Analysis of Oscillating Modality and Stability of U-shaped Aqueduct-fluid Coupling System

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ABSTRACT: U-shaped aqueduct is superior structure form, which is extensively used in the china's South-to-North Water Diversion Project. However the analysis of oscillating modality and stability for U-shaped aqueduct coupling system has not been studied in-depth. In the analysis of oscillating modality of this paper, potential fluid method is applied in the fluid model taking into account the surface vibration. With reference to different water depths, the variation law of natural frequency of U-shaped aqueduct - fluid coupling system along with the water depth is investigated. Finally the ALE finite element method is applied to solving nonlinear fluid-solid coupling problem of coupling system on the basis of numerical simulation of U-shaped aqueduct random wind field by applying autoregressive moving average (ARMA) model. The time history of overturning stress and moment for U-shaped aqueduct under different conditions is obtained. And through cross-comparison between the data, the curves of the overturning stress and overturning moment under different water depths are obtained to reflect the stability characteristics of aqueduct - fluid coupling system. Thus it further reveals vibration mechanism of U-shaped aqueduct under the action of the stochastic aerodynamic force. It provides a reference to expand the analysis of dynamic properties for U-shaped aqueduct – fluid coupling system.
KEYWORDS: U-shaped aqueduct - fluid coupling system; natural frequency; ALE finite element method; overturning stress; overturning moment

INTRODUCTION

The top of U-shaped aqueduct has large water weight, which is very sensitive to exterior incentives such as wind load. The overturning stress and overturning moment produced by water sloshing of U-shaped aqueduct could lead to structural collapse under external excitation load. Therefore comprehensive study of vibration characteristic and structural stability for U-shaped aqueduct under external excitation has become an important part of the dynamic response study.

At present, additional model or Housner model is often applied to the dynamic response of aqueduct structures. However, the water sloshing has be neglected in these models not to reflect the actual effects of water vibration and shaking. Through a lot of practice, ALE finite element method referenced by Hirt (1974) and Ramaswamy (1987) is verified to have high accuracy in the analysis of coupled dynamics of aqueduct under external excitation to reflect transient water sloshing waves. Therefore, ALE finite element method applied to solving dynamic response of aqueduct could provide reasonable accurate results. But the dynamic characteristics research of U-shaped aqueduct-water coupling system is not deep compared to other section forms of aqueduct. Thus detailed research of its dynamics has become urgent.

In this paper, the vibration characteristic of U-shaped aqueduct is analyzed based on considering liquid surface vibration. ALE finite element method is applied to get time history of overturning stress and overturning moment for U-shaped aqueduct under different conditions. The research results can provide reasonable theoretical reference in aqueduct design.

BASIC EQUATIONS

Coupling vibration equation of aqueduct

The aqueduct structure is simplified to get coupling plane vibration equation of the aqueduct as referenced in the work by Li (2002):

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{p\}_L+\{p\}_K \tag{1}$$

where [M] , [C] respectively stand for structural cohesion mass matrix, damping matrix; [K] is stiffness matrix; u is particle displacement relative to ground level; $\{P\}_K$ is horizontal force on the structure caused by external constraint spring binding; $\{P\}_L$ is horizontal force on the structural top caused by fluid in aqueduct. It can be expressed as follows :

$$F_L=-T\ddot{u}_N-\sum_{n=1} Q_n\int_0^t \ddot{u}_N(\tau)\sin w_{ln}(t-\tau)d\tau \tag{2}$$

First and second parts of Equation (2) stand for effects of the solid part and fluid on the structure respectively, above formula is equally applied to non-rectangular aqueduct such as U-shaped aqueduct.

Considering the free vibration system without damping, Equation (1) can be adjusted as follows :

$$(-\lambda[M]+[K]+k[A]-\xi(\lambda)[A])\{\bar{u}\}=\{0\} \tag{3}$$

where λ is the mode of feature vector , $\{\bar{u}\} = \int_{-\infty}^{\infty}\{u(t)\}e^{wt}dt$

Fluid equation of ALE finite element method

The key of dynamic analysis for aqueduct-fluid coupling system is to get the solution of interaction between structure and fluid. The essence is to solve the Native-Stokes equation with free surface. It can be expressed as follows (Wu, 2003; Wu, 2004)

$$\begin{aligned} &\rho\ddot{u}+\rho a\cdot\nabla\dot{u}-\nabla.\tau-f=0 \\ &\nabla\cdot\dot{u}=0 \end{aligned} \tag{4}$$

where ρ is density; u is displacement; f is external force; a stands for convection velocity, which is the superimposition of material velocity and grid velocity. τ is stress tensor expressed as:

$$\tau_{ij}=-p\delta_{ij}+2\mu e_{ij} \tag{5}$$

where p is pressure; μ is the fluid kinematic viscosity; δ_{ij} is Kronecker delta; e_{ij} is component of strain tensor.

Moving boundary conditions and dynamic boundary conditions must be satisfied by the grid nodes on the free surface:

$$\begin{aligned} &\frac{\partial S}{\partial t}+v_i S_{,i}=0 \\ &\tau_{ij}n_j=\left[-P_0+\sigma(\frac{1}{R_1}+\frac{1}{R_2})\right]n_i \end{aligned} \tag{6}$$

where v_i is grid nodes velocity for the free surface; $S_{,i}$ is partial derivatives of the variable I; P_0 is externally imposed pressure on the free surface ; σ is surface tension ;n_i, n_j is the unit normal vector of free surface ;R_1, R_2 is curvature radius of free surface.

The velocity and interaction of coupling boundary surface are continuous to satisfy the continuous boundary conditions:

$$\dot{u}_f = \dot{u}_s$$
$$S_f n_f = S_s n_s \qquad (7)$$

where S_f, S_s is respectively the stress of water and structure on coupling surface ; $n_{f,} n_s$ is respectively the normal vector of water and structure on coupling surface.

ANALYSIS OF VIBRATION CHARACTERISTICS

The potential fluid flow is applied considering the surface modes in this two-dimensional U-shaped aqueduct model. The free surface condition is defined. Model is shown as Figure 1. Its total height is 11.17 meters. The upper trough is 3.17 meters, with 1.6 meters of inside radius and 0.12 meters of thick trench shell. Seven water depths are used in analysis, which include 0.469, 0.8, 1.18, 1.377, 1.6, 1.9, 2.05 meters. Mesh density of aqueduct and fluid are both 0.25*0.25m^2 to coordinate the coupling computation of aqueduct and fluid.

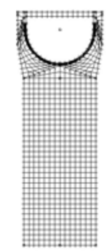

FIG.1. U-shaped aqueduct structure

After complete analysis of the different conditions, 50 vibration mode frequencies of U–shaped aqueduct-fluid coupling system are listed in Table 1 with 1.6 meter water depth. And frequency distribution curve of each condition is illustrated in Figure 2.

Table 1. U-shaped Aqueduct Coupling System Frequency (1.6 meters)

M	Frequency	M	Frequency	M	Frequency	M	Frequency	M	Frequency	M S
1	3.85E-08	11	1.58E+00	21	2.29E+00	31	3.15E+00	41	4.13E+00	
2	4.64E-01	12	1.66E+00	22	2.34E+00	32	3.25E+00	42	4.24E+00	
3	6.94E-01	13	1.73E+00	23	2.51E+00	33	3.35E+00	43	4.27E+00	
4	8.58E-01	14	1.81E+00	24	2.57E+00	34	3.44E+00	44	1.76E+01	horizontal
5	9.94E-01	15	1.88E+00	25	2.64E+00	35	3.54E+00	45	4.92E+01	horizontal
6	1.11E+00	16	1.95E+00	26	2.71E+00	36	3.64E+00	46	5.58E+01	vertical

7	1.22E+00	17	2.02E+00	27	2.78E+00	37	3.76E+00	47	7.75E+01	h+v
8	1.32E+00	18	2.09E+00	28	2.86E+00	38	3.87E+00	48	9.21E+01	vertical
9	1.41E+00	19	2.16E+00	29	2.95E+00	39	3.97E+00	49	1.31E+02	h+v
10	1.50E+00	20	2.23E+00	30	3.05E+00	40	4.06E+00	50	1.64E+02	vertical

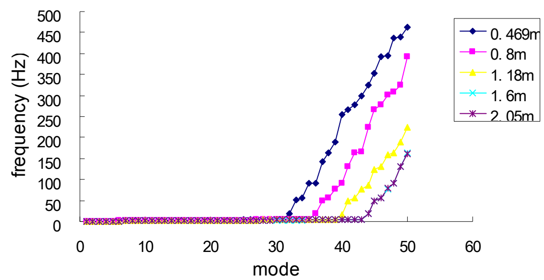

FIG. 2. Frequency distribution curve of U-shaped aqueduct –fluid coupling system

From Table 1 and Figure 2, it can be seen U-shaped aqueduct coupling system has lower natural frequencies. The natural frequency of U-shaped aqueduct - water coupling system presents gradually decreasing trend with the increase of water depth in aqueduct. The system oscillation modes include the structural mode and fluid oscillation mode. Orders before 43 are fluid oscillation modes, in which mode 1 is the rigid mode. The bands after 43 are the structural oscillation modes. The minimum stiffness direction is horizontal groove direction because the first-order structural oscillation mode of aqueduct is cross-slot mode direction. At the same time the greater distortion of structural higher order oscillation mode is caused by dynamic effect of weight focused on the pier top.

The total mass of the structure increases with the increase of water in coupling system. The natural frequencies of system occurs corresponding changes, but the trend direction of mode conditions do not change with the variation.

STABILITY ANALYSIS OF U-SHAPED AQUEDUCT

The great overturning stress and moment would be caused by water sloshing of aqueduct under external excitation load. Horizontal groove damage or failure of aqueduct bearing structure could be caused by these overturning effects. Models and conditions are based upon the text shown in the previous section. The aqueduct wall is divided into seven zones including 0°, 30°, 45°, 90°, 135°, 150°, 180°angles. ALE infinite method is applied to launch coupling operation. The viscous coefficient value is 0.001 in this numerical calculation.

The overturning stress extreme value of each aqueduct wall regions under various operating conditions was summarized and is shown in Figure 3:

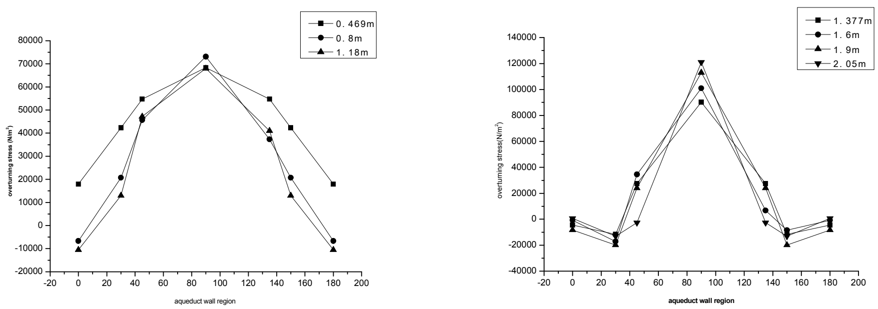

FIG. 3. Variation curve of overturning stress extreme value for aqueduct wall region

The overturning stress of each condition basically increases with the increase trend of aqueduct water. And the overturning stress of each condition all reaches maximum at the bottom. The overturning stress presents negative near surface region with the increase of water depth. The negative overturning stress may be due to water vortex caused by aqueduct water sloshing near surface, which is similar to air diversion.

With extraction of extreme overturning stress under different conditions, the variation of overturning stress with different water depth was analyzed and is shown in Figure 4.

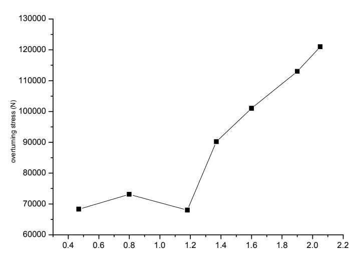

FIG. 4. Variation curve of overturning stress extreme value along with water depth

The overturning stress extreme value decreased at the 1.18 m water depth. The curve present increase – sharp mutation - rising trend. Incentive water depth does not exist compared with water sloshing extreme value variation.

For more intuitive research of overturning moment under different water depths, the overturning moment extreme value of each aqueduct wall regions under various operating conditions was summarized and is shown in Figure 5:

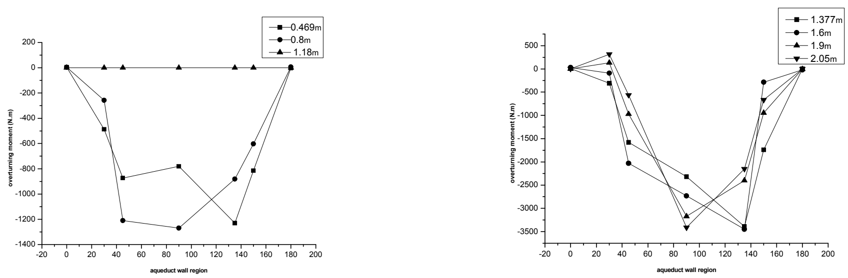

FIG. 5. Variation curve of overturning moment extreme value for aqueduct wall region

The overturning moment of each condition basically increases with the increase trend of aqueduct water. And the overturning moment of each condition all reaches maximum at the bottom. The overturning moment presents positive near surface region with the increase of water depth. However, most regions are mainly liable to produce overturning effects.

The overturning moment caused by water sloshing is only accounted small percentage of resistant overturning moment produced by gravity in this model. However, with the increase of depth-width ratio and water depth of aqueduct, overturning moment caused by water sloshing will become larger. Therefore, the overturning moment should be given enough attention in the design of large aqueduct. The reasonable depth-width ratio of aqueduct should be designed to control overturning moment.

With extraction of extreme overturning moment under different conditions, the variation of overturning moment with different water depth was analyzed and is shown in Figure 6.

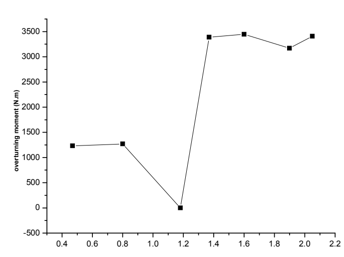

FIG. 6. Variation curve of overturning moment extreme value along with water depth

The overturning moment extreme value decreased at the 1.18 m water depth. The curve presents increase – sharp mutation - rising - mutation trend. Incentive water depth and TLD water depth exists which is similar with water sloshing extreme value variation.

CONCLUSION

In this paper, the curves of the overturning stress and overturning moment under different water depth are obtained to reflect the stability characteristics of aqueduct - fluid coupling system through cross-comparison between the data. Thus it further reveals vibration mechanism of U-shaped aqueduct under the action of the stochastic aerodynamic force. It provides a reference to further expand the analysis of dynamic properties for U-shaped aqueduct – fluid coupling system.

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Three-dimensional Dynamic Characteristics Analysis of Large Span Cable-stayed Aqueduct

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ABSTRACT: The cable-stayed aqueduct has become a new form of aqueduct after the beam and arched aqueducts. But the study focused on the dynamic characteristics of cable-stayed aqueducts is not enough. Based on the structure of cable-stayed aqueduct, the inner water is considered as added mass. The three-dimensional finite element model of fluid and the cable-stayed aqueduct is established to analyze the dynamic characteristics of aqueduct in different conditions. The results show that the dried modes and wet modes are analogous, but the natural frequencies of vibration are obviously different. The natural characteristics of vibration are three-dimensional and coupling with torsion mode. The cables have no effect on the fundamental frequency of lateral bending mode of vibration though they enhance vertical restraints to the aqueduct body.
KEYWORDS: cable-stayed aqueduct; 3D model; FEM; dynamic characteristics

INTRODUCTION

A cable-stayed aqueduct, as an important hydraulic structure in water conveying project, is beautiful, light and ingenious meanwhile adaptable to various terrains. Cable-stayed aqueducts usually cross terrains with complex geological conditions, such as canals, swales, valleys. Their capability of seismic and wind resistance is poor especially for aqueducts with large span and low depth-width ratio. So a dynamic analysis is particularly important. Research on dynamic characteristics is the basic study of seismic and wind resistance of structure. Up to now the study of dynamic characteristics of aqueducts focuses on beam and arch aqueducts. However, few researches have been conducted on this topic. We usually get references from researches and projects on cable-stayed bridges. Since the centroid and torsion center of aqueduct body do not coincided, and its section type is different from that of bridges, the vibrations are different between cable-stayed bridge and cable-stayed aqueduct.

The reasonable vibration unit of aqueduct is the foundation of the dynamic analysis (Chen, 2005). Torsion frequency of order one is greatly related to the critical flutter wind speed of cable-stayed aqueduct. A 2D model stiffens the aqueduct structures ignoring the thin wall structure of the aqueduct body which has lateral bending-torsion coupled vibration and constraining torsion deformation (Wu, 2005; Wang, 2000; Du, 2002). So it is necessary to establish 3D finite element model in order to represent a real structure of aqueduct. Due to the fact that the weight of water is usually larger than the weight of the aqueduct body, it is disadvantaged to seismic design. So effects of water should be considered in modal analysis of aqueducts. We establish water body and aqueduct structure model by using added mass method, then apply it to calculate and analyze dynamic characteristics of aqueducts in different working conditions.

BASIC THEORIES FOR DYNAMIC CHARACTERISTICS

Added mass method

The structure type of aqueduct is similar to liquid tank, so we can refer to the research results of liquid tank in order to study aqueduct dynamic characteristics. The added mass method neglects effects from large amplitude sloshing of water. Hydrodynamic pressure caused by liquid in the tank at a point is equivalent to the additional inertial force which is generated from the synergy movement of the liquid quality increase at this point on slurry wall. We do not consider the effects from other parts of the liquid at this point of hydrodynamic pressure on slurry wall. Added mass method attributes the effect of fluid on fluid-solid coupling systems to adding the mass matrix and stiffness matrix of the fluid in the dynamic equation of the overall structure (Ju, 1983; Pan, 2003; Bluetooth, 2001). According to Westergaard (1933), additional water mass can be computed by the following formula:

$$M_U = \left[3150 - 0.706\rho\sqrt{H(h-y)}\right](\cos\theta)^{\frac{1}{4}} \tag{1}$$

where ρ is the density of water, H is the height of aqueduct, h is the liquid level , y is the height variable of liquid, θ is the azimuth variable of slurry wall at a tangent.

Equations for dynamic characteristics

The vibration of the cable-stayed aqueduct is continuum; the displacement is continuous and the structure has an infinite number of degrees of freedom. So it needs to