. 2006 IBC and ASCE/SEI 7 Limitations on Second-Order Effects							
L/D	0.75		1.50		3		
P <sub>u</sub> /P <sub>s</sub>	1.15		1.11		1.08		
P-∆ effect	MF	BF	MF	BF	MF	BF	
B <sub>2S</sub>	1.42	1.33	1.42	1.33	1.42	1.33	
<b>B</b> <sub>2</sub>	1.51	1.40	1.48	1.38	1.47	1.37	

Equation 16-4) or 1.2D + 0.5L + 1.0E (from IBC Equation 16-5). For these LRFD load combinations and the normal range of live, *L*, to dead, *D*, load ratios found in most multi-level steel buildings, the second-order  $P-\Delta$  effect (*B*<sub>2</sub>) is limited to about 1.40 for steel braced frames and 1.50 for steel moment frames because of these seismic requirements. Higher values of *B*<sub>2</sub> would exist for gravity-only load combinations in both one-story and multi-level buildings.

The values,  $B_{2S}$  and  $B_2$ , given in Table 5-1 are determined as follows:

$$B_{2S} = \frac{1}{1 - \theta_{max}} \tag{5-2}$$

$$B_{2} = \frac{1}{1 - \frac{\theta_{max}(P_U / P_S)}{R_M}}$$
(5-3)

where

 $\theta_{MAX} \le 0.25$  (from ASCE/SEI 7 Eq. 12.8-17)  $P_S = D + 0.5L$   $P_U = 1.2D + 0.5L$  (from 2006 IBC Eq. 16-4 and 16-5)  $R_M = 0.85$  for moment frames (MF)  $R_M = 1.00$  braced frames (BF)

Equation 5-2 is based on ASCE/SEI 7 Section 12.8.7. Equation 5-3 is the AISC *Specification*  $B_2$  equation rewritten with ASCE/SEI 7 terminology. Several important observations can be made from these calculations:

- 1. The P- $\Delta$  effect is effectively calculated at the design earthquake lateral load level (see Figure 5-1) and not the inelastic or amplified lateral load level.
- 2. The P- $\Delta$  effect is based on gravity loads at the nominal (or lower) code load level and not the ultimate load level.
- 3. The code calculates the above P- $\Delta$  effects pertaining to the seismic load combinations using the reduced code level earthquake deflections, not the amplified or expected inelastic deflections.

4. The *P*- $\Delta$  amplifier expressed by Equation 5-1 does not account for any reduction in column stiffness in structural steel moment frames due to *P*- $\delta$  effects (these effects are accounted for by the use of *R*<sub>M</sub> = 0.85 in the AISC *Specification*).

On the surface, all of these assumptions appear to be unconservative. The structural responses due to static P- $\Delta$ effects and dynamic P- $\Delta$  effects are of course very different. For many frames subjected to seismic shaking, the structure may indeed see the larger inelastic drifts. However, the inertial effects associated with the dynamic response force the frame back in the opposite direction from a potential sidesway failure before collapse can occur.

The justification for the current code calculation of the  $P-\Delta$  effects is explained in the Commentary to FEMA 450-2 (FEMA, 2003), also called the NEHRP Provisions Commentary, which is the basis for the ASCE/SEI 7 code provisions. First, the procedure used is seemingly justified by the fact that there have not been many stability related failures observed during actual earthquakes. This can be attributed to the apparent overstrength beyond the code level design forces due to drift control. Also, the likelihood of a stability failure decreases with increased intensity of ground shaking because the stiffness of structures designed for extreme ground shaking is significantly greater than the stiffness of the same structure designed for a lower ground shaking or for wind alone. Low intensity earthquake damage is rare and there would likely be little observable damage. Secondly, consideration of the  $P-\Delta$  moments based on the inelastic drift, even using nominal gravity loads, would result in a large increase in the design forces. This appears unwarranted based on observations under actual earthquakes. For instance, suppose that the P- $\Delta$  amplification from Equation 5-1 is 1.18 for an intermediate moment frame with a  $C_d$  = 4.0. Therefore,  $\theta = 0.15$ . If one were to use  $C_d \theta = 0.60$  in Equation 5-1 to approximate the influence of the inelasticity, the *P*- $\Delta$  effect would increase to 2.50. Clearly, this would have a large impact on building costs and does not seem justified based on past performance. In addition, Equation 5-1 is truly just a static elastic P- $\Delta$  amplification factor. Use of  $C_d \theta$  in this equation amounts to the use of a secant stiffness tied to the inelastic drift, not a tangent stiffness associated with the inelastic response. Further discussion of steel frame

response under dynamic P- $\Delta$  effects can be found in Gupta and Krawinkler (2000).

### Summary of Design Recommendations:

- 1. The direct analysis method (DM) can be used along with the ASCE/SEI 7 load combinations, including the reduced seismic load effect, *E*, defined in Section 1605 of the 2006 IBC Code and Chapter 2 and Section 12.4 of ASCE/SEI 7.
- 2. The successful use of either the ELM or the DM is predicated upon satisfying all the seismic detailing requirements of the code and conforming to the prescribed drift limits defined in Section 12.12 plus the  $P-\Delta$  effect limits of ASCE/SEI 7 Section 12.8.7.
- 3. It is recommended that for steel structures the seismic load effect, *E*, whether from the ELF or the MRSA, be determined using the nominal properties of the members. Any subsequent static second-order analysis may be conducted by the DM using the reduced properties of the members and considering this load effect.

# 5.2 COMMON PITFALLS AND ERRORS IN STABILITY ANALYSIS AND DESIGN

Since the advent of the effective length concept in 1961, many articles and textbooks have been published about problems in the application of stability methods used in steel design. Some of these problems have led to refinements in various later editions of the AISC *Specification* and Commentary to alert the designer to common pitfalls. Some of these common problems are discussed in the following as an aid to the designer.

1. Improper Second-Order Analyses. Many errors in stability design can be traced to improper second-order analysis techniques. Since any stability analysis basically requires the consideration of equilibrium on the deformed geometry of the structure, it is important that the deflections of the frame be captured with sufficient accuracy. This means that all significant deformations of the structure must be considered, including flexural deformations of beams and columns, shear deformations in beams and columns, axial deformations of columns and braces, panel zone deformation in beam-column joints, differential foundation settlements and rotational restraint, and out-of-plumbness effects to name a few of the more important ones. The software used must capture all of the significant deformations and all of the considerations discussed here must be at least evaluated for their importance. Depending on frame geometry such as bay spacing and member proportions, as well as the

height and aspect ratio of the frame, the different effects take on different levels of importance.

The computer analysis should accurately capture the effect of individual column *P*- $\delta$  effects (reduction in column stiffness due to axial load) on the overall lateral drift of each story. For frames with large column axial loads ( $\alpha P_r$  larger than  $0.05\pi^2 EI/L^2$  in some cases), this effect may become significant and require additional nodes in the column length for the analysis. If the software does not accurately account for the *P*- $\delta$  effects in the formulation of its frame elements, the engineer may need to apply the  $B_1$  amplifier to approximate these effects or use multiple elements per member.

- 2. Neglect of Leaning Column Effect. It is important to properly include all gravity loads in determining the destabilizing effect on the lateral frame. The P- $\Delta$  effect of all gravity loads in the building must be accounted for in the analysis.
- 3. *Improper Use and Calculation of*  $K_2$ . The accurate calculation of  $K_2$  can be a challenge for complex frames with a large number of leaning columns or irregular frame geometry. This problem can be avoided with use of the direct analysis method (DM) or the first-order analysis method (FOA) where K = 1.0.
- 4. *Torsional Loading Effects in Frames.* Simplification of a building analysis into two dimensions can be problematic for eccentric code wind and seismic loading requirements, irregular building shapes that are not orthogonal, and when there is eccentricity between the center of stiffness and mass or lateral wind loading. This can lead to significant errors in internal frame forces.
- 5. Drift Control for Stability. Structures with very light wind loads and little or no seismic requirements can be very flexible in sidesway while still satisfying tight service drift requirements. Thus, drift control by itself is not sufficient to control the magnitude of second-order effects. Significant sidesway flexibility and large column loads and/or leaning column effects can lead to large second-order effects. Note that the  $B_2$  amplifier, and thus the second-order effects, becomes large when the total story gravity load is large when compared to the frame lateral buckling strength. Drift limits applicable to steel frame structures under seismic loading combinations are given in Section 12.12 and Table 12.12-1 of ASCE/ SEI 7. However, even with these drift limits, secondorder effects can be quite large for some gravity-only load combinations, even for frames that satisfy these requirements.

- 6. *Stiffness of Non-Steel Elements.* The stiffness of nonsteel elements such as cladding and partitions (particularly masonry) can both help and hurt a steel frame. It can help reduce frame drift and second-order effects at small amplitudes of lateral loading but it can also hurt the distribution of story shears within the frame resulting in unintended behavior (particularly torsional effects) under high wind and seismic loading. In such cases, the masonry infill should be isolated from the lateral load resisting frame using properly detailed "soft joints."
- 7. *Soft Stories.* A designer should strive to have nearly uniform stiffness at each story as encouraged by seismic codes. This will reduce demands on the frame under severe lateral loading and reduce high concentrated second-order effects.
- 8. Drift Control at Service Load Levels. Serviceability checks for drift under wind load should include

second-order effects whenever realistic drift limits that reflect actual potential damage are used. Torsional deformations in plan from eccentricity of loading, mass and stiffness should also be evaluated when checking story drift limits.

9. Enveloping Frame Stiffness in Mixed Frame Systems. When steel moment or braced framing is combined with concrete or masonry shear walls or composite columns or walls used in lateral frame resistance, consideration should be given to cracking of the concrete and masonry elements under different degrees of lateral loading. It is wise to assume various degrees of stiffness for these elements to check the sensitivity of the story shear participation of the steel elements and to design them for the worst effects from various degrees of stiffness. Guidance for stiffness of concrete elements can be found in the ACI 318-08 building code (ACI, 2008).

# APPENDIX A Basic Principles of Stability

# A.1 WHAT IS STABILITY?

The SSRC Guide (Ziemian, 2010) defines stability and instability as follows:

Stability: The capacity of a compression member, element or frame to remain in position and support load, even if forced slightly out of line or position by an added lateral force.

Instability: A condition reached during buckling under increasing load in a compression member, element or frame at which the capacity for resistance to additional load is exhausted and continued deformation results in a decrease in load resisting capacity.

Both of these terms are also defined within the AISC *Specification* as follows:

Stability: Condition reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry does not produce large displacements.

Instability: Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry produces large displacements.

Perhaps an even simpler definition for stability is equilibrium in a deformed position under the applied load. While theoretical buckling of a perfectly straight member or frame is a bifurcation behavior, real members and frame systems have an initial out-of-straightness such that application of a compressive load immediately results in transverse deflections relative to the initial crooked position. At the onset of instability, the effective lateral stiffness of the structure approaches zero. It is very fortuitous that this behavior often occurs in real structures, as it provides a warning of failure.

# A.2 FACTORS INFLUENCING FRAME STABILITY

There are many factors that can influence the stability of steel frame structures. The SSRC Guide (Ziemian, 2010) lists the primary factors in two tables of its Chapter 16: (1) Physical Attributes of the Structure and Loading and (2) Modeling Parameters and Behavioral Assumptions. The tables are repeated here for reference as Tables A-1 and A-2. All of these factors have been considered in the formulation of the AISC *Specification*. A discussion of the most significant of these factors is included in the AISC *Specification* 

Appendix 7 Commentary [2010 AISC *Specification* Chapter C Commentary]. A brief discussion of several of these factors follows to help provide insight into the AISC *Specification* requirements.

# A.2.1 Second-Order Effects, Geometric Imperfections, and Fabrication and Erection Tolerances

The AISC Specification provisions are based on the premise that member internal forces are determined using a secondorder elastic analysis, where equilibrium is satisfied on the deformed geometry of the structure. Two of the predominant second-order effects on frame members are the P- $\Delta$  and the  $P-\delta$  effects. Figure A-1 illustrates the fundamental meaning of these two effects. In tiered building structures, the P- $\Delta$ effect is usually considered as the effect of the vertical loads acting through the lateral sway displacements of the columns and other vertical load supporting elements. However, in general, this effect is simply the couple generated by the axial force acting through the relative transverse displacements of the ends of a given segment. Conversely, the P- $\delta$ effect is due to transverse displacement of a member cross section relative to a straight chord caused by the bending deformation of the member. Several attributes are important to note about the *P*- $\Delta$  and the *P*- $\delta$  effects:

- 1. If there is no sidesway of a member, then there are no member P- $\Delta$  moments and no member P- $\Delta$  effect.
- 2. If there is no curvature of the deformed member, that is, if the member remains straight (e.g., an ideal truss element), there are no P- $\delta$  moments and there is no P- $\delta$  effect.
- If bending deformations occur in a member due to sidesway, then the member is subjected to both *P*-Δ and the *P*-δ effects. For beam-columns under axial compression, the additional *P*-δ moments at a given cross section cause additional member bending deformations, and hence increase the member sidesway displacements, Δ. This increases the member *P*-Δ effect.
- 4. If a member is subdivided into multiple segments, e.g., if a member is modeled using multiple frame elements, the *P*- $\delta$  moments in each segment become smaller and smaller and the second-order effect in the plane of bending is captured by the *P*- $\Delta$  moments from the combination of all of the segments.

Consideration must be given to initial geometric imperfections in the structure due to fabrication and erection

Table A-1. Factors Affecting Steel Frame Stability – Physical Attributes of Structure and Loading				
Frame geometry and configuration Centerline framing dimensions Member geometry and material Connection details Foundation and support conditions Shear connections to slab Infill walls or secondary structural elements Finite member and joint size effects Out-of-plane bracing elements				
Material properties Elastic moduli Expected versus nominal strengths Ductility and fracture toughness				
Geometric imperfections Erection out-of-plumbness Member out-of-straightness Incidental connection or loading eccentricities				
Internal residual stresses From manufacturing/fabrication processes From erection fit-up From construction sequencing From incidental thermal loadings or support settlements				
Loadings Magnitude and distribution Loading rate and duration				

tolerances. In the traditional effective length method (ELM) and the first-order analysis method (FOM), the structure is assumed perfectly straight in the structural analysis model. The FOM is calibrated so that these effects are accounted for in the design of the frame; these effects are more implicit than explicit in the ELM. In the direct analysis method (DM), using the concept of notional loading, initial geometric imperfections are conservatively assumed to be equal to the maximum fabrication and erection tolerances specified by the AISC *Code of Standard Practice for Steel Buildings and Bridges* (AISC, 2005d). The user is free to modify these assumptions in the analysis based on evidence of stricter control. This is discussed in more detail later in this appendix. In ASTM A6/A6M (ASTM, 2012), for a W-shape with a flange width greater than 6 in. the member out-of-straightness works out to be approximately L/1000, where L is the member length in inches between bracing or framing points. This is explicitly stated in the AISC *Code of Standard Practice* as the frame out-of-plumbness tolerance of L/500, where L is the story height, subject to specified absolute limits.

## A.2.2 Residual Stresses and Spread of Plasticity

Residual stresses inherent in all rolled and built-up shapes during the rolling and fabrication process cause early yielding as the strength limit state is approached. This softening of the structure or spread of plasticity through the member cross section and along the member length is directly accounted for in the DM by reducing the axial and flexural properties of the members that are part of the lateral load resisting frame. In the ELM and the FOM, this effect is calibrated into the design process to account for this effect using the nominal properties of the members. Residual stresses also contribute to the stiffness reduction factors defined as  $\tau_a$ and  $\tau_b$  in the AISC *Specification* and Commentary.

# A.2.3 Member Limit States

Strength of members in the lateral load resisting frame may be controlled by cross-sectional yielding, local buckling, flexural buckling, and lateral-torsional or flexural-torsional



Fig. A-1. Second-order P- $\Delta$  and P- $\delta$  effects.

Table A-2. Factors Affecting Steel Frame Stability—Modeling Parameters and Behavioral Assumptions				
Linear elastic response				
Flexural, axial, and shear deformations of members				
Deformations of connections and beam-column panel zones				
Uniform torsion and/or nonuniform warping torsion deformations in members				
Foundation and support movement				
Dynamic and inertial effects				
Geometric nonlinear (second-order) response				
$P$ - $\delta$ effects: Influence of axial force on stiffness and internal moments in beam-columns				
$P-\Delta$ effects: Influence of relative joint displacements on forces and displacements				
Local buckling and cross section distortion				
Finite rotation effects (three-dimensional behavior)				
Material nonlinear response				
Member plastification under the action of axial force and blaxial bending (spread of plasticity versus plastic hinge				
Idealizations) Member plastification due to shear forces, uniform targion, and populatorm warping targion (bi memorts)				
Vielding in connection components and joint panels				
Tension runture of members and connections				
Strain hardening behavior				
Cvclic plasticity effects				
Load path effects, shakedown, and incremental collapse				

buckling. These limit states must be checked with separate member design equations from the various chapters of the AISC *Specification* and through the use of the beam-column interaction equations. A feature in the 2005 AISC *Specification* reducing some of the conservatism in the design allows compact-element section wide-flange members subjected to single-axis flexure and axial compression to be checked with interaction equations in AISC *Specification* Section H1.3.

## A.3 SIMPLE STABILITY MODELS

Many of the key aspects of frame stability can be demonstrated with simple stability models. Three such models are shown in Figure A-2. The models are described in the following.

*Model A:* Model A depicts the simplest of all moment frames—a cantilever column with a second pin-connected column carrying only gravity load, often called a "leaning column." This model is used to explain the basic principles of second-order effects in frames including the *P*- $\delta$  and *P*- $\Delta$  effect, the reduction in column stiffness from axial load (represented by the *C*<sub>L</sub> factor defined by LeMessurier (1977)) and the destabilizing effect of leaning columns.

*Model B:* Model B depicts a one-story braced frame, represented by a simple lateral spring support, along with a leaning column. This model is used to explain the same basic principles as Model A except in the context of a braced frame structure.

*Model C:* Model C depicts a subassemblage with one leaning column representing the gravity columns of a floor stabilized by a moment frame. The sub-assemblage may be

thought of as a typical bay of a moment frame consisting of a beam above and below a moment frame column (see Figure A-3). Each of the single beams at the top and bottom of the column may be thought of as representing two beams, one framing in from each side of the column. The beams in the analysis model are assumed pinned at the mid-length of the physical beams and the properties  $(EI_g)$  are doubled to properly mimic an interior subassemblage. This model is used to demonstrate some of the same basic principles of stability as Model A, but in a more realistic setting.

These models were also used to demonstrate the stability design procedure for the ELM (Chapter 2) and the DM (Chapter 3) covered in Chapter C and Appendix 7 of the 2005 AISC *Specification* [Appendix 7 and Chapter C of the 2010 AISC *Specification*].

## A.3.1 Model A

A simple cantilever column with and without leaning columns can be used to demonstrate many key principles of frame stability as demonstrated in Figure A-2 as Model A. This type of model was studied extensively by LeMessurier (1977) and a number of conclusions reached in that study are present in the building codes today.

Consider first the case without a leaning column. The first- and second-order moment diagrams are illustrated in Figure A-4. Analytical expressions for the second-order base moment and tip deflection in this problem are as follows (Timoshenko and Gere, 1961; Chajes, 1974; Chen and Lui, 1987):