the β_2 factor may be used to include these effects. If in doubt, the value of V_{uc} should be taken as zero.

C8.2.7.4 Reversal of loads and members in torsion

Where cracking can occur in regions that are normally in compression, such as in cases of load reversal or from torsion where cracks can be wide at ultimate, there is considerable doubt as to the reliability of values of V_{uc} specified elsewhere in Clause 8.2.7. In such cases, V_{uc} is to be assessed by more rigorous calculation or taken as zero.

C8.2.8 Minimum shear reinforcement

As shear cracks form suddenly and in a brittle manner, sufficient shear reinforcement is needed to guard against immediate failure at the onset of shear cracking. If the quantity of shear reinforcement is too small, the reinforcement will yield immediately after the inclined crack forms and the beam will fail. To avoid this, AS 3600—1988 adopted the ACI 318-71 (Ref. 28) provision, which required a minimum amount of shear reinforcement equal to (converted to metric units)—

$$A_{\rm sv.min} = 0.35 \frac{b_{\rm v}s}{f_{\rm sv.f}}$$

where b_v and s are in mm, f_{sy} is in MPa and $A_{sv,min}$ in mm².

For a value of $\theta_v = 45^\circ$, as used in the ACI Code, this minimum amount corresponds to a nominal force to be resisted by the reinforcement of $\tau_s b_v s$, where $\tau_s = 0.35$ MPa. As the stress needed to form the inclined crack increases with concrete strength, the limit on τ_s has been amended in AS 3600—2009 to be the greater of $0.06\sqrt{f'_c}$ and 0.35 MPa. The limit on τ_s for the standard strength grades are given in Table C8.2.8.

TABLE C8.2.8

NOMINAL SHEAR STRESS LIMIT FOR MINIMUM SHEAR REINFORCEMENT $\tau_s = A_{sv.min} f_{sv.f} / (b_v s)$

$f_{\rm c}'$, MPa	20	25	32	40	50	65	80	100
τ _s , MPa	0.35	0.35	0.35	0.38	0.42	0.48	0.54	0.6

C8.2.9 Shear strength of a beam with minimum reinforcement

The shear strength of a beam with minimum shear reinforcement is derived by adding the shear strength contributed by the minimum area of shear reinforcement $(A_{sv.min})$, given in Clause 8.2.8, to the shear strength of the concrete section without reinforcement (V_{uc}) . In calculating the minimum shear strength, to provide consistency with Clause 8.2.10(a), the strut inclination angle is taken as $\theta_v = 30^\circ$. With the area of shear reinforcement, $A_{sv.min} = \tau_s \ b_v s/f_{sy.f}$ where $\tau_s = \max \left[(0.06 \sqrt{(f_c')}, 0.35) \right]$ MPa, the minimum shear strength becomes—

$$V_{\text{u.min}} = V_{\text{uc}} + \max(0.06\sqrt{f_{\text{c}}'}, 0.35)b_{\text{v}} \ z \ \cot 30^{\circ}$$

where z is the internal lever arm between the flexural compressive force and the flexural tensile force.

In the Standard, the approximation $z \approx d_0$ is used and the horizontal projection of the inclined shear crack is taken as $d_0 \cot 30^\circ$.

C8.2.10 Contribution to shear strength by the shear reinforcement

The strength of a beam with shear reinforcement has been the subject of extensive research (Ref. 29). The increase in strength with shear reinforcement ratio is shown in Figure C8.2.10.

The assumption that the concrete contribution to the shear force (V_{uc}) remains constant after inclined cracking, and equal to its value immediately before cracking, is convenient from a design point of view as it eliminates any discontinuity in the design. In reality, the concrete contribution decreases with higher shear forces and this effect is included in the CEB-FIP method (Ref. 23).

Truss theories (e.g. Ref. 30) usually give a range for the truss angle, which becomes more restricted with higher shear forces. In more severe cases, the truss angle is limited to about 45° . In AS 3600, a procedure was adopted where V_{uc} is taken as constant but the truss angle is explicitly stated and increases with increasing shear.

The method is a hybrid but gives reasonable results as shown in Figure C8.2.10. Experimental results were obtained from Ref. 19.

It is noted that the inclination of the compressive strut θ_v is expressed as a function of the design shear (V^*). This is most convenient in a design situation when the amount of shear reinforcement (A_{sv}) is being calculated. If, on the other hand, the shear strength of an existing beam with known transverse reinforcement quantities is to be calculated, the following expression may be more useful:

$$\theta_{\rm v} = 30^{\circ} + 15^{\circ} \left[\frac{A_{\rm sv} - A_{\rm sv.min}}{A_{\rm sv.max} - A_{\rm sv.min}} \right]$$

where

$$A_{\text{sv.max}} = \frac{b_v s}{f_{\text{sy.f}}} \left[0.2 f_c' - \frac{V_{\text{uc}}}{b_v d_o} \right] \text{ and}$$
$$A_{\text{sv.min}} = \max\left(0.06\sqrt{f_c'}, 0.35\right) \frac{b_v s}{f_{\text{sy.f}}}$$



FIGURE C8.2.10 EFFECT OF SHEAR REINFORCEMENT ON SHEAR STRENGTH—COMPARISON OF PROPOSALS

C8.2.11 Hanging reinforcement

When the support is at the soffit of a beam or slab, as shown in Figure C8.2.11(A)(a), the diagonal compression passes directly into the support. When the support is at the top of the beam, as shown in Figure C8.2.11(A)(b), the diagonal compression will need to be carried back up to the support via an internal tie as shown. It is essential that adequately anchored reinforcement be included to act as the tension tie (termed *hanging reinforcement* in the Standard) and that the reinforcement pass into and be anchored within the support.



FIGURE C8.2.11(A) SUPPORT POINTS (Ref. 17)

Consider the suspended slab supported from above by the upturned beam shown in Figure C8.2.11(B). The horizontal component of the diagonal compression being delivered at the support of the slab has to be resisted by the bottom slab steel. The vertical component of the diagonal compression (i.e. the reaction from the slab) has to be carried in tension up to the top of the upturned beam. This tension force has to be carried across the unreinforced surface indicated in Figure C8.2.11(B)(a). The concrete on this unreinforced surface may not be able to carry this tension and, if cracking occurs, premature and catastrophic failure could occur. The detail shown in Figure C8.2.11(B)(b) overcomes the problem. The diagonal compression from the slab is now resisted by the stirrups in the upturned beam (acting as *hanging reinforcement*). No longer is there an unreinforced section of concrete required to carry tension. The vertical and horizontal members of the analogous truss have been effectively connected.



FIGURE C8.2.11(B) SUSPENDED SLAB SUPPORTED FROM ABOVE BY UPTURNED BEAM (Ref. 17)

Where a primary girder supports a secondary beam, and the secondary beam frames into the side of the primary girder, hanging reinforcement is required. Consider the connection shown in Figure C8.2.11(C). The reaction from the secondary beam (R^*) is delivered to the primary girder at the level of the bottom steel. This reaction should be carried by hanger or suspension reinforcement up to the top of the girder where it can be resolved into diagonal compression in a similar way to that of any other load applied to the top of the girder. For the reasons discussed in the previous paragraph, the bottom reinforcement in the secondary beam should always pass over the bottom reinforcement in the primary girder. The reinforcement details of the primary girder, together with its truss analogy, are shown in Figures C8.2.11(C)(b) and C8.2.11(C)(c).

The suspension reinforcement is additional to the transverse reinforcement required for shear in the primary girder and has to be located within the secondary beam/primary girder connection. The area of additional hanging reinforcement (A_{sr}) required to carry the factored reaction R^* is given by—

$$A_{\rm sr} = \frac{R^*}{\phi_{\rm st} f_{\rm sy}}$$

where $\phi_{st} = 0.7$ (for strut and tie action) and f_{sy} is the characteristic yield stress of the hanger reinforcement.



(iii) Primary girder - Truss analogy



C8.2.12 Detailing of shear reinforcement

C8.2.12.1 Types

The types of reinforcement that may be used as shear reinforcement are restricted in this Clause. Specifically, bent up bars are not allowed because of difficulties in anchorage, potential crack control problems and the likelihood of the concrete splitting in the plane of the bends (Ref. 15).

The strut and tie model for shear adopted in the Standard assumes that all bars crossing a crack are at yield and, for all bars to be at yield, some will be required to undergo large plastic deformation. The possibility of fracture of one or more of the bars before adjacent bars have yielded has to be avoided.

C8.2.12.2 Spacing

The requirement for maximum spacing ensures that a potential failure surface (inclined crack) intersects one or more stirrups and reduces the concentration of compressive forces in the web strut. For lower shears, the failure surface is flatter (Ref. 31) and the inclined compressive forces are more moderate, thus a relaxation of the limit to 0.75D or 500 mm is permitted when $V^* \leq \phi V_{u.min}$.

In regions of high shear, it is desirable to use multi-leg stirrups when more than two longitudinal tensile bars are used. Multi-leg stirrups should be used in members with wide webs to avoid the undesirable distribution of diagonal compression shown in Figure C8.2.12.2. For this reason, the maximum spacing of shear reinforcement across the width of a member is the smaller of 600 mm and D.



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(a) Elevation

(b) Cross-section

FIGURE C8.2.12.2 UNDESIRABLE DISTRIBUTION OF DIAGONAL COMPRESSION DUE TO WIDE FITMENTS (Ref. 32)

C8.2.12.3 Extent

The Clause provides in part for possible inaccuracies in analysis and atypical failure mechanisms, as well as considerations arising from truss-analogy theory. It is consistent with the requirements of Clause 8.1.10.1 (see also C8.1.10.1).

C8.2.12.4 Anchorage of shear reinforcement

It is essential that shear reinforcement be adequately anchored. This Clause states the minimum requirements for this purpose.

The flow of internal forces in a beam can be idealized as a parallel chord truss as illustrated in Figure C8.2.6. The compressive top chord and the diagonal web strut are the concrete portions of the truss while the tensile bottom chord and vertical web ties are the steel reinforcement. The diagonal compression (in the concrete web strut) can only be resisted at the bottom of the beam at the intersection of the horizontal and vertical reinforcement (that is, at the pin-joints of the analogous truss). It is evident that the tension in the vertical tie is constant over its entire height (that is, from the pin joint at the bottom chord to the pin joint at the top chord). Therefore, adequate anchorage of the stirrups will need to be provided at every point along the vertical leg of the stirrup. When calculating the shear strength provided by the stirrups, it is assumed that every vertical stirrup leg crossed by an inclined crack is at yield, irrespective of whether the inclined crack crosses the stirrup at its middepth or close to its top or bottom. The anchorage of the vertical leg of a stirrup may be achieved by a standard hook or cog complying with Clause 13.1.2.7 or by welding of the fitment to the longitudinal bar or by a welded splice.

Ideally, stirrups should be anchored in the compression zone where anchorage conditions are most favourable. At ultimate loads, when diagonal cracks have developed, the compression zone may be relatively small. Therefore, stirrup hooks should be as close to the compression edge as cover requirements allow. Stirrups depend on this transverse pressure for anchorage. It is common practice to locate the stirrup hooks near the top surface of a beam, even in negative moment regions. When the top surface is in tension, the discontinuity created by a stirrup and its anchorage may act as a crack initiator. Therefore, a primary crack frequently occurs in the plane of the stirrup hook and anchorage is lost. As a consequence, in these regions, the beam may possess less than its required shear strength.

It is good practice to show the location of the stirrup hooks on the structural drawings and not to locate the hooks in regions where transverse cracking might compromise the anchorage of the stirrup.

Stirrup hooks should always be located around a larger diameter longitudinal bar, which disperses the concentrated force at the anchorage and reduces the likelihood of splitting in the plane of the anchorage. Longitudinal bars are in fact required in each corner of the stirrup to distribute the concentrated force applied to the concrete at each corner. It is

essential that the stirrup and stirrup hook fit snugly and be in contact with the longitudinal bars in each corner of the stirrup.

In Figure C8.2.12.4, some satisfactory and some incorrect stirrup arrangements are shown. Stirrup hooks should be bent through an angle of at least 135°. A 90° bend (a cog) will become ineffective should the cover be lost, for any reason, and will not provide adequate anchorage. The Standard states that fitment cogs are not to be used when the cog is located within 50 mm of any concrete surface.

In addition to carrying diagonal tension produced by shear, and controlling inclined web cracks, closed stirrups also provide increased ductility by confining the compressive concrete. The open stirrups depicted in Figure 8.2.12.4(b) are commonly used, particularly in post-tensioned beams where the opening at the top of the stirrup facilitates the placement and positioning of the post-tensioning duct along the member. This form of stirrup does not provide confinement for the concrete in the compression zone and is undesirable in heavily reinforced beams where confinement of the compressive concrete may be required to improve ductility of the member.



FIGURE C8.2.12.4 INCORRECT, UNDESIRABLE AND SATISFACTORY FITMENT ANCHORAGES (Ref. 17)

C8.2.12.5 End anchorage of mesh

(No Commentary).

C8.3 STRENGTH OF BEAMS IN TORSION

C8.3.1 General

This Clause has been written to cover the design of beams subjected to torsion and any combination of bending and shear. The strengths in bending and in shear, without torsion, are determined from Clause 8.1 and Clause 8.2, respectively. It is noted that when torsion is present in beams requiring shear reinforcement, the contribution of the concrete to the shear strength has to be ignored.

C8.3.2 Secondary torsion

Compatibility torsion (called secondary torsion in the Standard) is treated in the manner proposed by Collins and Mitchell (Ref. 30).

In a statically indeterminate structure where alternative load paths exist and the torsional strength of a member is not required for equilibrium, the torsional stiffness of the members

may be disregarded in analysis. In the real structure, where members do have torsional stiffness, torsion may develop. Provided the member is ductile, redistribution will occur when torsional cracks develop and the compatibility torsion in the member will reduce significantly. Compatibility torsion may be treated in design by providing sufficient compatibility reinforcement to ensure torsional cracks are controlled at service loads. The minimum quantity of reinforcement for compatibility torsion is specified in Clause 8.3.7 and the detailing requirements are specified in Clause 8.3.8.

A common example of compatibility torsion occurs in a spandrel beam supporting the edge of a monolithic floor slab in a building structure. The floor loading causes torsion to be applied along the length of the beam. The Standard permits a designer to disregard the torsional stiffness of the spandrel beam in the structural analysis, and therefore disregard torsion in the spandrel, and to rely on redistribution of internal forces to find an alternative load path. In the real structure, torsion will develop in the spandrel beam before cracking. When torsional cracking occurs, the torsional stiffness of the spandrel drops significantly, and therefore the restraint provided to the slab edge is reduced. Additional rotation of the slab edge occurs, reducing the negative moment in the slab and the torsion in the spandrel; however, full redistribution will only occur if the structure possesses adequate ductility and it may be accompanied by excessive cracking and large local deformations. Hence, the need for a minimum quantity of ductile torsional reinforcement (Refs 4, 33 and 34).

C8.3.3 Torsional strength limited by web crushing

A simple upper limit $T_{u.max}$, consistent with the shear limit, is placed on the torsional moment to avoid web crushing. This limit is conservative. For combined shear and torsion, a linear interaction is assumed. The formula for J_t for solid rectangular sections has been amended from the previous edition of the Standard.

C8.3.4 Requirements for torsional reinforcement

The components T_{uc} and T_{us} are the contributions of the concrete and steel reinforcement, respectively, to the strength of the member in torsion without any shear force. Likewise, V_{uc} and V_{us} are the strengths of the member in shear without any torsion. The values obtained from Clause 8.2 for V_{uc} and V_{us} and from Clause 8.3.5 for T_{uc} and T_{us} , depending on the absence or presence of torsional reinforcement, are as follows:

- (a) Where torsional reinforcement is not required, the linear interaction given is more conservative than other theories.
- (b) Where torsional reinforcement is required, the conservative assumption is made that the concrete contribution to the torsional strength is zero. Sufficient torsional reinforcement consisting of closed and anchored stirrups of bar cross-sectional area (A_{sw}) at spacing (s) calculated in accordance with Clause 8.3.5(b) is required such that $\phi T_{us} \ge T^*$. The required torsional reinforcement is in addition to the reinforcement required for shear calculated in accordance with Clause 8.2 (remembering that when torsion is present V_{uc} should be taken as zero).

C8.3.5 Torsional strength of a beam

- (a) The torsional strength of a concrete beam without torsion reinforcement is largely related to onset of torsional cracking, which is deemed to occur when the maximum principal tensile stress exceeds the tensile strength of the concrete.
- (b) For a beam with closed fitments, the torsional strength is calculated from a variable angle truss formulation with the angle of the torsional compressive struts taken as equal to θ_v , as determined from Clause 8.2.10. This represents a change from AS 3600—2001 where the angle of the torsional struts was independent of that for shear. While the theory of plasticity allows for such apparent contradiction in the summation of the component solutions (flexure, shear and torsion), in the three dimensional truss model on which the approach is based only one angle exists.

C8.3.6 Longitudinal torsional reinforcement

The expressions given in this Clause for the additional longitudinal steel required for torsion (additional to that required for bending and/or axial force) are obtained from the variable angle truss formulation (see Refs 21 and 30).

C8.3.7 Minimum torsional reinforcement

The minimum quantity of closed ties and longitudinal reinforcement has to be provided in order to maintain some torsional capacity of the section if cracking does occur and to provide crack control at the serviceability limit states.

C8.3.8 Detailing of torsional reinforcement

Torsional reinforcement has to be detailed to ensure that a minimum number of legs of the hoop reinforcement cross potential torsional cracks and that the full yield capacity of these bars can be developed.

The longitudinal reinforcement should be detailed consistently with the design truss model, with the longitudinal stringers located in the corners of the section.

C8.4 LONGITUDINAL SHEAR IN COMPOSITE AND MONOLITHIC BEAMS

C8.4.1 General

This Clause covers longitudinal shear design at the interface of precast concrete sections and cast-in-place flanges or toppings and at flange to web connections of integrally cast beam and slab construction.

C8.4.2 Design shear stress

The design longitudinal shear stress taken at the junction of a web and flange (τ^*), may be determined with reference to Figure C8.4.2 and the following:

(a) For the case where the concrete compressive stress block lies within the flange, the longitudinal design shear stress through section 2-2 may be approximately obtained by taking moments about 'O' on the left-hand face of the element and gives—

$$\Delta C = V * \frac{\Delta L}{z} \qquad \dots \text{ C8.4.2(1)}$$

where ΔC is the change in the compressive force on the section over the length of ΔL ; V^* is the factored design shear force acting on the section; and z is the internal lever arm between the centroids of the tensile and compressive forces.

The longitudinal design shear force per unit length (q^*) through section @-@ is then—

$$q^* = \Delta C / \Delta L = V^* / z \qquad \dots \text{ C8.4.2(2)}$$

and the longitudinal design shear stress is-

$$\tau^* = \frac{q^*}{b_{\rm w}} = \frac{V^*}{b_{\rm w} z} \qquad \dots \ \text{C8.4.2(3)}$$

(b) Where the compressive stress block lies within the web, the design longitudinal shear force per unit length through the interface of the flange and web is approximated as that given by Equation C8.4.2(3) multiplied by β , where β is the ratio of the component of the compressive force that lies above the web-flange interface relative to the total compressive force acting on the section.

When the flange is in tension, τ^* is calculated also from Equation 8.4.2(3) but with the proportionality factor β calculated in terms of the tensile forces.



FIGURE C8.4.2 SHEAR STRESS ON A LONGITUDINAL SECTION OF LENGTH, ΔL (SHOWN FOR FLANGE IN COMPRESSION)

C8.4.3 Shear stress capacity

The design shear strength is due to-

- (a) the clamping effects produced by the shear reinforcement;
- (b) the direct shear resistance provided by dowel effect and aggregate interlock; and
- (c) transverse pressure across the interface.

The clamping effect assumes that a crack occurs along the interface and shear transfer is achieved by reinforcement crossing the crack. Any relative longitudinal displacement is accompanied by a perpendicular displacement, which is proportional to the roughness of the interface. This creates a tensile strain locally in the reinforcement and a force normal to the shear plane. The maximum value of this force is equal to the area of the reinforcement (A_{sf}) over a spacing (s) times the yield strength of the reinforcement ($f_{sy.f}$).

Reinforcement supplied for the purpose of this Clause will need to be fully anchored either side of the shear plane to enable the yield strength to be achieved. Shear and torsion reinforcement supplied to meet the requirements of Clauses 8.2 and 8.3 and which crosses the shear plane may be taken as part or all of the area A_{sf} .

The shear resistance is equal to the clamping force times the coefficient of friction (μ) . Frictional shear resistance may be increased by direct transverse pressure provided by permanent unfactored distributed loads (g_p) . Shear resistance, in addition to the frictional shear resistance, is provided by interlocking aggregates placed in direct shear along the interface. The two forms of shear resistance are reflected by the coefficients μ and k_{co} in the shear strength equation. The design shear strength equation provides for both longitudinal shear components. Resistance provided by dowel action of the shear reinforcement is not easily calibrated and may be assumed to be incorporated in the first term.

The maximum yield strength of the shear reinforcement should be limited to 500 MPa as there is no research evidence available to show that the full clamping force can be developed when higher yield strength shear reinforcement is used. Very close spacing of large diameter shear reinforcement may have the same effect as high yield shear reinforcement. That is, the clamping force equal to the yield capacity of the shear reinforcement may not be developed. Again, there is no research evidence on the effect of size and spacing of shear reinforcement on the clamping force.

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