CHAPTER 4

Hydrologic Designs for Extreme Events under Nonstationarity

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Abstract: *Infrastructure built for protecting communities from the impacts of extreme events have largely been designed based on concepts such as return period and risk assuming stationarity. In view of increased attention to the effects of anthropogenic and climate variability and change, traditional methods of hydrologic designs are being extended to deal with nonstationarity of future extremes. The nonstationary design methods discussed in this paper are based on the concepts of (a) Expected Waiting Time (EWT); (b) Risk; and (b) Expected Number of Events (ENE). These methods are applied and compared using two examples of increasing floods and extreme sea levels. In addition, the effect of uncertainty introduced by the projection of nonstationarity into the future along with potential options for dealing with it are discussed. The importance of developing adaptive pathways based on flexible designs is also emphasized.*

Keywords: *Nonstationarity; hydrologic design; return period, risk, and frequency of extremes; flexible designs.*

1 INTRODUCTION

Hydrologic design of infrastructure for protection from the impacts of extreme events have been traditionally based on a wide range of statistical techniques which assume that the hydrologic regime is stationarity. In simple terms, stationarity implies that past observations provide an indication as to what is to be expected in the future. A stochastic process representing time series of extremes is stationary when its probability distribution function (PDF) is invariant with respect to time *t*. A more formal definition of a strictly stationary time series, Z_t, t $= \ldots -1, 0, 1, \ldots$

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is one where the joint statistical distribution of $\{Z_{t1}, Z_{t2}, \ldots, Z_{tn}\}$ is the same as the joint distribution of $\{Z_{t1+m}, Z_{t2+m}, \ldots, Z_{tn+m}\}$ for all values of m and n (Kendall et al. 1983). For extreme events of hydrologic variables such as precipitation amounts, flood magnitudes, and sea levels, commonly PDFs such as Log-Normal (LN), Log-Pearson Type III (LP3), and Generalized Extreme Value (GEV) have been employed, although applications of other alternatives can be found in the literature. The past observations at a site or its vicinity are typically used for fitting a selected extreme value distribution for designing a project for an assumed life span of n years (e.g., 50 years) which may initiate its operation following construction at some time t_0 (Figure 1). Traditionally, the concept of "return period", T, has been used as a criterion for sizing a hydraulic structure (e.g., spillway or a sea wall). In a stationary framework, a selected PDF with constant parameters are estimated from historical data, and the design quantile (also known as return level) for a desired return period is determined. A fundamental assumption is that the same PDF will remain unchanged in the future.

Some hydrologists argue that hydrologic processes have never been stationary or never will be. Hydrologic observations typically consist of both deterministic and stochastic components, and therefore the justification for the above argument may be made easily because both components are likely to change with time due to anthropogenic influence in the environment or climate variability and change. Detection of spatio-temporal change is difficult due to many reasons including: (a) often hydrologic records are too short to determine the absence or presence of statistically significant trends and (b) in some cases, the low-frequency components of the climatic process may be misinterpreted as spurious, persistent trends. Likewise, observations exhibiting trends, may be the result of land use changes in the river basin of interest, which in turn may require methods beyond those based on the stationarity assumption. In some cases, the effects of land use changes, such

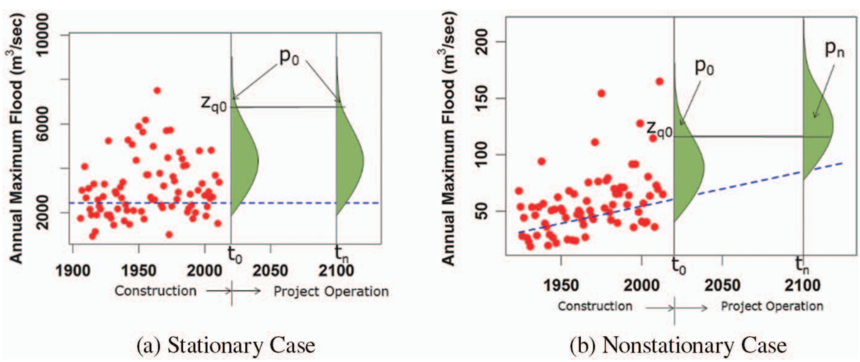


Figure 1. Examples of stationary and nonstationary annual maximum floods for: (a) Umpqua river basin in Elkton, Oregon and (b) Assunpink Creek, Trenton, New Jersey. The dashed line in each figure is the fitted location parameter using a stationary and nonstationary GEV respectively for each case. The variables t_0 *and* t_n *denote the beginning and end of the project design life,* n, *respectively. In addition,* z_{q0} *is the design quantile, and* p*'s denote exceedance probabilities.*

as urbanization, on the hydrologic regime has been considered by using adjustments so that the observed record becomes stationary (e.g., Moglen, 2003; Gilroy and Mc Cuen, 2012). In addition to known changes in the landscape of river basins, the growing evidence of human influence on the climate system and their potential effect on the hydrologic cycle justifies seeking newer, nonstationary approaches for hydrologic designs.

In view of the effects of anthropogenic and climatic variability and change on the hydrologic cycle, Milly et al. (2008) published a thought-provoking paper entitled, “Stationarity Is Dead: Whither Water Management?” and suggested the need for finding a suitable successor to the stationary approach. This paper drew major attention worldwide with pros and cons and critical commentaries from many, as exemplified by papers with such titles as, “Stationarity: Wanted Dead or Alive? (Lins and Cohn 2011), “Comment on the Announced Death of Stationarity” (Matalas 2012), “Negligent Killing of Scientific Concepts: the Stationary Case” (Koutsoyiannis and Montanari 2014), “Modeling and Mitigating Natural Hazards: Stationarity is Immortal!” (Montanari and Koutsoyiannis 2014), and “Stationarity is Undead: Uncertainty Dominates the Distribution of Extremes” (Serinaldi and Kilsby 2015). It is beyond the scope of this paper to discuss the critical commentaries of the paper by Milly et al. (2008). Clearly, many hydrologists believe that the declared death of stationarity was premature. In response to some of the above criticisms, Milly et al. (2015) agreed with many arguments advanced in support of keeping the stationarity concept alive, but reiterated the need to consider nonstationarity in the representation of hydrologic processes in the 21st century. They also suggested that the simplicity of the message helped convey the need to consider nonstationarity in the future and doubted that readers of Milly et al. (2008) misunderstood the essence of what was implied by the phrase, “Stationarity is Dead.”

While the scientific debate continues regarding the future role of stationarity, the practicing engineering community has been searching for need novel methods for hydrologic design for extreme events where some form of nonstationarity is observed or projected for the future, whether it is induced by land-use change and/or climate variability and change. Of course, one needs to be aware of the difficulties and many uncertainties involved in predicting both deterministic and stochastic changes in the hydrologic process of interest. In this paper, we review recent advances in extending the stationary concepts and methods for evaluating the performance of hydraulic projects considering a nonstationary paradigm. First, we briefly summarize the current concepts associated with the stationarity approach. Next, we provide a summary of extensions to the concepts of Expected Waiting Time (EWT) which is same as the Return Period, Expected Number of Events (ENE), and Risk (R), suggesting how they could be used for hydrologic design under nonstationarity. We illustrate the foregoing concepts using two examples, one for increasing floods and another for increasing sea level extremes. The paper concludes with a discussion of uncertainties and some remarks.

2 BRIEF REVIEW OF STATIONARY METHODS

The stationary approach assumes that extreme values (e.g., floods, sea levels) are independent and identically distributed (i.i.d) random variables with a specified probability distribution. Henceforth, we will consider the case of annual maxima (each value represents the maximum value over a block-length of one year). Denoting the random variable of extremes as Z, assume that is has a Cumulative Distribution Function (CDF) denoted by $F_Z(z, \boldsymbol{\theta})$ where $\boldsymbol{\theta}$ is its parameter set. Consequently, for a given cumulative probability, q, the corresponding value of the variable, Z, denoted as z_q is called the q-th quantile. In addition, sometimes the notation z_p is utilized, where p denotes the exceedance probability, i.e. $p = 1 - q$. Likewise, traditionally, the concept of Return Period, T, has been used in which $T = 1/p$ (e.g., Gumbel 1941) and in this case the quantiles z_q or z_p as defined above are also written as z_T. Furthermore, in some recent literature, the referred quantiles have been called "return level" (e.g., Coles 2001). In the remainder of the paper however, we will use the names quantiles, design quantiles, or design levels.

Figure 1(a) shows schematically the hydrologic design problem using the annual maximum flood data for the Umpqua river near Elkton, Oregon which is assumed to be stationary over the historical period. The design life of n years is assumed to start from time t_0 when the project operation begins following construction. When using the Return Period T as the design criteria, it is useful to remember that T is the expected waiting time (EWT) and it can be extended easily to the nonstationary condition. The waiting time, X, is the time it takes from time t_0 for the first flood event exceeding the design quantile, say, z_{q0}, which implies that all other prior annual maxima after t_0 are less than z_{q0}. It can be shown that, X, is a random variable that follows the geometric distribution (e.g., Mood et al, 1974) in which its expected value is $\mathrm{E}[X] = 1/p_0$ where p_0 is the exceedance probability of the design level z_{q0} and it can be determined from $p_0 = 1 - F_Z(z_{q0}, \boldsymbol{\theta})$. It follows that the design return period, T which is equal to $1/p_0$ is the Expected Waiting Time (EWT) for the extreme event to occur. It is noted that the Return Period is not related to the design life, n. The return period concept may be interpreted as, "*the expected waiting time for the T-year event is T years*." In addition, the variance of X is $\mathrm{Var}(X) = (1 - p_0)/p_0^2$.

Another measure that is important in evaluating and designing projects is the hydrologic risk which incorporate the design life, n. It may be shown that the number of events exceeding z_{q0} in a n-year period is a random variable, Y, which has a Binomial Distribution (BD) (e.g., Bras, 1990)

$$P[Y = y] = \binom{n}{y} (p_0)^y (1 - p_0)^{n-y} \qquad y = 0, 1, \ldots n \tag{1}$$

where y is the number of such events over the design life, n. Risk, R, may be defined as the occurrence of one or more extremes exceeding the design return level or equivalently $\mathrm{P}[Y \geq 1]$. Consequently, R is given by

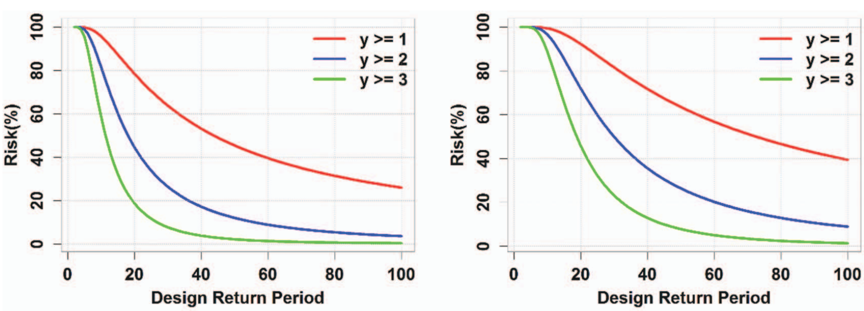
$$R = P[Y \geq 1] = 1 - P[Y = 0] = 1 - (1 - p_0)^n \tag{2}$$

Other quantities of interest for project evaluation and design include: (a) the expected number of events exceeding the design event over the n-year period, i.e. $E[Y] = np_0$ and (b) the risk of say, y or more floods in the n-year period, i.e. $P[Y \geq y]$. The latter can be computed using the BD given in Eq. (1). Note that Eq. (2) suggests that risk, R, is only a function of $T = 1/p_0$ and n and computing it does not require the knowledge of the underlying extreme value distribution. Using some typical values of $n = 30$ and 50 years, the variation of risk, R as a function of the design return period, T, is shown in Figure 2.

An often ignored fact is that, for a typical design return period of say, $T = 100$ years, the risk of one or more events during the design life can be substantially high. For instance, in case of $n = 30$ (years) design life, the risk of a project designed for $T = 100$ years is as high as 26% which is the probability of one or more events exceeding the design quantile over the n-year period. When $n = 50$ years, the risk increases to about 40%. Note that the risks of 2 or more or 3 or more events over the design life decreases rapidly from these magnitudes. For example, when $y \geq 2$ and $y \geq 3$, the 26% risk reduces to about 3.6% and 0.3%, respectively for $n = 30$ years. In situations where projects can sustain repetitive extreme events that may exceed the design event, knowledge of the foregoing risk for higher values of y may be relevant.

3 NONSTATIONARY METHODS

We will now extend the project evaluation and design methods outlined in the previous section into conditions of nonstationarity. Figure 1(b) illustrates a situation of nonstationarity where the annual flood maxima of the Assunpink



(a) Design Life, $n = 30$ years (b) Design Life, $n = 50$ years

Figure 2. Variation of Risk, R*, as a function of design return period,* T*, for design life,* n *equal to (a) 30 years, and (b) 50 years. In both cases the number of exceeding events considered are* y = 1, 2, *and 3 over* n *years.*

Creek near Trenton, New Jersey show an increasing pattern. This data set have been used extensively by Obeysekera and Salas (2014, 2016) to illustrate the quantification of uncertainty and the recurrent flooding concepts under non-stationarity. At this location, the annual maximum floods have been increasing over years due to increasing urbanization as indicated by a doubling of population from 1930s to 1990s (Dow and DeWalle 2000). Unlike the stationary case where the probability p_0 is expected to remain constant in the future, the probability exceeding the design quantile z_{q0} will increase over time from p_0 to p_n at the end of the design life, n (Fig. 1b). Consequently, since the exceedance probability, p_t increases for $t = 1, 2, \ldots, n$ the traditional geometric distribution with constant p is not applicable. The time-varying p can be obtained readily from a fitted time varying model as $p_t = 1 - F_Z(z_{q0}, \boldsymbol{\theta}_t)$ where the subscript t in $\boldsymbol{\theta}$ indicates that the underlying PDF of annual maxima changes with time, and hence nonstationary (note that we assume that the type of PDF is the same, but the parameters vary with covariates that evolve with time). In this case, the waiting time, X, for the first occurrence of an event exceeding z_{q0} follows a nonhomogeneous geometric distribution (Mandelbaum et al. 2007; Salas and Obeysekera 2014). As in the stationary case, we will use the Expected Waiting Time (EWT) concept but now with time varying probabilities, p_t.

The derivation of EWT under nonstationarity may be found in Cooley (2013) and Salas and Obeysekera (2014). This leads to a convenient formula for EWT, which we denote as T given by

$$T = 1 + \sum_{x=1}^{\infty} \prod_{t=1}^{x} (1 - p_t) \tag{3}$$

In practice, the increasing values of p_t will converge the product to zero quickly and a finite, but somewhat large value of x, say x_{max} may be adequate instead of the infinite summation shown in Eq. (3). Since, $p_t = \mathrm{f}(z_{q0}, \boldsymbol{\theta}_t)$ and the initial design return period is $T_0 = 1/p_0$, a curve of T versus T_0 can be constructed for a given sequence of p_t values. We call this a "Return Period Curve" and it is a convenient design tool for nonstationary situations. It can be used to answer questions such as "What should be the design T_0 if the desired EWT, T, is say, 50 years?" Clearly, for increasing extreme events, $T < T_0$. We will illustrate this case with an example in the next section.

The risks under nonstationarity can be derived using an approach similar to the stationary case but it is somewhat more complex. Assuming, once again, Y is the number events exceeding the design quantile z_{q0} over the design life n with possible values $y = 0, 1, 2 \ldots n$, the probability mass function (equivalent to the Binomial Distribution in the stationary case) is given by the Poisson Binomial Distribution (Obeysekera and Salas, 2016),

$$\mathrm{P}[Y = y] = \sum_{\mathrm{A} \in \mathcal{F}_y} \prod_{j \in \mathrm{A}} p_j \prod_{i \in \mathrm{A}^c} (1 - p_i), \quad y = 0, 1, \ldots, n \tag{4}$$

where $\mathcal{F}_y$ is the set of all subsets of y integers that can be selected from $\{1, 2, 3, \ldots, n\}$, and A^c is the complement of A with respect to $\{1, 2, \ldots, n\}$. While Eq. (4) can be used to determine the risk $R = P(Y > 0)$, as mentioned in Obeysekera and Salas (2016), the computation of the PMF given by Eq. (4) is cumbersome, particularly for large n. Instead one may use the simple nonstationary risk formula (Salas and Obeysekera 2014),

$$R = P[Y \geq 1] = 1 - P[Y = 0] = 1 - \prod_{t=1}^{n} (1 - p_t) \tag{5}$$

Since the time varying probabilities, p_t can be determined for a given initial design quantile, $z_{q0,}$ the computation of the risk, R, using Eq. (5) is straight forward. As in the stationary case, the Risk of multiple events during the design life n can be computed using the expression $P[Y \geq y]$, where $y = 2, 3, \ldots$ but that will require the use of Eq. (4).

Another quantity of interest under nonstationarity is the time-varying frequency of extreme events. In case of increasing probabilities of exceedance, the frequency of extreme events exceeding the initial design level z_{q0} will increase with time. The expected number of events, ENE, over the design life n is a measure that may be used for evaluating existing projects or as a design criteria for future projects (Obeysekera and Salas 2016). Although the computation of PMF given by Eq. (4) is cumbersome, the expected value and the variance of Y are simpler and can be determined from

$$E[Y] = \sum_{t=1}^{n} p_t \tag{6}$$

$$Var[Y] = \sum_{t=1}^{n} p_t (1 - p_t) \tag{7}$$

It is straightforward to show that, under stationarity conditions, i.e. $p_t = p_0$ for all t, then $E[Y] = np_0$ and $Var[Y] = np_0(1 - p_0)$ which correspond to the first two moments of the Binomial Distribution as stated above in the previous section.

4 ILLUSTRATIVE EXAMPLES

4.1 Increasing Annual Floods

For illustrating the evaluation and design concepts presented above considering nonstationarity, the annual peak flows of the Assunpink Creek watershed in Trenton, New Jersey for the period 1924–2013 are analyzed. As indicated above, this watershed is located in a highly urbanized area and the observations of maximum annual floods exhibit an increasing trend as shown in Figure 1(b).

Although the methods described in Section 3 is general in that any extreme value distribution (e.g LP3) could be fitted to the data, we illustrate the application by fitting a nonstationary Generalized Extreme Value (GEV) to the annual maxima data. Obeysekera and Salas (2014) fitted a variety of stationary and nonstationary GEV models and demonstrated the statistical significance of an apparent trend as shown in Figure 1(b). The chosen model was a GEV distribution with the location parameter varying with time and constant scale and shape parameters. The corresponding exceedance probability for the nonstationary GEV model is given by,

$$p_t = 1 - exp\left\{-\left[1 + \frac{\varepsilon}{\sigma}(z_{q0} - \mu_t)\right]^{-\frac{1}{\varepsilon}}\right\} \tag{8}$$

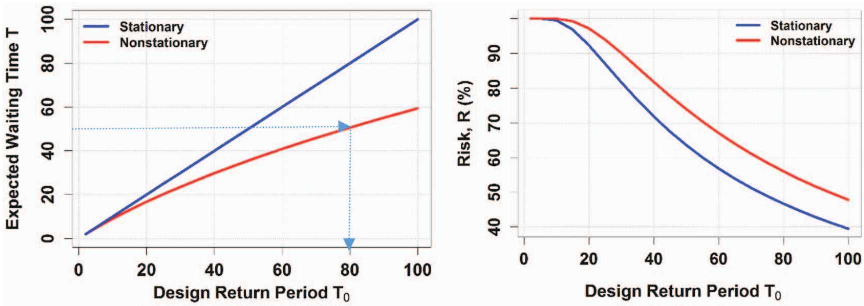
where μ_t is the location parameter in year t, and σ and ε are the scale and shape parameters respectively, and they are assumed to be time-invariant. The R-package extRemes (Gilleland et al. 2016) was used to estimate the parameters of the nonstationary GEV model by applying the method of maximum likelihood, which give: $\mu_t = a_0 + a_1(t - 1968.027)$ with $a_0 = 44.587\frac{m^3}{sec}$, $a_1 = \frac{0.306\frac{m^3}{sec}}{year}$, $\sigma = 16.617\frac{m^3}{sec}$, and $\varepsilon = 0.136$. The time variable, t, has been centered around its own mean (1924 to 2013) for computational efficiency reasons. Using the above parameters, the time varying probabilities, $p_{t,}$ may be computed from Eq.(8) for future years given the design quantile, z_{q0}.

In order to illustrate the application of the nonstationary EWT criterion (Expected Waiting Time for the first occurrence), let us assume that a design is required for a project with an initial year of operation in the year 2020 (i.e. t_0 = 2020), and a design life, n = 50 years. First, various values of design quantiles z_{q0} corresponding to a range of values of T_0, say 5 to 100 years in increments of 5 years, are computed using the GEV quantile equation,

$$z_{q0} = \mu_0 - \frac{\sigma}{\varepsilon}\left\{1 - \left[-\ln\left(1 - \frac{1}{T_0}\right)\right]^{-\varepsilon}\right\} \tag{9}$$

where μ_0 is the location parameter corresponding to the initial year 2020, and the parameters σ and ε have been estimated from data as discussed above. For each value of z_{q0}, the time varying probabilities, p_t, may be computed from Eq. (8). Then, using the values of p_t, the nonstationary EWT, T, is determined from Eq. (3). For this example, a sufficiently large value x_{max} = 1000 was used for the infinite summation shown on the right side of Eq. (3). Then a Return Period Curve is developed using the above steps as shown in Figure 3(a). And a Risk Curve for $n = 50$ under nonstationarity is determined using Eq. (5) as shown in Figure 3(b). The corresponding curve for the stationarity condition is also shown in Fig. 3(b) for comparison.

The Return Period Curve shown in Figure 3(a) is the primary design metric utilized based on EWT. It provides information of the size of the project to be built in Year 2020 in terms of the Design Return Period, T_0 for a desired level of



(a) Return Period Curve (b) Risk Curves

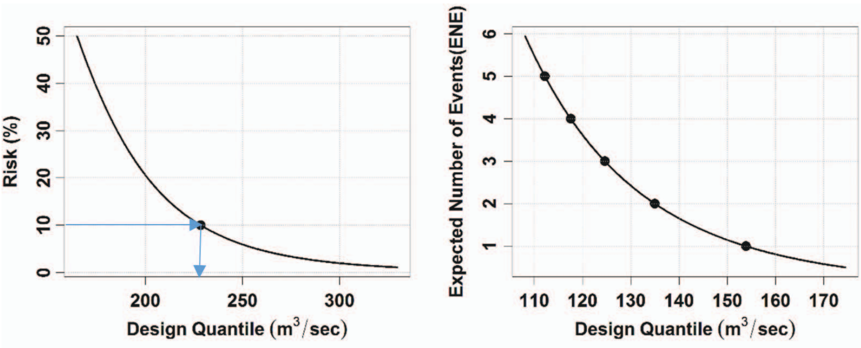
Figure 3. (a) Return Period Curve and (b) Risk Curve for nonstationary floods. In (b), the stationary risk curve is also shown (dashed line) for comparison.

protection under nonstationarity specified by the metric, T (EWT). For instance, if the value $T = 50$ years is considered, then the project should be designed for T_0 of about 80 years as shown in Figure 3(a). The increase in the return period is attributed to the nonstationary behavior of annual floods which is expected to continue beyond 2020.

The risk of one or more floods exceeding the design flood quantile z_{q0} is shown in Figure 3(b). It is a useful measure for assessing the performance of a given project. For instance, when $T_0 = 80$ years (corresponding to $T = 50$ years), for a design life, $n = 50$ years, the Risk, R, assuming stationarity is about 47%, while under nonstationarity, the risk increases to about 56% (i.e. about 9% increase of risk due to nonstationarity). Note the large values of the risk of "failure" obtained under the given assumptions of project design. The results imply that the design of the project z_{q0} may be too small and suggests that smaller risks can be obtained by increasing the design quantiles z_{q0} i.e. increasing the return periods T_0.

Next, we illustrate the two other design criteria that were introduced in the previous sections, namely: (a) Risk based design and (b) Expected Number of Events (ENE) design. In the case of risk based design, tolerable risk (of one or more events exceeding the design level) over the design life n is used to determine the initial design in terms of its return period (T_0) or quantile (z_{q0}). This method is similar to the Design Life Level (DLL) concept suggested by Rootzen and Katz (2013). To implement this approach, Eq. (5) is applied to determine an appropriate value of z_{q0} (note that the values of $p_t,\ t = 1, 2, \ldots n$ are related to z_{q0} from Eq. (8)) for the specified value of risk, R. Since there is no explicit solution to find z_{q0} for the specified value of R a numerical solution of Eq. (5) can be followed. Alternatively, an indirect practical procedure which is easy to implement can be used. This is accomplished by creating a curve of R versus z_{q0} first and then finding the value of the design quantile, z_{q0} for a specified value of R via interpolation. For illustration, such a curve derived by using Eqs. (5) and (8) for a design life, $n = 50$ years starting in year 2020 is shown in Figure 4(a). As an example, assuming the allowable level of Risk, is say 10%, the design level can be obtained from this figure by interpolation, which gives 228 m^3/sec (shown as the point on the curve).

Using Figure 4(a), the return level, z_{q0} was computed for a range of Risk values as shown in Figure 5 (continuous solid blue line). Once z_{q0} is determined, the corresponding return period, T_0 is computed using the fitted GEV model as shown in Figure 5 (continuous solid red line). For comparison, equivalent curves assuming stationarity are also shown (dashed lines). The above are design curves



(a) Risk versus Design Quantile (z_{q0}) (b) ENE versus Design Quantile (z_{q0})

Figure 4. (a) Risk versus Design Quantile curve for determining z_{q0} *for a desired level of risk* R *and (b) ENE versus Design Quantile curve for determining* z_{q0} *for a specified value of ENE.*

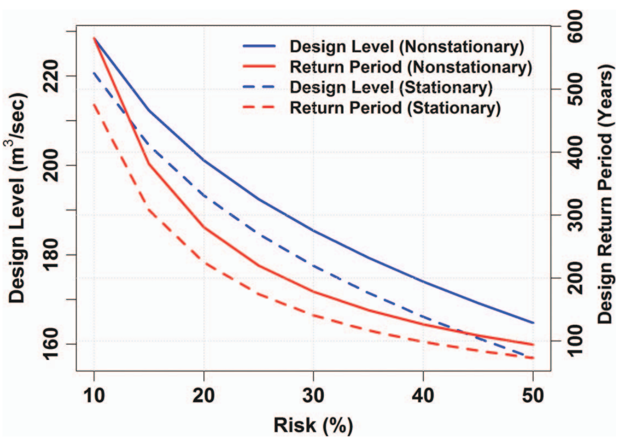


Figure 5. Risk-based design curves for a design life, n = *50 years starting in year 2020. The blue lines (solid and dashed) show the design level (*z_{q0}*) for a specified Risk,* R *(nonstationary and stationary) and the red curves show the same but for design return period,* T_0*.*