

groundwater-surface water interaction, and the migration of chemical contaminants. In such cases, the simulation model is executed repeatedly to achieve an objective such as a specified recharge to or discharge from an aquifer. Optimal management alternatives are unlikely to be found using only simulation models. Rather, a need exists for a simulation model combined with an optimization model (S/O), one that considers both surface water and groundwater system behavior and determines the best operating policy given prescribed objectives and restrictions.

Simulation models can represent the details and interrelationships of the hydrologic cycle. Linear and non-linear programming techniques can be used to supplement simulation models to optimize either discharge or recharge, or both. This is accomplished by using common software to quickly estimate a response to a hydrologic stress in a given flow field using data from the calibrated simulation model.

S/O model setup and execution are affected by the degree of linearization of the hydrologic response. In strictly confined aquifers, the saturated thickness does not vary with pumping or recharge; hence, the head response varies linearly with either, making linear programming attractive. In contrast, in unconfined aquifers, the saturated thickness and transmissivity do change with pumping or recharge. Therefore, the head in an unconfined and layered aquifer varies non-linearly with hydraulic stress. Although non-linear programming models are available, linear programming can still be used within specific limitations (Maddock, 1974; Lall and Lin, 1991). This also applies to situations where there are hydraulic interactions between the pumping or recharge locations. In these cases, the principle of superposition may be used through unit response matrices. If the drawdown or mounding is small relative to the initial saturated thickness (at least an order of magnitude less), the head response is nearly linear. Thus linear programming gives an excellent first approximation.

The following case study demonstrates how linear and non-linear programming (optimization) was used as a highly efficient, condensed simulation tool to evaluate complex management problems in an unconfined aquifer.

### **Case Study**

The focus of this groundwater modeling and optimization effort was to evaluate means for maintaining a sustainable water supply to meet the present as well as future needs of east-central Palm Beach County. By coupling groundwater flow simulations and linear and non-linear programming techniques, an optimization matrix was developed and used to optimize groundwater discharge and recharge. To develop this matrix, an integrated hydrologic (surface water-groundwater flow) model in east-central Palm Beach County (CDM, 1998) was used to simulate water table drawdown at designated monitoring points around seven

regional wellfields. This existing model was composed of the USGS MODFLOW groundwater model coupled with the surface water WETLANDS package (Restrepo and Montoya, 1997). Flow simulations were conducted to develop a relationship between hydraulic stresses and the responses at these monitoring locations. From this, a simple relationship was developed between pumpage at each wellfield and area drawdown. Using regression analysis, linear and/or non-linear relationships were developed for mounding due to loading of a recharge area.

## Development Of Optimization Tool

### *Unit Response Matrix*

Model simulations were conducted to obtain a relationship of wellfield pumpage to drawdown and applied recharge to mounding at 95 monitoring points placed throughout the model area. These monitoring points were located near existing and proposed wellfields and surrounding lakes, wetlands, and landfills within the model area. **Figure 1** indicates monitoring locations as well as wellfields modeled in the study.

The cumulative effect of pumpage at various wellfields on each monitoring point was then computed. The output heads from the pumpage simulations were then compared to the output heads from a base simulation where no pumpage and only background recharge was simulated. The drawdown at each control point at any pumping rate due to pumpage from a given wellfield was calculated by determining the relationship between drawdown for each control point to the pumping rate at an individual wellfield. This was accomplished by running a base simulation with no pumping from any wellfields and then running maximum pumping simulations for each of the wellfields represented in the optimization matrix. The maximum pumping rate for each wellfield was usually defined as the wellfield maximum pumping capacity. Thus, the resulting equation is valid through the range of zero to the maximum capacity of each wellfield. Assuming a linear relationship, the relative drawdown at each monitoring location due to a given wellfield was calculated by Equation 1.

$$RD_{ki} \text{ (ft/MGD)} = [h(SIM0)_i - h(SIMk)_i] / [P(SIMk) - P(SIM0)] \quad \text{(Equation 1)}$$

Where,

$SIMk$  = the groundwater flow simulation of pumpage from wellfield  $k$ .

$SIM0$  = the groundwater flow simulation of background conditions, with no pumpage from any wellfields, and background recharge.

$h(SIMk)_i$  = the groundwater elevation in feet NGVD at monitoring location  $i$  due to maximum pumpage from wellfield  $k$ .

$h(SIM0)_i$  = the groundwater elevation in feet NGVD at monitoring location  $i$  under background conditions.

$P(SIMk)$  = the simulated maximum pumpage rate from wellfield  $k$  (MGD).

$P(SIM0)$  = the simulated pumpage rate under background conditions = 0 MGD.

$RD_{ki}$  = the relative drawdown in feet at monitoring location  $i$  due to pumping from wellfield  $k$  (feet/MGD).

Similarly, the mounding response of each monitoring location due to applied recharge loading at individual recharge areas was utilized to develop a relationship of mounding to loading. Recharge areas evaluated included the proposed recharge areas to the Wetland Based Water Reclamation Project (WBWRP), including the Wetland Reuse Site and the Standby Wellfield, and area lakes impacted by pumping. The cumulative effect of loading at these recharge areas on each monitoring point was computed by comparing the output heads from the recharge simulations to the output heads from the base simulation. By determining the relationship between mounding for each control point to the loading rate at a recharge area, the mounding at that control point at any loading rate from that recharge area was calculated. Due to inherent non-linear nature of surface water relationships, regression analysis was used to determine the relationship of mounding due to loading of a recharge area. An illustration of the type of non-linear relationship developed for mounding response due to loading at certain monitoring locations within the study area is presented in **Figure 2**. The overall analysis of mounding response due to loading yielded both linear and non-linear relationships. Depending upon the linearity of the relationship developed, the relative mounding at each monitoring location due to a given loading and a given recharge area was calculated by one of the following equations.

$$RM_{ri} \text{ (ft/MGD)} = (\Delta h / \Delta L) \quad \text{(Equation 2)}$$

For a linear relationship:

$$\Delta h = [h(SIMr)_i - h(SIM0)]$$

$$\Delta L = [L(SIMr) - L(SIM0)]$$

$$RM_{ri} \text{ (ft/MGD)} = [h(SIMr)_i - h(SIM0)] / [L(SIMr) - L(SIM0)] \quad \text{(Equation 2A)}$$

For a non-linear relationship:

$$\Delta h = [a L(SIMr)^2 + b L(SIMr)]$$

$$\Delta L = [L(SIMr) - L(SIM0)]$$

$$RM_{ri} \text{ (ft/MGD)} = [a L(SIMr)^2 + b L(SIMr)] / [L(SIMr) - L(SIM0)] \quad \text{(Equation 2B)}$$

Where,

$\Delta h$  = Change in head (feet).

$\Delta L$  = Change in loading rate (MGD).

$SIMr$  = the groundwater flow simulation of recharge from recharge area  $r$ .

- $SIM0$  = the groundwater flow simulation of background conditions, with no pumpage from any wellfields, and background recharge.  
 $h(SIMr)_i$  = the groundwater elevation in feet NGVD at monitoring location  $i$  due to loading of recharge area  $r$ .  
 $h(SIM0)_i$  = the groundwater elevation in feet NGVD at monitoring location  $i$  under background conditions.  
 $L(SIMr)$  = the simulated loading rate from recharge area  $r$  in MGD.  
 $L(SIM0)$  = the simulated loading rate under background conditions = 0 MGD.  
 $a$  = regression analysis coefficient.  
 $b$  = regression analysis coefficient.  
 $RM_{ri}$  = the relative mounding in feet at monitoring location  $i$  due to recharge from area  $r$  (feet/MGD).

These equations were developed to be valid over the range of zero loading to a predetermined maximum loading (usually the physical limits of the system).

#### ***Development of Spreadsheet Optimization Model***

Utilizing the drawdown and mounding relationships described by Equation 1 and Equation 2A and/or 2B, an optimization model was developed. The model was developed utilizing the Solver package in Microsoft EXCEL. The optimization matrix utilizes the relative drawdown,  $RD_{ki}$  equations developed at each monitoring point plus the  $RM_{ri}$  equations to optimize the total pumping rate within the model area. The objective of the optimization solver is to maximize the sum of all wellfield-pumping rates. The total drawdown of each control point is determined for any number of simulations by the following equation.

$$S_i = [RD_{ki} \times P_k + RD_{(k+1)i} \times P_{k+1} + \dots + RD_{k+n} \times P_{k+n}] - [RM_{ri} \times L_r + RM_{(r+1)i} \times L_{r+1} + \dots + RM_{r+n} \times L_{r+n}]$$

$$S_i = \sum_{k=1}^n RD_{ki} \times P_k - \sum_{r=1}^m RM_{ri} \times L_r \quad (\text{Equation 3})$$

Where,

- $S_i$  = total drawdown at monitoring location  $i$ , feet.  
 $RD_{ki}$  = the relative drawdown in feet at monitoring location  $i$  due to pumping from wellfield  $k$ , feet/MGD.  
 $P_k$  = pumping rate from wellfield  $k$ , MGD.  
 $n$  = number of wellfields.  
 $RM_{ri}$  = the relative mounding in feet at monitoring location  $i$  due to recharge from area  $r$ , feet/MGD.  
 $L_r$  = loading rate for recharge area  $r$ , MGD.  
 $m$  = number of recharge areas.

**Table 1** illustrates the general structure of the optimization model and is not representative of a particular scenario. In this case, the optimization solution is the total regional pumpage. Changing the variable cells, which correspond to wellfield pumpage at each wellfield, optimizes this value. The constraint values are the limits imposed on the individual pumping rates (minimum and maximum) and the total drawdown. In general, total drawdown constraint limits were set at a maximum of 1 foot of total drawdown in wetland areas, and underneath landfills and a maximum of 2 feet at area lakes. Wellfield pumping constraints corresponded to a maximum of wellfield capacity, and in some scenarios a minimum of existing allocation. Constraint values were also developed for maximum heads due to mounding at lakes and wetlands. Wetland water elevations were constrained at a maximum of 8 inches above the seasonal normal elevation. Maximum water elevations at lakes were constrained to the lake normal pool elevation.

#### *Verification of Optimization Model*

Verification of the optimization matrix was completed by comparing the heads predicted by the matrix for a given pumping and recharge scenario to the output heads from the integrated surface water/groundwater model under the same conditions. Head values were recorded at each control point for various groundwater simulations and optimization runs. A plot of the computed heads from the optimization matrix versus the computed heads from the groundwater model is presented in **Figure 3**. A total of 475 data points were used to verify the accuracy of the optimization matrix. A mean residual of 0.2 feet with a standard deviation of 0.6 feet was computed.

#### **Results**

Two series of optimization runs were performed to evaluate the maximum pumpage rates for multiple wellfields while being constrained to the minimum drawdown levels criteria established by a local regulatory agency. Four optimized solutions are presented. The first series of optimization runs compared the optimal regional pumpage while constraining the minimum pumpage rates at each wellfield to their current allocations. Two scenarios were evaluated for this run. The first scenario determined the maximum allowable regional pumpage that met the prescribed drawdown constraint criteria without initiating a recharge program. The second scenario evaluated the impact of initiating a recharge program on the maximum optimal regional pumpage. As illustrated by **Figure 4**, regional wellfield pumpage could be increased by 23 percent through the initiation of a recharge program. A feasible optimal solution could not be found without a recharge program.

The second series of optimization runs consisted of eliminating the minimum pumpage criteria of current allocation at each wellfield, allowing the model to find a fully optimized solution. The first run in this series determined the

maximum optimal regional wellfield pumpage without a recharge program. As illustrated on Figure 4, regional allowable pumpage was increased by 22 percent even without a recharge program. With the initiation of a recharge program-and relaxing the minimum pumpage criterion, regional pumpage could be increased by 61 percent. The execution time for each optimization simulation in the Excel spreadsheet was 10 seconds. This is compared to the average run-time of the integrated hydrologic model, which was 3 hours per simulation due to the large size of the model. In relative terms the spreadsheet optimization matrix was over 1000 times faster than running the groundwater flow model.

### Summary and Conclusions

The development of a simulation model is a necessary step in any modern analysis of a groundwater management problem. However, simulation alone is often not enough because the problems of aquifer management do not involve prediction alone. Rather, they involve both simulations for prediction- and optimization to develop the best operating policy for a particular objective taking into account those restrictions that exist on a project-specific basis.

As this analysis shows, the optimization procedure outlined provides for quick, efficient, comparative analyses of simulations. The advantages and uses of this approach in integrated resource management are numerous. The limitations of this approach are imposed by the complexity of actual surface water - groundwater interaction and associated non-linearities.

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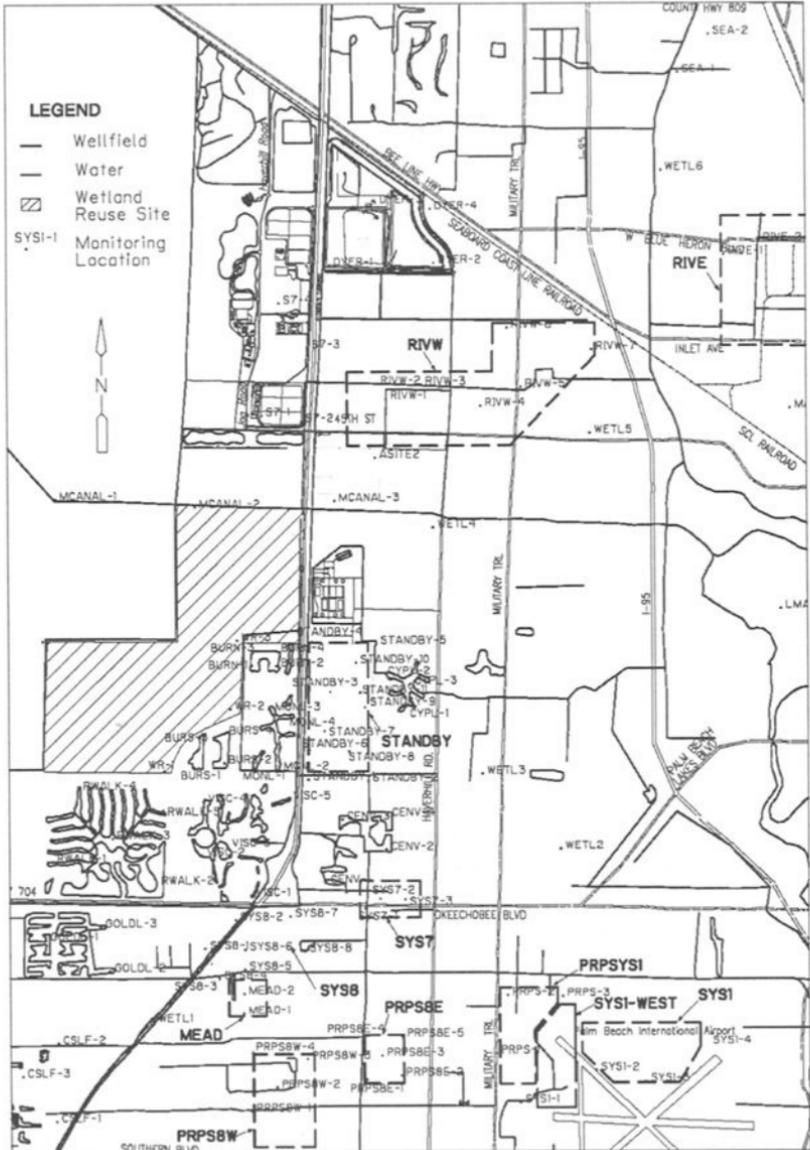


Figure No. 1 Study Area

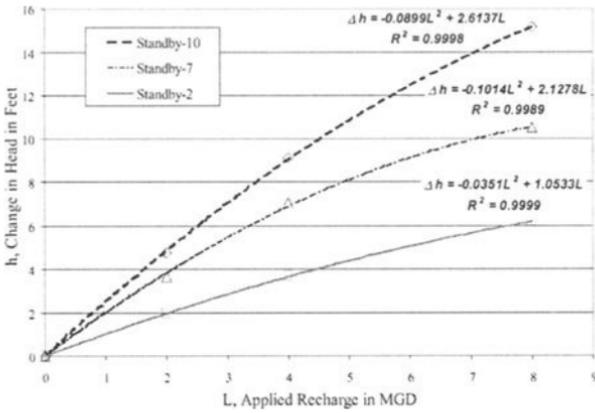
Table No. 1  
Optimization Matrix

Constraint Location	Feet Drawdown per MGD Pumping				Feet Mounding per MGD Applied Recharge				Total Drawdown		
	SYS1 (ft/MGD)	SYS7 (ft/MGD)	SYS8 (ft/MGD)	Standby (ft/MGD)	DHN	Standby	Century Village	Vista Center	Constraint Value (feet)	Constraint Limit (feet)	Limiting Foot Feet
CENV-2	0.0265	0.3328	0.1104	0.2342	0.0148	0.2066	1.6866	0.1079	0.35	2.00	1.65
CENV-3	0.0186	0.2451	0.1135	0.3221	0.0209	0.2951	1.5400	0.1693	-0.70	2.00	2.51
CENV-4	0.0233	0.2589	0.0638	0.3035	0.0174	0.2794	1.6422	0.1058	-0.70	2.00	2.70
CSLF-1	0.0008	0.0061	0.0229	0.0021	0.0003	0.0021	0.0070	0.0447	0.51	1.00	0.49
CYPL-1	0.0032	0.0364	0.0202	0.6602	0.0330	0.5074	0.1832	0.0477	-3.75	1.00	4.75
CYPL-2	0.0001	0.0007	0.0050	0.8810	0.0374	0.4879	0.1270	0.0322	-4.07	1.00	5.07
CYPL-3	0.0001	0.0065	0.0085	0.5966	0.0313	0.4292	0.1304	0.0305	-3.30	1.00	4.30
BURGS-1	0.0035	0.0271	0.0602	0.2739	0.0671	0.1463	0.1258	0.2666	-0.80	2.00	2.80
BURGS-2	0.0036	0.0327	0.0638	0.3657	0.0732	0.2078	0.1661	0.2939	-1.32	2.00	3.32
BURGS-3	0.0035	0.0250	0.0529	0.2958	0.0769	0.1512	0.1188	0.2265	-1.05	2.00	3.05
BURGS-4	0.0031	0.0265	0.0509	0.4279	0.0939	0.2302	0.1514	0.2359	-1.89	2.00	3.89
MONT-5	0.0043	0.0489	0.0724	0.5057	0.0659	0.3263	0.2521	0.3461	-2.08	2.00	4.08
MONT-6	0.0050	0.0618	0.0690	0.8674	0.0667	0.4663	0.3012	0.2931	-3.14	2.00	5.14
BURGN-9	0.0025	0.0145	0.0128	0.7687	0.2209	0.3448	0.0858	0.0714	-4.51	2.00	6.51
BURGN-12	0.0022	0.0115	0.0118	0.8520	0.1943	0.4283	0.0964	0.0718	-4.82	2.00	6.82
RWALK-1	0.0020	0.0149	0.0549	0.0281	0.0068	0.0139	0.0274	0.0647	0.96	2.00	1.02
RWALK-3	0.0030	0.0230	0.0745	0.0599	0.0143	0.0307	0.0521	0.1380	1.11	2.00	0.89
STANDBY-4	0.0012	0.0101	0.0085	1.2848	0.1469	0.6823	0.0939	0.0508	-6.25	1.00	7.25
STANDBY-5	0.0037	0.0614	0.0585	0.9258	0.0622	0.8750	0.3643	0.2170	-7.27	1.00	8.27
...	...	...	...	...	...	...	...	...	...	...	...
VISC-4	0.0011	0.0106	0.1003	0.1820	0.0346	0.1206	0.1736	0.6544	-0.15	1.00	1.15
VISC-5	0.0007	0.0266	0.0991	0.3745	0.0371	0.3129	0.4567	0.7605	-1.99	1.00	2.99
WETL2	0.0564	0.1044	0.0405	0.9848	0.0058	0.0573	0.1056	0.0206	0.77	1.00	0.23
WETL3	0.0228	0.1149	0.0420	0.2592	0.0143	0.1857	0.2165	0.0383	-0.47	1.00	1.47
WETL4	0.0068	0.0039	0.0018	0.1873	0.0238	0.0786	0.0173	0.0053	0.80	1.00	0.20
WR-1	0.0033	0.0219	0.0595	0.1717	0.0450	0.0866	0.0909	0.2171	-0.09	1.00	1.09
WR-2	0.0021	0.0143	0.0266	0.5300	0.1384	0.2481	0.1221	0.1533	-2.86	1.00	3.86
WR-3	0.0009	0.0038	0.0052	0.5246	0.3120	0.1984	0.0515	0.0472	-4.48	1.00	5.48
WR-5	0.0000	0.0000	0.0000	0.1910	0.0903	0.0656	0.0119	0.0061	-0.73	1.00	1.73

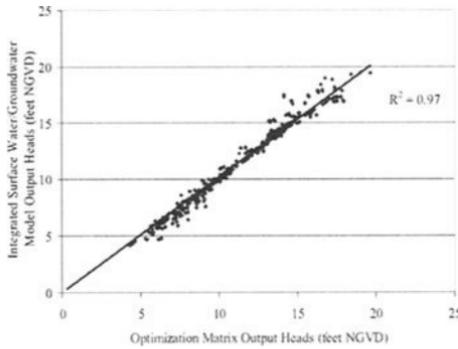
Wellfield	Minimum and Maximum Pumpage Constraints (MGD)				Minimum and Maximum Recharge Constraints (MGD)			
	MIN	MAX	SYS7	SYS8	DHN	Standby	Century Village	Vista Center
MIN	0.17	3.00	0.91	12.90	10.00	8.00	0.50	1.25
MAX	10.00	3.00	3.00	20.00	10.00	8.00	0.50	1.25

Wellfield	Optimized Wellfield Pumpage (MGD)			
	MIN	MAX	SYS7	SYS8
Wellfield	3.25	0.91	0.91	12.90
Optimized Pumpage	3.25	0.91	0.91	33.15

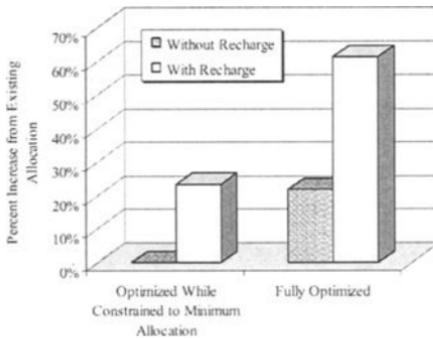
\* + values indicates feet drawdown  
- indicates feet mounding



**Figure No. 2 Relative Mounding Non-Linear Relationship**



**Figure No. 3 Verification of Optimization Matrix**



**Figure No. 4 Benefits of Optimization**

## On Simulating Canal, Overland, and Groundwater Flow in South Florida

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### Abstract

We present a multi-dimensional finite element numerical model (COSFLOW) to simulate 1D canal, 2D overland, and 3D subsurface flow in South Florida. The developed model was designed for use in both regional scale analyses of various water resources projects and for the detailed design of specific projects. The diffusion wave approach was used to determine water flows on overland and in each canal reach controlled by hydraulic structures at its two ends. The Richard's equation was solved to compute the subsurface flow in both saturated/unsaturated zones. The flow governing equations were discretized with the Galerkin finite element method, where the interaction between surface and subsurface waters is handled numerically through a coupling process. The developed numerical code has been incorporated into a modified GMS (Groundwater Modeling System) graphical user environment to allow accurate construction of computational domains and efficient use of the code for the user. From the developed hydrogeologic conceptual model, the South Dade Model that covered the area from just west of L-67 Extension eastward to the coast in South Florida was calibrated with water elevation data from observation wells in 1995. In the South Dade Model, the canal system consisted of all major canals in South Dade County where each canal reach contained many canal elements whose lengths were about 600 meters. The 2D overland flow mesh consisted of 4,720 nodes, which was formed by the surface layer of nodes and elements in the 3D subsurface mesh. The 3D finite element mesh

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