Discussion on Calculation Method for Side Friction of Mixing Pile with Stiffness Core Under Subgrade

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ABSTRACT

Based on the load transfer mechanism about the bi-level model, this paper studies on the influence about frictional force in side-pile of cement soil plug which pre-stressed pipe pile is implanted into a large diameter bidirectional cement mixing pile in a concentric manner. On the basis of the analysis of soil-compacting direction, theory of cavity expansion has been adopted to analyze the disturbance on the side soil in the pile driving process of pre-stressed pipe pile in MPSC. According to the lower core pile, the radial stress of pile-soil contact surface before and after the complete occlusion of cement soil plug is calculated combining the shear strength of foundation soil after disturbance. The eigenvalues of lateral friction before and after complete occlusion of soil plug are adopted according to comparison between the length of soil plug and outer pile. Considering extruding densification of core pile, improvement coefficient is implied in modifying the ground soil and outer pile, the calculation Equation for lateral friction of MPSC has been developed.

Keywords: MPSC, bi-level model, cement soil plug, theory of cavity expansion, side friction of pile

As a country with a vast territory, China has complicated geological conditions and in the southeast coastal zone such as Zhejiang, Fujian, Guangdong and other areas, soft soil of marine sediment is deep. When dealing with the region, in order to develop economic and reasonable treatment for soft ground with advanced technology, the seniors in geotechnical engineering have already done a lot of in-depth researches. Long-short-pile, barrette, Y-shaped and X-shaped tube-sinking cast-in-situ pile and compound types, have been used for strengthening of deep soft foundation. As a typical new treatment form composite foundation, MPSC is paid more and more attention ,because of its characters about mechanism and load transfer. The MPSC (such as graph 1-2) described in the paper is a new compound pile in which pre-stressed pipe pile is implanted into a large diameter bidirectional cement mixing pile in a concentric manner by static pressure. The larger side surface area of mixing pile can be used to strengthen the shallow ground, and the implantation of pre-stressed pipe pile can enhance the endurance of single pile, and can transfer the load to deep ground, and enhance the transferring depth of additional stress, controlling the complex foundation settlement further.

Although geotechnical workers do a series of field tests and theoretical research on MPSC in the recent years, and get some results, MPSC is in the phase of popularization and application, and is mainly applied in the treatment section of expressway bridgehead. In the design calculation, the calculation formula for compound foundation of regular rigid piles is used usually, and it can not reflect the load transfer of MPSC. In this paper, the cement soil plug produced in the implantation process of core pile of MPSC and its compaction on soil around pile and the effect on lateral friction of pile have been used to analyze the lateral friction of MPSC, and the calculation formulas for lateral friction of MPSC before and after the complete occlusion of soil plug have been developed.

Fig.1: The Rigid-Flexible Pile

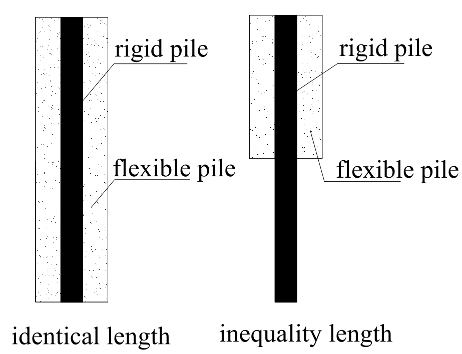

Fig.2: Sketch of the Rigid-Flexible Pile

Study on the Load Transfer Mechanism of MPSC

The study of DONG suggests that when the reinforced mixing pile receives upside load, the load transfer appears “bi-level model”. The composition of single pile of MPSC is similar to reinforced mixing pile, but the length of core pile of MPSC is much longer than reinforced mixing pile, and the load-bearing mechanisms are different. Reinforced mixing pile is cement-soil mixing pile with reinforced module and strength of pile, and is a kind of friction pile or end-support friction pile, but the MPSC is more like a friction and end-bearing pile.

Bi-level model (such as graph 3) can be used in the load transfer of MPSC: first, the load is concentrated to the compound pile by the pile-soil deformation, and the core pile undertakes most of the load. When the load is transferred along the core pile down, in the scope of L_1, the length of outer cement-soil mixing pile, it is transferred to the outer cement-soil pile by lateral friction; in the length between L_1 (length of outer cement-soil mixing pile) and L_2 (length of core pile), the load is transferred to soil between piles by lateral friction and end resistance. This is the first level of load diffusion model of MPSC. In the length of L_1 outer pile, the upside load directly borne by cement-soil mixing pile and the load transferred from core pile can be diffused to the soil between piles by lateral friction and end resistance of pile. Considering that the cement-soil mixing pile is a flexible pile and the full exertion of lateral friction, the end resistance of pile is little, and can be used for safety margin, not to be considered and the outer cement-soil mixing pile is regarded as a pure friction pile. This is the second level of load diffusion of MPSC.

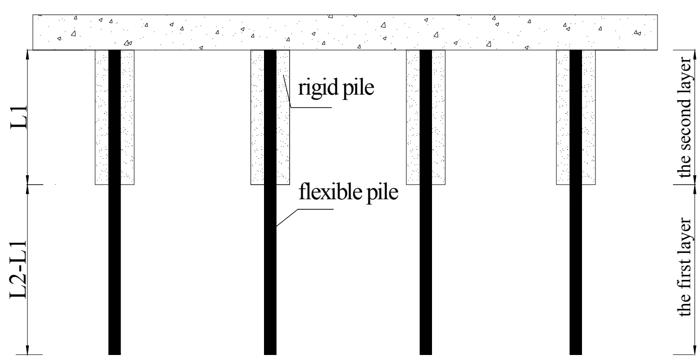

Fig.3: The Load Diffusion Bi-level Model of MPSC

Determination of Radial Stress at the Pile-Soil Contact Surface Before and After Complete Occlusion of Soil Plug

In the theory of cavity expansion, the soil is regarded as an elastic-perfectly plastic material and its bend complies with Mohr-Coulomb or Tresca rule, and elastic and plastic scope and stress displacement solution are given for the elastic area within radius of R in the penetration process of initiate stress σ_0 of pile, which can simulate the compaction effect in saturated cohesive soil well. In the implantation process of core pile, the soil around pile is extruded outside, and partial soil is extruded upward, but the majority is extruded outward in radial direction. Therefore, the expansion of cavity is selected to simulate the compaction effect of soil around pile.

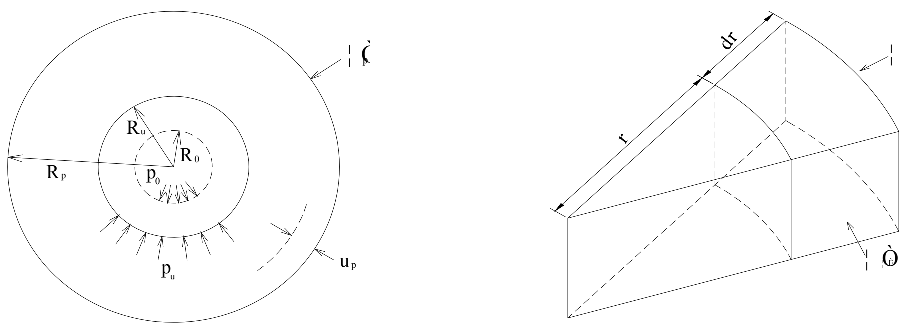

Fig. 4: Calculation Model for Soil in Cavity Expansion

Symmetric equilibrium differential equation of plane strain axis of cavity expansion in polar coordinates is:

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}+\frac{\sigma_r-\sigma_\theta}{r}=0 \qquad (1)$$

Geometric equation is:

$$\varepsilon_r=\frac{\mathrm{d}u_r}{\mathrm{d}r}\ \ \varepsilon_\theta=\frac{u_r}{r} \qquad (2)$$

Physics equations of elastic area are:

$$\varepsilon_r=\frac{1-\nu^2}{E}(\sigma_r-\frac{1-\nu}{\nu}\sigma_\theta)\ ,\ \varepsilon_\theta=\frac{1-\nu^2}{E}(\sigma_\theta-\frac{1-\nu}{\nu}\sigma_r) \qquad (3)$$

Select the stress function of axisymmetric problem in polar coordinates $\psi = A_0 \ln r + B_0 r^2$, the radial stress σ_r and toroidal stress σ_θ of elastic area are:

$$\sigma_r = \frac{1}{r}\frac{\mathrm{d}\psi}{\mathrm{d}r} = \frac{A_0}{r^2} + 2B_0 \quad , \sigma_\theta = \frac{\mathrm{d}^2\psi}{\mathrm{d}r^2} = -\frac{A_0}{r^2} + 2B_0 \tag{4}$$

Substitute the boundary conditions: $\sigma_r\big|_{r=R\mathrm{p}} = \sigma_{\mathrm{rp}}$; $\sigma_r\big|_{r\to\infty} = \sigma_0$, into Equation (4), $2B_0 = \sigma_0$, $A_0 = (\sigma_{rp} - \sigma_0)R_p^2$, can be gotten, and then

$$\sigma_r = (\sigma_{rp} - \sigma_0)\frac{R_p^2}{r^2} + \sigma_0 \quad , \sigma_\theta = -(\sigma_{rp} - \sigma_0)\frac{R_p^2}{r^2} + \sigma_0 \tag{5}$$

The radial displacement u_r of elastic area can be obtained from Equation (2) and Equation (3):

$$u_r = r\varepsilon_\theta = r\frac{1-\nu^2}{E}(\sigma_\theta - \frac{\nu}{1-\nu}\sigma_r) \tag{6}$$

The soil in the plastic area ($R_u \le r \le R_p$) complies with Mohr-Coulomb yield criterion, namely,

$$\sigma_\theta = \frac{\sigma_r(1-\sin\varphi)}{1+\sin\varphi} - \frac{2C\cos\varphi}{1+\sin\varphi} \tag{7}$$

Equation (7) is substituted into differential equation (1):

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \frac{2\sigma_r \sin\varphi}{r(1+\sin\varphi)} + \frac{2C\cos\varphi}{r(1+\sin\varphi)} = 0 \tag{8}$$

Make $A = \dfrac{2\sin\varphi}{1+\sin\varphi}$ and $B = \dfrac{2C\cos\varphi}{1+\sin\varphi}$, Equation (8) can be modified as:

$$\frac{d\sigma_r}{dr} + A\frac{\sigma_r}{r} + \frac{B}{r} = 0 \qquad (9)$$

With boundary condition: $\sigma_r|_{r=R_u} = \sigma_u$, the radial stress of plastic area can be obtained from Equation (9):

$$\sigma_r = (\sigma_u + \frac{B}{A})(\frac{r}{R_u})^A - \frac{B}{A} \qquad (10)$$

At the boundary of elastic and plastic areas ($r = R_u$), it can obtained from Equation (5) and Equation (7)

$$\sigma_{rR} = \sigma_0 + \sigma_0 \sin\varphi + C\cos\varphi \;\; \sigma_{\theta R} = \sigma_0 - \sigma_0 \sin\varphi - C\cos\varphi \qquad (11)$$

Substituted into Equation (10):

$$\sigma_0 + \sigma_0 \sin\varphi + C\cos\varphi = (\sigma_{ru} + \frac{B}{A})(\frac{r}{R_u})^A - \frac{B}{A} \qquad (12)$$

A and B are substituted, then the radial stress σ_{ru} at $r = R_u$, ultimate radius of cavity expansion, is:

$$\sigma_{ru} = (1+\sin\varphi)(\sigma_0 + C\cos\varphi)(\frac{R_u}{R_p})^{\frac{2\sin\varphi}{1+\sin\varphi}} - C\cot\varphi \qquad (13)$$

With the construction, soil plug is progressive development.In the process that incomplete occlusion of soil plug becomes complete occlusion, the degrees of compaction on soil around pile are different, namely, the volumes of cavity expansion are different. Provided that when the piling length is L_a, the incomplete occlusion of soil plug becomes complete occlusion, ZHENG and et al. assume when the length of pile driving $L < L_a$, the extruding soil volume of pile driving is equal to the total of volume change of soil around pile and volume of soil plug, namely, the radius before cavity expansion is the internal radius R_0 of open pipe pile, and the radius after cavity expansion is the external radius R_u of open pipe pile, so the following Equation can be obtained:

$$\pi R_u^2 L - \pi R_0^2 L = \pi R_p^2 L - \pi(R_p - u_p)^2 L + \pi R_u'^2 h \qquad (14)$$

Where, L—length of pile driving of open pipe pile; h—height of soil plug before complete occlusion, R_u' —radius of soil plug, and because $R_u' = R_0$, and when the high order term u_p^2 is omitted, the Equation (14) can be modified as

$$\frac{R_u^2}{R_p^2} = \frac{L}{L-(L+h)m^2} \cdot \frac{2u_p}{R_p} \qquad (15)$$

Where $m = R_0 / R_u$, namely the ratio of internal radius and external radius of pre-stressed pipe pile.

When the soil plug becomes complete occlusion ($L \geq L\text{a}$), the compaction effect of open pipe pile become similar to that of closed pipe pile, and the initiate radius of cavity expansion is 0, and the terminal radius of cavity expansion is the external radius of pipe pile, and provided that the volume of cavity expansion is equal to the total of volume change of elastic area and volume change of plastic area, it can be obtained that:

$$\pi R_u^2 = \pi R_p^2 - \pi(R_p - u_p)^2 + \pi(R_p^2 - R_u^2)\Delta \qquad (16)$$

Where Δ=strain of average volume of plastic area. Simply Equation (16), and both sides of equation are divided with R_{p}^2, and the high order term u_p^2 is omitted, obtaining

$$\frac{R_u^2}{R_p^2} = 2\frac{u_p}{R_p} + (1 - \frac{R_u^2}{R_p^2})\Delta \qquad (17)$$

The u_p can be obtained from Equation (6) and (17) as

$$u_p = R_p \frac{1-\nu^2}{E}[\sigma_0 - \sigma_0 \sin\varphi - C\cos\varphi - \frac{\nu}{1-\nu}(\sigma_0 + \sigma_0 \sin\varphi + C\cos\varphi)] \qquad (18)$$

namely,

$$u_p = R_p \frac{1+\nu}{E}[(1-2\nu)\sigma_0 - \sigma_0 \sin\varphi - C\cos\varphi] \qquad (19)$$

Substitute Equation (19) into Equation (15) and (17) to obtain R_u^2 / R_p^2 , and substitute it into (13), the ultimate radius of cavity expansion σ_{ru} is:

$$\sigma_{ru} = (1+\sin\varphi)(\sigma_0 + C\cos\varphi)\{\frac{2L}{L-(L+h)m^2}\cdot\frac{1+\nu}{E} \cdot[(1-2\nu)\sigma_0 - \sigma_0\sin\varphi - C\cos\varphi]\}^{\frac{\sin\varphi}{1+\sin\varphi}} - C\cot\varphi \quad (L < L_a) \quad (20)$$

$$\sigma_{ru} = (1+\sin\varphi)(\sigma_0 + C\cos\varphi)\{\frac{2\frac{1+\nu}{E}[(1-2\nu)\sigma_0 - \sigma_0\sin\varphi - C\cos\varphi] + \Delta}{1+\Delta}\}^{\frac{\sin\varphi}{1+\sin\varphi}} - C\cot\varphi \quad (L \geq L_a) \quad (21)$$

Calculation Formula for Lateral Friction of MPSC

The radial pressures of side soil on the external wall of extruding pile when soil plug is transmitted from incomplete soil plug to complete soil plug has been given in Equation (20) and (21), considering the cavity expansion of soil plug in pile driving process. The eigenvalue of lateral friction q'_{si} is calculated with the soil shear strength at pile-soil contact surface:

$$q'_{si} = C + \sigma_{ru}\tan\varphi \quad (22)$$

Where: σ_{ru} — normal stress of outer surface of soil at pile-soil interface on pre-stressed pipe pile, namely, the radial stress at ultimate radius of cavity expansion, calculated from Equation (20) and (21); the implantation of core pile of MPSC has a compaction effect on the outer pile, and improvement coefficient ξ is used to calculate the lateral friction of outer pile, and if the radius of core pile is bigger and the length of soil plug is shorter, the value of ξ is bigger. When calculating the lateral friction of lower core pile of compound pile and soil around pile, the eigenvalues of lateral friction q'_{si} before and after complete occlusion of soil plug are adopted according the comparison of length of soil plug and outer pile.

The calculation Equation for lateral friction of MPSC can be expressed as:

$$Q = u_1 \sum_{0 \le l \le L_1} \xi q_{si} l_i + u_2 \sum_{L_1 \le l \le L_2} q'_{si} l_i \quad (23)$$

Where u_1 and u_2 — the circumferences of sections of cement-soil mixing pile and pre-stressed pipe pile; L_1 and L_2 — the lengths of cement-soil mixing pile and pre-stressed pipe pile; ξ —the improvement coefficient of lateral friction of cement-soil mixing pile, considering the compaction effect on cement-soil mixing pile in the driving process of pre-stressed pipe pile, and ξ=1.05 ~ 1.10; q_{si} —the lateral friction of ground soil of level i and outer pile at the depth of $0 \sim L_1$; q'_{si} —the lateral friction of ground soil of level i and core pile at the depth of $L_1 \sim L_2$, calculated from Equation (22) in the paper.

Through the site measurement of MPSC in some highway in Zhejiang, the height of soil plug formed by core pile of 25m is 2 ~ 5m, shorter than 10m, the length of outer pile. Preliminary analysis suggested that it was caused by the instant insertion by dead-weight of core pile within 2 hours after the completion of outer pile when the strength of pile was very low. In this case, the determination of the eigenvalue of lateral friction q'_{si} was conducted according to the condition of the complete occlusion of soil plug.

CONCLUSION

According to the character about combination form of MPSC, the paper gives the load transfer and diffusion bi-level model, and discusses the load transfer and working mechanism on each layer foundation of the bi-level model;

The cement soil plug will affect the disturbance and compaction of side soil. The radial stresses of pile-sole contact surface before and after complete occlusion of cement soil plug have been calculated, to indicate the different effects on side soil.

The eigenvalue of lateral friction has been determined by adopting the shearing strength. Considering the compaction effect of core pile implantation on outer side, calculation Equation for lateral friction of MPSC has been proposed.

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