

Figure 1. Geometry and notation.

Numerical analysis of the undrained uplift capacity of buried pipelines have generally considered an idealised clay, with uniform or linearly increasing shear strength (e.g. Ballard and Smith 2015; Martin and White 2012). The idealisation of clay as a linear elastic- or rigid-perfectly plastic material may not allow important physical characteristics to be captured. Natural clays often show inter-particle bonding, or structure, which can significantly affect soil response under load (Leroueil and Vaughan 1990) but cannot be simulated by simple elasto-plastic models. Furthermore, spatial variability is an inherent feature of soil deposits both on- and off-shore (e.g. Lacasse and Nadim 1996; Phoon and Kulhawy 1999). Probabilistic studies using random fields to represent spatial variability have shown that the mechanical behaviour and bearing capacity of offshore foundation types such as skirted foundations (Charlton and Rouainia *In press*) and spudcans (Li et al. 2016) in undrained clay can be substantially different from deterministic predictions.

This paper presents finite element analysis of the uplift capacity of a buried pipeline in undrained clay using an advanced critical state constitutive model, formulated within a framework of kinematic hardening, to capture the degradation of clay structure as the pipeline is loaded to its ultimate capacity. The disturbed clay backfill is modelled by a continuum approach, with a random field used to represent the spatial variability of clay structure around the pipeline. Statistics of the uplift capacity are computed by Monte Carlo simulation and a parametric study is undertaken to identify the effect of the coefficient of variation (COV) and autocorrelation length of clay structure on uplift behaviour.

#### **CONSTITUTIVE MODEL**

The rate-independent kinematic hardening structure model (KHSM) developed by Rouainia and Muir Wood (2000) is used to describe the effect of clay structure on pipeline uplift capacity. The KHSM extends the conventional Modified Cam Clay (MCC) model by including an initial amount of structure that is progressively destroyed until a fully remoulded state is reached, represented by the reference MCC yield surface. The KHSM introduces a kinematic hardening bubble and an outer structure surface to describe the state and effect of soil structure. The bubble, reference and structure surface have the same elliptical shape.

The essential feature of the KHSM is the degradation of structure in the soil as plastic strain accumulates. The degree of structure is firstly specified by the parameter  $r_0$ , which controls the initial size of the structure surface in relation to the reference surface ( $r_0 \ge 1$ ). The degradation of this initial structure is modelled by the following damage law:

$$r = 1 + (r_0 - 1)exp\left[\frac{-k\varepsilon_d}{(\lambda^* - \kappa^*)}\right]$$

where *r* is the current structure in the soil. The damage law is a monotonically decreasing function of a damage strain,  $\varepsilon_d$ , and the minimum value of *r* is 1 when structure and reference surfaces coincide. The parameter *k* controls the rate of structure degradation, while  $\lambda^*$  and  $\kappa^*$  are respectively the slope of the normal compression line and swelling line in a volumetric strain-logarithmic mean stress plot. Damage strain is calculated from the volumetric ( $\varepsilon_v^p$ ) and shear ( $\varepsilon_q^p$ ) components of plastic strain by a relationship between the strain rates:

$$\dot{\varepsilon}_{d} = \left[ (1 - A)\dot{\varepsilon}_{v}^{p2} + A\dot{\varepsilon}_{q}^{p2} \right]^{1/2}$$

where A is a dimensionless parameter that determines how  $\varepsilon_v^p$  and  $\varepsilon_q^p$  contribute to the damage strain and the degradation of structure in the soil.

# NORRKÖPING CLAY

The KHSM parameters are calibrated to triaxial test results from Norrköping clay, an inorganic clay of low sensitivity. The calibration procedure was undertaken and reported by Rouainia and Muir Wood (2000), where further details of the model parameters may be found. The calibrated parameters are given in Table 1. An effective unit weight of  $\gamma' = 8$ kN/m<sup>3</sup> is considered throughout and the initial stress conditions are generated by  $K_0$  consolidation, with  $K_0$  equal to 0.4489. The clay is normally consolidated as this is typical of seabed sediments and represents the conditions relevant in the reconsolidated backfill material in a pipeline trench.

Material property	Value
Slope of swelling line, $\kappa^*$	0.0297
Slope of normal compression line, $\lambda^*$	0.252
Poisson's ratio, v	0.29
Critical state stress ratio, M	1.35
Ratio of size of bubble and reference surface, R	0.145
Stiffness interpolation parameter, B	1.98
Stiffness interpolation exponent, $\psi$	1.547
Initial degree of structure, $r_0$	1.75
Destructuration strain parameter, A	0.494
Destructuration parameter, k	4.16

 Table 1. Calibrated soil parameters for Norrköping clay.

### **REPRESENTATION OF SPATIAL VARIABILITY**

The spatial variability of clay structure is considered by modelling the KHSM parameter  $r_0$  as a random field. The choice of probability density function (PDF) and autocorrelation structure of  $r_0$  is not straightforward as there is limited information available in the literature about the statistics of clay structure. For the analysis of the undrained capacity of a buried pipeline the effect of clay structure can be understood by its influence on the undrained shear strength,  $s_u$ . Figure 3 shows the results of simulated undrained triaxial compression tests on isotropically consolidated samples of Norrköping clay. The effect of increasing  $r_0$  is to increase the peak value of  $s_u$ . The sensitivity of the clay is also greater, as  $r_0$  does not affect the size of the MCC yield surface and so the remoulded  $s_u$ , simulated by setting  $r_0 = 1$ , remains the same. The monotonic relationship between  $r_0$  and  $s_u$  indicates that basing the statistics of  $r_0$  on those of the undrained shear strength is a reasonable choice.



Figure 2. Effect of  $r_{\theta}$  on the  $s_u$ -axial strain response in simulated undrained triaxial tests on Norrköping clay.

Lacasse and Nadim (1996) found that a lognormal distribution is appropriate for modelling  $s_u$  and Ching and Phoon (2012) also observed that several properties of structured clays, including peak and remoulded  $s_u$ , could be satisfactorily represented by a lognormal distribution. However, the minimum value for  $r_0$  is 1 so a shifted lognormal distribution is considered here:  $r_0 \sim ln \mathcal{N}(\alpha, \beta^2, \delta)$ . The cumulative distribution function (CDF), *F*, of the shifted lognormal distribution is:

$$F(r_0; \alpha, \beta, \delta) = \Phi\left(\frac{\ln(r_0 - \delta) - \alpha}{\beta}\right)$$

where  $\Phi$  is the Gaussian CDF with zero mean and unit variance. The first two distribution fitting parameters,  $\alpha$  and  $\beta$ , are respectively the mean and standard deviation of the natural logarithm of  $r_0$ . A third parameter  $\delta$  is introduced in order to move the lower bound of the distribution;  $\delta$  is taken to be 1 so that  $r_0$  cannot take values inadmissible in the KHSM.

The calibrated value of  $r_0$  is considered to be the mean value ( $\mu_{r_0} = 1.75$ ). This reflects the fact that the spatial variability of structure is inherent to the intact clay as well as the backfilled material. Keaveny et al. (1989) found that the autocorrelation,  $\rho$ , of  $s_u$  can be represented by either an exponential or square exponential function. In this study, a square exponential function is chosen with autocorrelation distances  $\theta_x$  and  $\theta_y$  in horizontal (x) and vertical (y) directions respectively.

The shifted lognormal random field of  $r_0$  can be generated as follows:

$$r_0(x, y) = \delta + \exp(\alpha + \beta G(x, y))$$

where G(x, y) is a correlated Gaussian random field of zero mean and unit variance and is discretised on a rectangular grid using the expansion optimal linear estimation (EOLE) method (Li and Der Kiureghian 1993). The shifted lognormal distribution is strongly non-Gaussian and the mapping from a Gaussian to non-Gaussian marginal distribution distorts the autocorrelation structure, particularly when the COV is high. Fortunately, the autocorrelation in the underlying Gaussian space can be found analytically in this case (Liu and Der Kiureghian 1986). The correlation distortion is such that the autocorrelation matrix of G(x, y),  $\Sigma^{G}$ , was not positive-semi definite in all analysis cases considered in this paper. This can introduce serious errors into the correlation and marginal distribution of the non-Gaussian random field, so a strategy of computing

#### FINITE ELEMENT MODEL

A plane strain finite element model (Plaxis 2012) is used to compute the uplift capacity pipeline of diameter D = 0.25m buried at a depth H = 0.875m. The dimensionless embedment ratio H/D = 3.5. This is chosen as being representative of a typical pipeline diameter and embedment. The finite element mesh is shown in Figure 6 and consists of 539 15-noded triangular elements, with the mesh refined in the region around the pipeline. Here, the pipe is assumed to have a rough surface, as might be the case if a concrete coating was applied, and the pipe and soil are attached with an infinite tensile capacity. The random field is generated on a fine stochastic mesh, separate from the finite element mesh, and values of  $r_0$  are mapped to the finite element Gauss points using linear shape functions.



### **DETERMINISTIC ANALYSIS**

Figure 4(a) shows the load-displacement curves for a uniform  $r_0$ . For structured clay ( $r_0 = 1.75$ ), a peak force is reached before softening occurs as plastic strains develop and structure is degraded. In contrast, in remoulded clay ( $r_0 = 1$ ) the load continues to increase until an ultimate value is reached. This ultimate force is equal to that of the structured clay at large displacements due to complete remoulding of the clay along the failure planes.

The failure mechanisms are shown in Figure 4(b) and in both cases a deep, flow-around mechanism forms. An exact solution has been derived for this mechanism, with  $N_c = Q_u / Ds_u =$  11.94 (Randolph and Houlsby 1984). Table 2 gives the capacity factors for both structured and remoulded clay. The reference undrained shear strength,  $s_{u,ref}$ , is taken at the centre of the pipe and calculated by simulating a plane strain compression test using the in-situ stress conditions. A good match is observed in the remoulded clay, with the value of  $N_c$  overestimated by less than 3%. In structured clay, the capacity factor is predicted to be 25% less than the exact value. However, the plasticity solution does not take into account degradation of structure, which reduces the capacity factor due to a progressive failure as plastic strains develop.



Figure 4. (a) Load-displacement curves for uniform  $r_{\theta}$  and (b) shadings of incremental displacements at 0.15m displacement.

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Case	$s_{u,ref}(kPa)$	$Q_u$ (kN/m)	$N_c$
Structured ( $r_0 = 1.75$ )	3.52	7.79	8.85
Remoulded ( $r_0 = 1.00$ )	2.42	7.42	12.26

# **EFFECT OF COV**

A parametric study was undertaken with  $\text{COV}_{r0} = \{0.1, 0.3, 0.5\}$  to cover the typical range of variability of  $s_u$  (Phoon and Kulhawy 1999). For these analyses,  $\theta_x$  and  $\theta_y$  are fixed at 1.5m (6D) and 0.25m (1D) respectively. Load-displacement plots from 500 simulations are presented in Figure 5 for  $\text{COV}_{r0} = 0.1$  and 0.5. Figure 5(a) shows that when  $\text{COV}_{r0}$  is low, the load-displacement responses are very similar to the deterministic analysis with a uniform  $r_0 = \mu_{r_0} = 1.75$ . In each case a peak load is reached before softening occurs and the load reduces to the remoulded value. It can be inferred that a flow-around mechanism governs the response.



Figure 5. Load-displacement curves for 500 simulations with (a)  $COV_{r0} = 0.1$  and (b)  $COV_{r0} = 0.5$ .

In contrast, as evident in Figure 5(b) when  $\text{COV}_{r0} = 0.5$  there is a much wider range of loaddisplacement responses. Importantly, the spatial distribution of clay structure can be seen to influence the type of failure. Figure 6 shows the failure mechanism from a simulation with  $\text{COV}_{r0} =$ 0.5; the corresponding load-displacement response is indicated in Figure 5(b). For this case, a reverse bearing capacity mechanism forms involving a block of soil being lifted upwards with the pipe rather than the flow-around mechanism that occurs when  $r_0$  is uniform or the spatial distribution has a low variability (e.g.  $\text{COV}_{r0} = 0.1$ ). Instead of post-peak softening behaviour, the load-displacement curve shows an increase to a constant ultimate load that, for this specific simulation, is higher than the peak force observed with  $r_0 = 1.75$ .



Figure 6. Failure mechanism from a simulation with  $COV_{r0} = 0.5$ .

The effect of  $\text{COV}_{r0}$  on the mean ultimate load ( $\mu_{Qu}$ ) is shown in Figure 7(a). Confidence intervals are computed by 10,000 bootstrap samples. When the variability of  $r_0$  is low,  $\mu_{Qu}$  is close to the deterministic analysis with a uniform r0. Further investigation is required to determine whether the higher  $\mu_{Qu}$  at  $\text{COV}_{r0} = 0.1$  relative to the uniform capacity is a result of a physical mechanism or model non-linearity, or can be attributed to statistical error. The effect of increasing  $\text{COV}_{r0}$  is to reduce  $\mu_{Qu}$ ; while there are cases of very high peak loads when  $\text{COV}_{r0} = 0.5$ , as discussed previously, they must occur fairly infrequently and the flow-around failure is the predominant mechanism. The extreme peak loads contribute to the increase in the COV of the ultimate load with  $\text{COV}_{r0}$ , as shown in Figure 7(b).



Figure 7. Effect of  $COV_{r0}$  on (a) Mean ultimate load,  $\mu_{Qu}$ , and (b) COV of ultimate load,  $COV_{Qu}$ .

#### EFFECT OF AUTOCORRELATION DISTANCE

A further parametric study was carried out to identify the effect of autocorrelation distance on pipeline uplift capacity. Correlation of soil parameters is generally anisotropic, with correlation in the horizontal distance often an order of magnitude greater than in the vertical direction (Lacasse and Nadim 1996). The parameter ranges investigated in this study are  $\theta_x = \{0.25m (1D), 0.75m (3D), 1.5m (6D), 2.5m (10D)\}$  and  $\theta_y = \{0.125m (0.5D), 0.25m (1D), 0.5m (2D), 1m (4D)\}$ . The horizontal autocorrelation distance is chosen to be similar to the width of a typical, steep-sided pipeline trench (DNV 2007). The default autocorrelation distances are  $\theta_x = 6D$  and  $\theta_y = D$ , while COV<sub>r0</sub> = 0.3. For all cases, 500 simulations were run to characterise the response.

The autocorrelation distance has less influence on  $\mu_{Qu}$  than  $\text{COV}_{r0}$ , but important effects were observed on the load-displacement behaviour. Figure 8 presents the load-displacement curves for the extreme values of  $\theta_x$  and  $\theta_y$ . From Figure 8(a) and (b) it can be seen that the increase in  $\theta_x$ from 1D to 10D leads to a greater spread in the load-displacement curves from the end of the elastic range to a total displacement of 0.15m (0.6D). The tendency of some curves to reach a plateau or show only limited softening behaviour is indicative of a reverse bearing capacity mechanism. As evident in Figure 8(c), these curves are present when the vertical correlation is short but for  $\theta_y = 4D$ , shown in Figure 8(d), the load-displacement behaviour shows a peak and subsequent softening with nearly all cases reducing to the remoulded capacity at large displacements.



Figure 8. Load-displacement curves for (a)  $\theta_x = 0.25m$  (1D), (b)  $\theta_x = 2.5m$  (10D), (c)  $\theta_y = 0.125m$  (0.5D) and (d)  $\theta_y = 1m$  (4D).

The effect of the autocorrelation distance on the failure characteristics of the pipeline can be further assessed by considering the displacement at which the peak load occurs, denoted  $d_{peak}$ . In Figure 9(a), the mean of  $d_{peak}$  ( $\mu_{d,peak}$ ) is shown as  $\theta_x$  is increased from 1D to 10D. With a longer autocorrelation distance in the x-direction,  $\mu_{d,peak}$  also increases suggesting that the reverse bearing

capacity mechanism becomes more frequent. On the other hand, Figure 9(b) shows that when  $\theta_y$  increases,  $\mu_{d,peak}$  is reduced, particularly when the vertical autocorrelation length is similar to the pipeline diameter. This reflects the load-displacement behaviour observed in Figure 8 and suggests that the flow-around failure becomes the principal mechanism. This conclusion is strengthened by the fact that, as shown in Figure 9(d), the COV of  $d_{peak}$  decreases with longer  $\theta_y$ , implying that the response is increasingly governed by one type of failure mechanism. In contrast, Figure 9(c) shows that a longer  $\theta_x$  results in higher COV<sub>d,peak</sub>, suggesting a range of possible failure modes involving flow-around of the clay or lifting of a block of soil with the pipe.



Figure 9. Effect of autocorrelation length on (a)-(b) mean displacement at peak load,  $\mu_{d,peak}$ , and (c)-(d) COV of the displacement at peak load,  $COV_{d,peak}$ .

### CONCLUSION

A study of pipeline uplift capacity in an undrained clay with spatially variable structure has been conducted. The effect of clay structure on mechanical behaviour was described using the KHSM, an advanced kinematic hardening constitutive model. The model has been implemented in a finite element code and coupled with a random field representation of the parameter  $r_0$ , which describes the initial degree of structure. In a deterministic analysis with a uniform spatial distribution of  $r_0$ , a flow-around failure mechanism formed and the load-displacement curve showed a peak load before softening. The failure mechanism was unaffected if the spatial variability of  $r_0$  is low but for higher COV<sub>r0</sub> failure could be influenced by the lifting of a block of soil with the pipe. With increasing COV<sub>r0</sub>, the mean of the ultimate load reduced but the variability increased as a result of the different mechanism. The study demonstrates that the spatial variability of clay structure has an important effect on the uplift behaviour of a pipeline.

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## Effect of Spatial Variability on the Earth Pressure of a Rigid Retaining Wall

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#### Abstract

Present study investigates the influence of spatial variability of soil properties on the lateral thrust on a rigid retaining wall in active condition. Friction angle of the soil is modelled as a lognormal anisotropic random field in two dimensions, using the Cholesky decomposition technique to study the effect of horizontal and vertical scale of fluctuation on the lateral thrust. The Monte Carlo simulation approach is used in association with FLAC<sup>3D</sup> to highlight the worst-case spatial variability configuration. The study revealed that deterministic analysis employing mean friction angle underestimates the lateral thrust on the wall, as compared to that obtained using a spatially variable soil. For satisfactory performance of a wall, a scaling factor of around 1.5 is required to be used, if lateral active thrust obtained from deterministic analysis is employed in the design calculations. It is concluded from the reliability analysis that between the two practically possible scenarios, high horizontal and vertical scales of fluctuation scenario exhibit higher lateral thrust.

# **INTRODUCTION**

Retaining walls are an essential part of almost all infrastructure projects, to support vertical or near vertical backfills. As the sectional dimensions of retaining wall are the function of lateral thrust on wall, so a precise estimation of thrust on wall is crucial step for deciding the sectional dimensions of retaining wall. Calculation of thrust on retaining wall has usually been done using Coulomb's or Rankine's theories of lateral earth pressures. These theories involve homogeneity of the soil as a primary assumption, which is not the case in actual field. In practice, characteristic value of the soil parameters are estimated by averaging the soil parameters of collected soil sample from few locations. A true estimate of the soil parameters, depends on the sample size, larger the size sample data, better is the estimate but collection of large size sample data involves ample amount of cost and time. To get the large amount of virtual sample data, random field modeling plays a good role without involving much cost and time. This well established method becomes helpful for geotechnical engineers due to its straightforwardness. These virtual data obtained from random field modeling, take into account many configurations of soil variability which might exist in the field. It assists geotechnical engineers to design the foundations of a structure for the worst possible soil configuration. Random field modeling serves as