be Mohr — Coulomb and Drucker — Prager type respectively. Yield function $F_s$ corresponding to shear hardening is defined as:

$$F_s = -\eta I_1 + \sqrt{J_2} / g(\theta) - k_1 = 0 \quad (12)$$

where $I_1$ is the first stress invariant, $J_2$ is the second stress deviator, $g(\theta)$ is the lode angle function, defined as:

$$g(\theta) = \frac{3 - \sin\varphi_{mob}}{2\sqrt{3}\cos\theta - 2\sin\theta\sin\varphi_{mob}} \quad (13)$$

$\eta$ is the deviatoric stress in $\pi$ plane when $\theta = 30°$, which controls the growth of shear yield surface with hardening parameter $W_s^{p*}$ and is related to mobilized angle of internal friction, namely:

$$\eta = \frac{2\sin\varphi_{mob}}{\sqrt{3}(3 - \sin\varphi_{mob})} \quad (14)$$

$\varphi_{mob}$ is mobilized angle of internal friction, and the expression of $\sin\varphi_{mob}$ is :

$$\sin\varphi_{mob} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{\sigma_v' - \sigma_h'}{\sigma_v' + \sigma_h'} = t / s = X_s - r \cdot \ln(s / p_a') \quad (15)$$

Plastic potential function $G_s$ corresponding to shear hardening is expressed as:

$$G_s = -\alpha' I_1 + \sqrt{J_2} - k_2 = 0 \quad (16)$$

For sand, $k_2$ and $k_1$ are all equal to zero. In plane stain conditions:

$$\alpha' = \frac{\tan\psi_{mob}}{\sqrt{9 + 12\tan^2\psi_{mob}}} \quad (17)$$

where $\psi_{mob}$ is the mobilized dilatancy angle of sand, and could be given by dilatation relation. Considering the effect of both shear hardening and volumetric hardening, the mobilized dilatancy angle can be expressed as:

$$\psi_{mob} = \arcsin\left[-\left(\mathrm{d}\varepsilon_{vol}^p\right)_s / \mathrm{d}\gamma^p\right] \quad (18)$$

where $\left(\mathrm{d}\varepsilon_{vol}^p\right)_s$ is plastic volumetric strain increment caused by shear hardening (=total plastic volumetric strain increment- plastic volumetric strain increment caused by volumetric hardening). According to Yasin and Tatsuoka (2000), the average dilatancy relation can be expressed as:

$$t/s = m\left[-\left(\mathrm{d}\varepsilon_{vol}^{p}\right)_{s} / \mathrm{d}\gamma^{p}\right] + c \tag{19}$$

where $m$ and $c$ are material constants, and $m = 0.636$ and $c = 0.635$ for Toyoura sand.

For volumetric hardening, the yield functions and plastic potential function have the same form, that is $F_c = G_c$. The volumetric yield function is derived from volumetric hardening function:

$$F_c = \beta\left(\frac{\sigma_1' + \sigma_3'}{2p_a'}\right)^{\alpha} + k_1 - W_c^{p*} / p_a' = 0 = G_c \tag{20}$$

For Toyoura sand, $\alpha = 0.7$, $\beta = 1.564\times10^{-4}$, $k_1 = -\beta\left[s_0 / p_a'\right]^{\alpha}$, $s_0$ is $s$ in initial state.

## MODEL VALIDATION

The proposed modified plastic work double-hardening constitutive model is applied into the numerical simulation of plane strain compression tests with different stress paths on Toyoura sand to validate the model. FIG. 7(a) and (b) show the comparisons between numerical simulation results and physical plane strain compression experimental results along different stress paths. It can be seen from FIG. 7(a) and (b) that results from numerical simulation are well accordance with those from physical tests. The double-hardening model proposed in this paper could not only simulate the effect of stress path properly, but also reflect the strength and deformation properties of sand reasonably.

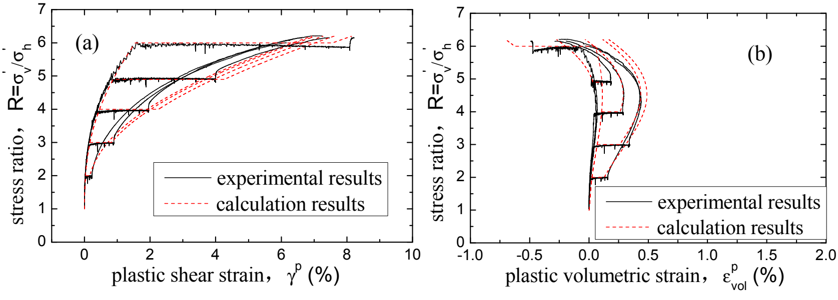


**FIG.7 Comparisons between calculation results and physical experimental results: (a) Relation between plastic shear strain and principal stress ratio; (b) Relation between plastic volumetric strain and principal stress ratio.**

## CONCLUSIONS

(1) A couple of hardening parameter and function respectively for shear and volumetric plastic strain for sandy soils is proposed, which is stress path-independent.
(2) An elastoplastic double-hardening constitutive model incorporating the above hardening parameter and function is formulated. By the comparisons between results from numerical calculation and those from physical plane strain compression tests, the proposed constitutive model is validated. It is shown that the proposed model can reasonably reflect the strength and deformation characteristics of sand as well as the effect of stress path.

## ACKNOWLEDGMENTS

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# Clay Subjected to Cyclic Loading: Constitutive Model and Time Homogenization Technique

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**ABSTRACT:** Numerical modeling of the behavior of geotechnical structures subjected to cyclic loading requires the consideration of two main aspects: the constitutive relationship and numerical strategy. The constitutive model has to be representative of the clay behavior, whereas the numerical strategy should be efficient in order to reduce the computation time. To simulate undrained triaxial cyclic tests on clay, a method of time homogenization is applied to a bounding surface plasticity model. This method of homogenization is based upon splitting time into two separate scales. The first scale relates to the period of cyclic loading and the second to the characteristic time of the material. Simulations of undrained triaxial cyclic tests on normally consolidated clay under one-way cyclic loading are carried out. The performance of time homogenization is numerically validated. The sensitivity of the time increment is also investigated.

## INTRODUCTION

Structures such as wind power plants, offshore installations, embankments, railways and tunnels are subjected to a large number of loading cycles. The phenomenon of fatigue in soils is identified and characterized (Andersen 2009), but design tools for these geotechnical problems are missing. Two main problems have to be considered.

First, the constitutive model has to be able to reproduce with good accuracy the cyclic behavior of soil. During the last decades, multi-surface models have been developed. Most of them belong to kinematic hardening plasticity theory (Mroz 1967) or to bounding surface plasticity (Dafalias and Popov 1975). The latter receives significant attention because of its simplicity and efficiency. This type of model is selected for this study and described in the following section.

Second, the conventional simulation of a structure subjected to a large number of cycles is time consuming. Thus different strategies have to be developed which could require only the simulation of a few cycles, the other cycles being treated differently according to the strategies considered (e.g. Wichtmann 2005).

In this paper, one of these strategies, called time homogenization, is selected. Time homogenization, as spatial homogenization, is derived from mathematical perturbation theory. Bensoussan et al. (1978) followed by Sanchez-Palencia (1980) introduced these methods for periodic structures. This paper presents the principles upon which time homogenization is based and the conclusions are applied to the outlined model in order to simulate undrained triaxial cyclic tests.

## CONSTITUTIVE MODEL

The constitutive model refers to the theory of bounding surface plasticity. The implementation of the model is based on the works of Dafalias and Herrmann (1986) and Manzari and Nour (1997). This model is a bounding surface version of the Modified Cam-Clay with Hooke's elasticity. It assumes two surfaces: the yield surface (called bounding surface) and the subyield surface (called loading surface). They are homothetic with the respect to a projection center. Figure 1a represents both surfaces and the projection center corresponds to the origin of the coordinates. The current stress point $c$ defines the loading surface. When the current stress point is on the loading surface, an "image" stress $\bar{c}$ is defined as the projection of the current stress $c$ on the bounding surface with respect to the projection center (see Fig. 1a).

The relation between image and current stresses is called the mapping rule. The flow rule is respected and the plasticity parameter is given by:

$$\lambda = \frac{1}{\overline{K}_P}\left(\frac{\partial F}{\partial \overline{\sigma}}\dot{\overline{\sigma}}\right) = \frac{1}{K_P}\left(\frac{\partial F}{\partial \overline{\sigma}}\dot{\sigma}\right) \tag{1}$$

where $F$ is the potential surface. The plastic modulus at the current stress point $K_P$ is a function of the plastic modulus at the image stress point $\overline{K}_P$ and of the distance between bounding and loading surfaces. Based on the relation proposed by Manzari and Nour (1997), the plastic modulus is given by:

$$K_P = \overline{K}_P + H_0 \cdot \frac{(1+e_0)}{\rho} \cdot {p'_{c0}}^3 \cdot (\beta - 1) \tag{2}$$

where $\beta$ is equal to the ratio $p'_{c0} / p'_{c_\sigma}$ (see Fig. 1a). When the current stress point $c$ is on the bounding surface ($\beta = 1$), Eq. (2) is reduced to the same form as the conventional Modified Cam-Clay model.

The constitutive model has the parameters of the Modified Cam-Clay model (Young's modulus $E$, Poisson's ratio $\nu$, critical state slope $M$, consolidation pressure $p'_{c0}$, $\rho = \lambda - \kappa$ with $\lambda$ slope of the normal consolidation line and $\kappa$ slope of the swelling line, and initial void ratio $e_0$), and one additional parameter, namely the hardening parameter $H_0$. Figure 1b represents the simulation of an undrained triaxial test on a normally consolidated clay under one-way cyclic loading in (p',q) plan. The values of the model parameters are given in Table 1 and $H_0$ is taken equal to 1000.

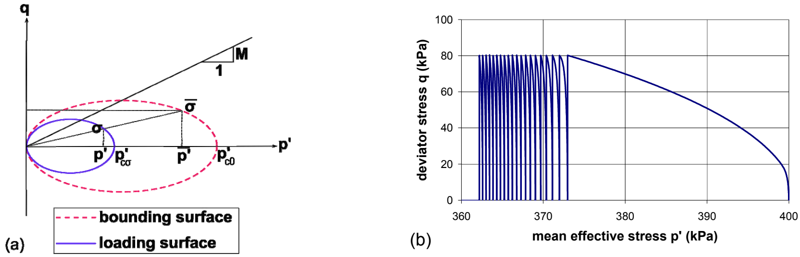


**FIG. 1. (a) Representation of bounding and loading surfaces in (p', q) plan. (b) Simulation of an undrained triaxial test on a normally consolidated clay under one-way cyclic loading in (p',q) plan.**

## TIME HOMOGENIZATION

Fatigue of a material subjected to cyclic loading evolves slowly in comparison to the duration of one single loading cycle. It cannot be detected by considering only few cycles, but by considering a large number of cycles. Therefore the evolution during one single load cycle needs to be described by an accurate time scale, whereas a coarser one is enough for the fatigue evolution.

The aim of time homogenization is to separate the effects that take place at the two time scales in order to use small time increments only when it is necessary and so reduce time computation. To do so, we define the time characteristic of the cycles $\tau_0$ (the period) and the time characteristic of the fatigue phenomenon $t_r$ which depends on material properties. On a graph representing the evolution of a variable related to fatigue (e.g. permanent strain or pore pressure) as a function of the time, $t_r$ can be defined as the time corresponding to the intersection of the line of the asymptotic behavior and the tangent at the origin. $\tau_0$ and $t_r$ can be characterized by two different units. The ratio of these units is noted $\zeta$ From this small positive ratio and the natural time scale describing the long-term fatigue, a second time scale describing the loading can be derived: $\tau = t/\zeta$ The time homogenization method, as it is explained further, consists in approximating displacements and stresses by using asymptotic expansions with respect to the ratio $\zeta$ Since we consider the first order of the asymptotic expansions, the smaller the ratio $\zeta$ the better the results.

Although the selected constitutive model is a time-independent model which does not involve any notions of time, we can still apply this method by considering a fictitious time related to the number of cycles. Mathematical proof is given by Guennouni (1988), who considers an elastoplastic problem as the limit of a series of elastoviscoplastic problems. As shown by Guennouni (1988) and Yu (2002), the homogenization method makes possible the separation of the two time scales so that the original initial-boundary value problem (Pb) can be divided into two problems (Pb*) and (Pb**).

$$
(\mathrm{Pb})\quad \begin{cases} \sigma_{ij,j} + b_i(x) = 0 \\ \sigma_{ij} = C_{ijkl} : (\varepsilon_{kl} - \varepsilon_{kl}^p) \\ \dot{\varepsilon}_{kl}^p = \lambda \dfrac{\partial F}{\partial \sigma_{kl}} \\ \dot{p}'_c = p'_c \dfrac{1+e}{\lambda - \kappa} \dot{\varepsilon}_v^P \end{cases} \quad \text{and} \quad \begin{cases} \varepsilon_{ij} = \dfrac{(u_{i,j} + u_{j,i})}{2} \\ u_i(x, t=0) = \tilde{u}_i(x) \\ u_i = \overline{u}_i(x,t) \\ \sigma_{ij} n_j = f_i(x,t) \end{cases} \tag{3}
$$

where $b_i$ corresponds to volumetric forces; $\sigma_{ij}$, $\varepsilon_{ij}$ and $\varepsilon^P_{ij}$ are respectively the components of stress, strain and plastic strain tensors; $C_{ijkl}$ the components of elastic tensor; $u_i$ the component of displacement vector. The displacement, stress, plastic strain and hardening variable are approximated by asymptotic expansions in the form of:

$$
\alpha^{\varsigma}(x,t) = \sum_{M=0,1,..} \varsigma^M \alpha_M(x,t,\tau) \tag{4}
$$

where $\alpha_M$ $\tau$-periodic. Using Eq. (3) and (4) and considering the first-order term of the asymptotic expansions, it appears that the plastic strains and the hardening variable are $\tau$-independent, so that their evolution is noticed only in the long term. Moreover, the global behavior of the material can be divided into the non-oscillatory long-term behavior and the oscillatory short-term behavior. The first part is defined as the average over a cycle of the first-order term of the asymptotic expansions; the second part as the remaining term.

$$
\begin{aligned} u_0(x,t,\tau) &= \langle u_0(x,t,\tau)\rangle + \chi_0(x,t,\tau) \\ \sigma_0(x,t,\tau) &= \langle \sigma_0(x,t,\tau)\rangle + \phi_0(x,t,\tau) \\ \varepsilon_0(x,t,\tau) &= \langle \varepsilon_0(x,t,\tau)\rangle + \psi_0(x,t,\tau) \end{aligned} \qquad \text{where } \langle \bullet \rangle = \frac{1}{\tau_0}\int_0^{\tau_0} \bullet \, d\tau \tag{5}
$$

The non-oscillatory and oscillatory terms are respectively the response fields of the problems (Pb*), called the macro-chronological problem, and (Pb**), called the micro-chronological problem. It is worth noting that (Pb**) is a linear elastic problem, which can be solved at the beginning of the calculation. The constitutive relation of (Pb*) is called the homogenized constitutive relation and depends on the behavior of the material. Eq. (6) and (7) give both problems (Pb*) and (Pb**) as follows:

$$
(\mathrm{Pb}^*)\quad \begin{cases} \langle \sigma_{0ij} \rangle_{,j} + b_i(x) = 0 \\ \langle \sigma_{0ij} \rangle_{,t} = C_{ijkl} : (\langle \varepsilon_{0kl} \rangle_{,t} - \varepsilon_{0kl,t}^p) \\ \varepsilon_{0kl,t}^p = \langle \lambda \dfrac{\partial F}{\partial \sigma_{0kl}} \rangle \\ p'_{c0,t} = p'_{c0} \dfrac{1+e}{\lambda-\kappa} \varepsilon_{0v,t}^P \end{cases} \quad \text{and} \quad \begin{cases} \langle u_{0i} \rangle (x, t=0) = \tilde{u}_i(x) \\ \langle u_{0i} \rangle = \langle \overline{u}_i(x,t,\tau) \rangle \\ \langle \sigma_{0ij} \rangle n_j = \langle f_i(x,t,\tau) \rangle \end{cases} \tag{6}
$$

$$
(\mathrm{Pb}^{**})\quad \begin{cases} \phi_{0ij,j}(x,t,\tau) = 0 \\ \phi_{0ij,\tau}(x,t,\tau) = C_{ijkl} : \psi_{0kl,\tau}(x,t,\tau) \end{cases} \quad \text{and} \quad \begin{cases} \chi_{0i}(x, t=\tau=0) = 0 \\ \chi_{0i}(x,t,\tau) = \overline{u}_i(x,t,\tau) - \langle \overline{u}_i(x,t,\tau) \rangle \\ \phi_{0ij} n_j = f_i(x,t,\tau) - \langle f_i(x,t,\tau) \rangle \end{cases} \tag{7}
$$

## NUMERICAL VALIDATION

The model with homogenization is implemented in a FORTRAN routine which is used to simulate undrained triaxial tests and to estimate the reduction of time computation. An undrained triaxial test on a normally consolidated clay under one-way cyclic loading (q = 80 kPa) is assumed for the numerical validation. The permanent strain is defined as the strain remaining after a cycle. The values of the model parameters corresponding to a kaolinite clay are summarized in Table 1. The first ten cycles are simulated conventionally because of the important rate of permanent strains at the beginning of the calculation (see Fig. 1b).

**Table 1. Model parameters.**

| E (kPa) | $\nu$ | M | $p'_{c0}$ (kPa) | $\kappa$ | $\lambda$ | $e_0$ | $H_0$ |
|---|---|---|---|---|---|---|---|
| 30000 | 0.3 | 0.7 | 400 | 0.04 | 0.19 | 1.5 | 1000; 3000;10000 |

The time increment corresponds to $N_c$ cycles. Simulations are carried out for three values of $N_c$: 50, 100 and 200. The homogenized calculation is about $N_c$ times faster than the conventional one. Figures 2(a-b) show the influence of $N_c$ over 2000 cycles. The maximal error for permanent strain (83%) due to homogenization appears for $N_c$ = 200 at the 210$^{th}$ cycle. However, the error is reduced to 14% after 2000 cycles and becomes less than 1.2% after 30000 cycles (see Fig. 2c).

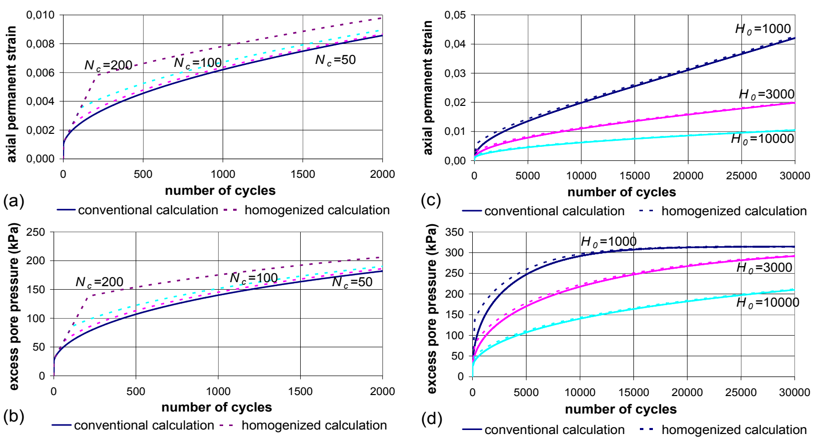


**FIG. 2. Comparison of conventional and homogenized calculations: (a) permanent strain / (b) excess pore pressure for $H_0$ = 1000 and $N_c$ = 50, 100, 200; (c) permanent strain / (d) excess pore pressure for $H_0$ = 1000, 3000, 10000 and $N_c$ = 200.**

Figures 2(c-d) compare the conventional and homogenized calculations with different values of the hardening parameter. The characteristic time $t_r$ depends on this parameter, as shown by the different curvature of the curves, and it explains why the calculation with $H_0 = 1000$ is at first the most affected by the homogenization strategy.

## CONCLUSIONS

The example of cyclic modeling outlined in this paper shows to what extent time homogenization can be used to efficiently and accurately model cyclic behavior of soils. When applied to a bounding surface plasticity model, this strategy can significantly improve computational efficiency and guarantees over a large number of cycles that the same results will be obtained as by conventional calculations. Particular attention should be paid to the choice of the time increment. The paper presents the numerical validation for the time homogenization technique applied to a bounding surface model. For further studies, it will be interesting to validate the approach by using real experimental testing results on soil samples as well as boundary value problems.

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**Modeling Anisotropic, Debonding and Viscous Behaviors of Natural Soft Clays**

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**ABSTRACT:** A new viscoplastic model is developed extended from overstress theory of Perzyna. A scaling function based on the experimental results of constant strain-rate oedometer tests is adopted, which allows viscoplastic strain-rate occurring wherever the stress state is inside or outside of the reference surface. The inherent and induced anisotropy is incorporated using the formulations of yield surface and its rotation of S-CLAY1. The bonds are progressively destroyed by plastic straining by incorporating the concept of an intrinsic yield surface. No additional test is needed for determining all model parameters compared to the Modified Cam Clay model, and the parameters determination is straightforward. The experimental verification is carried out with reference to the constant strain-rate and creep tests on two natural clays.

## INTRODUCTION

The soft and sensitive clay deposits formed after the Pleistocene period cover most of the densely populated low-lying coastal areas in the world. The construction on soft soil deposits has become increasingly important in last decades. A safe and economical design and construction on soft soils becomes an important issue.

The behavior of soft sensitive clay is very complicated. It exhibits several features: (a) a significant degree of anisotropy developed during their deposition, sedimentation, consolidation history and any subsequent straining; (b) some apparent bonding which will be progressively lost during straining; (c) time-dependent stress-strain relationship which has a significant influence on the shear strength and the preconsolidation pressure.

In order to describe all these features, Yin & Karstunen (2008) proposed an elasto-viscoplastic model based on overstress theory of Perzyna (1966) with an elastic region assumed. This hypothesis leads the present model to a less effort in parameter determination.

In this paper, we propose a new model extended from overstress theory of Perzyna (1966) with no elastic region exist in the stress space. The proposed model is based on the strain-rate effect on preconsolidation pressure. It is different from creep models (i.e., Leoni et al. 2008) based on the concept of instant and delayed compression of