# Void Bulk Modulus Reduction of Porous Asphalt Mixture Based on Porous Linear Elastic Continuum Theory

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**ABSTRACT:** The void bulk modulus reduction of porous asphalt mixtures was evaluated using the Gassmann's equation to describe the relationship between the various bulk moduli and the percentage of air voids in porous asphalt mixtures along with the theory of linear elastic material with voids. A continuum interpretation of the porosity of the material was presented and used in the paper. The void bulk modulus was considered to represent the air voids reduction of the porous asphalt mixtures. The data showed a larger reduction in the void bulk modulus for the dry state when compared to the saturation state. The void bulk modulus under the dry state decreased by 8.4% with the 25% reduction in the initial air voids. However, the void bulk modulus decreased by 7.8% under the saturation state.

**Key words:** porous asphalt mixture; linear elastic theory; air voids, bulk modulus; Gassmann's equation

## INTRODUCTION

The percentage of air voids in porous asphalt mixtures is generally about 20% which is obviously higher than dense graded asphalt mixtures. Consequently, water on the surface of the porous asphalt mixture can effuse through the interconnected air voids in time providing the mixture good drainage properties. The porous asphalt mixture is composed of single-sized aggregates with 70~85% of coarse-graded aggregate resulting in high percentage of air voids.

The permanent deformation in asphalt mixture under traffic loading has two mechanisms: densification and plastic flow. It has been shown that the plastic flow mechanism predominates while the reduction in air voids is less than 3% (Murali Krishnan and Rengaraju 1999). Densification due to traffic occurs in asphalt mixtures with 6% air voids until about 50% reduction in initial air voids is reached when consolidation changes from densification into plastic flow. However, the investigation of the porous asphalt mixtures for heavy traffic in France showed that the percentage of air voids reduced from 20% to 16% after the first 28 months, and then gradually stabilized at 15% (Xu 1994). Hence, the densification of the porous asphalt mixtures changed into a plastic flow process after a 25% reduction in initial air voids.

Research on the void bulk modulus of the porous asphalt mixtures has been somewhat limited. However, there have been several researches on the bulk modulus and its reduction in other areas. Wang et al. studied the size-induced reduction of transition pressure and the enhancement of bulk modulus of Aluminum nitride (AlN) nanocrystals. Researchers found that the bulk modulus of the nanosized wurtzite was larger than that of the bulk AlN crystals (Wang et al. 2004). Rivadulla et al. studied the reduction of the bulk modulus at high pressure in Chromium Nitride (CrN) and showed an unexpected 25% reduction in the bulk modulus (Rivadulla Francisco et al. 2009). Pei et al. predicted the reduction of bulk modulus for porous asphalt pavement using the Gompertz model (Pei and Chang 2009).

The void bulk modulus is proposed as an indicator for the compressive properties of air voids under traffic loading. This paper evaluated the changes in void bulk modulus under different conditions. The linear elastic material theory with voids is used in this paper and the concept of void bulk modulus is given based on Gassmann's equation (Gassmann 1951). In addition, the void bulk modulus reduction of porous asphalt mixture is studied under dry and saturated states.

## THEORETICAL METHODS

### Continuum linear elastic material theory with voids

The basic concept of the linear elastic material theory with voids is that the bulk density $\rho$ of a material can be written as the product of two parameters (Cowin and Nunziato 1983): the density filed of the matrix material ($\gamma$) and the percentage of air voids ($\phi$):

$$\rho = \gamma(1-\phi) \tag{1}$$

The kinematic variable in the linear theory is defined as the change in volume fraction $\phi(x,t)$ and can be expressed as follows:

$$\phi(x,t) = \phi(x,t) - \phi_R \tag{2}$$

where, $x$ is the spatial position vector in cartesian coordinates, $t$ is the time, and $\phi_R$ is the volume fraction in the reference configuration of the porous asphalt mixture. According to Passman (1984) the restriction on the volume fraction magnitude is: $-1 \leq \phi(x,t) \leq 1$.

From a reference volume fraction $\phi_R$ to a state when the voids have completely collapsed and the material becomes a perfectly elastic material, the change in volume fraction can be written as:

$$\phi(x,t) = \phi_R(x) \tag{3}$$

For a constant strain, the change in matrix volume fraction is also uniform. Consequently, $\phi(x,t)$ only varies with time and is independent of $x$. Thereby, the volume fraction $\phi(x,t)$ in equation (2) can be rewritten as follows (Krishnan and Rengaraju 1999):

$$\phi(t) = \phi(t) - \phi_R \tag{4}$$

**Reciprocal work theorem and the conceptual model of porous asphalt mixture**

The reciprocal work theorem states that for a linear elastic structure subject to two sets of forces $\{P_i, i=1,\ldots, m\}$ and $\{Q_j, j=1,\ldots, n\}$, the work done by the set P through the displacements produced by the set Q is equal to the work done by the set Q through the displacements produced by the set P.

The Marshall specimens of porous asphalt mixture with 20% percentage of air voids are scanned longitudinally by the X-ray computed tomography (CT) scanning technique with 2mm space. Fig. 1 shows the air voids as a discontinuous phase and the asphalt mixture as a continuous phase. The X-ray CT image is used to model the porous asphalt mixture under traffic loading (see Fig. 2).

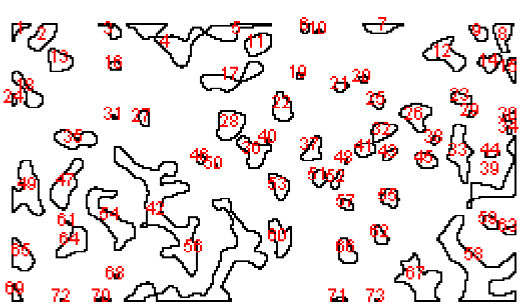


**FIG. 1. Marshall specimen image by CT scanning.**

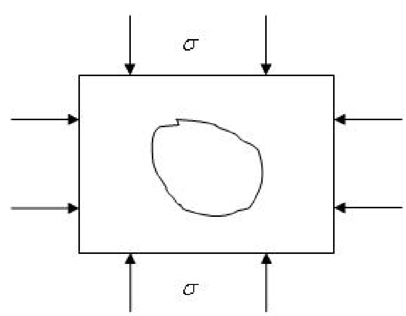
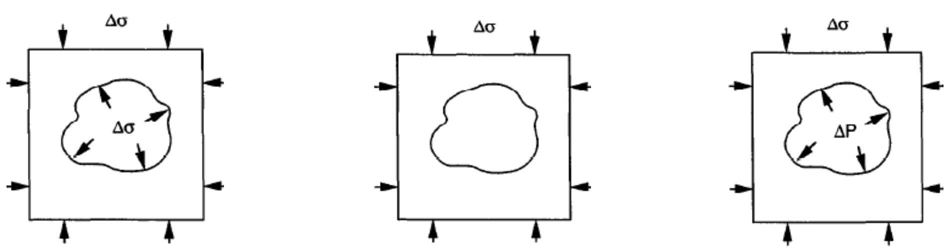



**FIG. 2. Conceptual model of porous asphalt mixture. ($\sigma$ is the loading stress)**

The porous asphalt mixture can be abstracted as double-phase elastic medium, namely the solid phase (asphalt mixture) and the gas phase (filling with air in voids) under the dry state, the solid phase (asphalt mixture) and the liquid phase (filling with fluid in voids) under the saturation state. The irregular domains are abstracted as air voids and the remanent area is abstracted as asphalt mixture matrix. Both, air voids and asphalt mixture matrix are modeled as a continuum medium. Therefore, the conceptual model of asphalt mixture can be analyzed by the work-energy reciprocal theorem.

**Solving of the void bulk modulus for porous asphalt mixture**

The void bulk modulus' solving process of porous asphalt mixture based on Gassmann's equation makes use of the work-energy reciprocal theorem. The porous asphalt mixture is abstracted as two-phase elastic medium and analyzed in a dry state with and without gas pressure and in a saturation state (Mavko and Mukerji 1995). Fig. 3 shows the forces applied to the system in all three cases. The voids space can be of any geometric shape and size.




**(a) dry state with gas pressure (b) dry state without gas (c) saturation state**

**FIG. 3. Force system in three cases.**

According to Li and Wang (2007) the compression factor $\lambda$ can be derived in terms of the bulk modulus of the porous asphalt mixture:

$$\lambda = \frac{d\varepsilon_V}{d\sigma} = \frac{1}{V}\frac{dV}{d\sigma} = \frac{1}{K} = \frac{3(1-2\nu)}{E} \tag{5}$$

where, $\varepsilon_v = \Delta V/V$ is the volumetric strain, $k$ is the mixture bulk modulus, $E$ is the mixture Young's modulus, and $\nu$ is the mixture Poisson's ratio.

In the case of dry state with and without gas pressure, the following equation can be derived using the work-energy reciprocal theorem with the inner normal direction of the air voids being the positive direction (Mavko and Mukerji 1995):

$$\Delta\sigma\Delta V_d - \Delta\sigma\Delta V_p = \Delta\sigma\Delta V_0 \tag{6}$$

where, $\Delta\sigma$ is the stress change of the porous asphalt mixture matrix and dry voids, $\Delta V_d$ is the voids bulk volume change of porous asphalt mixture in (b), $\Delta V_p$ is the voids volume change in (b), and $\Delta V_0$ is the voids bulk volume change of porous asphalt mixture in (a).

Using Equation (5) and the relationships between compressibility coefficients and bulk modulus in (a) and (b), the following relationship can be derived:

$$1/K_d = 1/K_0 + \phi/K_\phi \tag{7}$$

where, $\phi = V_p/V$ is the percentage of air voids of porous asphalt mixture, $K_d$ is the bulk modulus of dry porous asphalt mixture, $K_0$ is the matrix bulk modulus of porous asphalt mixture, and $K_\phi$ is the void bulk modulus.

According to (a) and (c) in Fig. 3, the void bulk modulus of porous asphalt mixture can be inferred under the saturation state in a similar way:

$$\frac{1}{K_s} = \frac{1}{K_0} + \frac{\Phi}{K_\Phi + \frac{K_0 K_f}{K_0 - K_f}} \tag{8}$$

The geometry compatibility condition is (Shi et al. 2000):

$$(\Delta\sigma - \Delta P)/K_\Phi + \Delta P/K_0 = \Delta P/K_f \tag{9}$$

where, $K_s$ is the bulk modulus of porous asphalt mixture under the saturation state, and $K_f$ is the fluid bulk modulus of porous asphalt mixture under the saturation state.

Equations (7) and (8) are the Gassmann's equations, and the bulk moduli of porous asphalt mixture are obtained by these two equations under the dry and saturation states.

## RESULTS AND DISCUSSION

### Reduction model of void bulk modulus for porous asphalt mixture

Because a porous asphalt mixture is a discontinuous medium with complex air voids distribution with no precise measurements in calculating the total air voids, it is difficult to interpret the air voids reduction in space and time. However, it can be solved by the continuum assumption of poroelasticity theory. Combined with dynamics variable, the void bulk modulus could be used to interpret the densification process. Equations (7) and (8) under the dry and saturation states can be transformed to:

$$\begin{aligned} K_\phi &= A\phi \text{ (dry state)} \\ K_\phi &= B\phi - C \text{ (saturation state)} \end{aligned} \tag{10}$$

where, A = $K_0K_d/(K_0 - K_d)$, B = $K_0K_s/(K_0 - K_s)$ and C = $K_0K_f/(K_0 - K_f)$.

By substituting Equation (4) into Equation (10), then Equation (11) could be obtained:

$$K_\phi = A(\phi + \phi_R) \text{ (dry state)}$$
$$K_\phi = B(\phi + \phi_R) - C \text{ (saturation state)} \quad (11)$$

**Example analysis**

An initial percentage of 20% was assumed for the air voids of the porous asphalt mixture. Taking the un-weathered granite as the skeleton material for the mixture, the skeleton bulk modulus was assumed to be 10% of the granite bulk modulus, i.e. 5773MPa. The fluid is mainly water in porous asphalt mixture under the saturation state. A water density of $1\times10^3$kg/m$^3$ and a longitudinal wave velocity in water of $1\times10^3$m/s were used in this study. Then a bulk modulus of 2250MPa was used for water. The relationships between the skeleton bulk modulus and the void bulk modulus under the dry and saturation states are shown in Fig. 4 and Fig. 5.

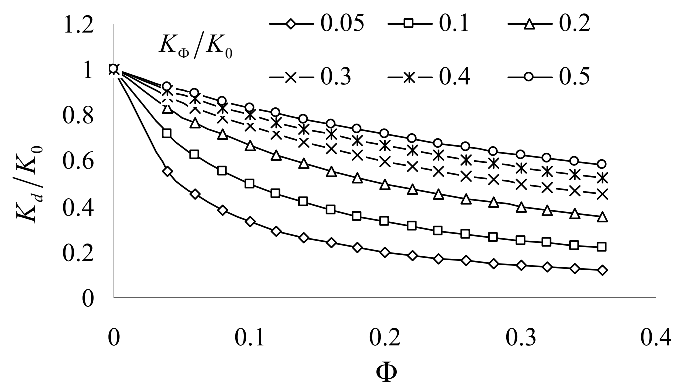



**FIG. 4. Skeleton bulk modulus versus void bulk modulus for dry state.**

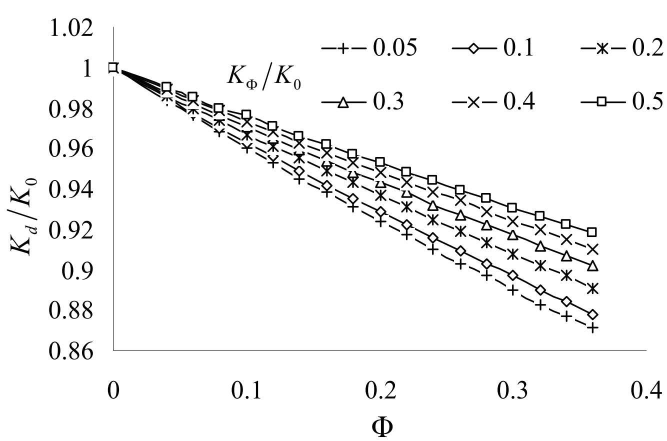



**FIG. 5. Skeleton bulk modulus versus void bulk modulus for saturation state.**

The curves from top to bottom correspond to $K_\phi/K_0$ of 0.5, 0.4, 0.3, 0.2, 0.1 and 0.05. For the dry state (Fig. 4), the bulk modulus of asphalt mixture increases as the void bulk modulus increases. The bulk modulus of the asphalt mixture reduces gradually with air voids towards a constant value. The inflection points can also be observed in Fig.4, which indicates that the air voids reduction can reach a constant value. For the saturation state (Fig. 5), the change laws of bulk modulus are similar to those in Fig. 4. but inflection points were not observed.

For $\phi = 0$, it is assumed that the percentage of air voids in porous asphalt mixture is 20%, $K_d$=0.5$K_0$, namely $\phi$=$\phi_R$=20%, $K_d$=2887MPa, and $K_s$=4660MPa. The known parameters are introduced into Equation (11). The void bulk modulus of porous asphalt mixture under dry state is $K_\phi$=1155MPa and that of under the saturation state is $K_\phi$=1147MPa. The results indicate that the void bulk moduli are mainly the same under two states. The air voids reduction is in the preliminary stage when the percentage of air voids is 20%.

According to Fig. 4, when the air voids reduction is 25% (i.e. the current percentage of air voids is 15%) for the porous asphalt mixture and for $K_d$=0.55$K_0$, then $K_d$ =3175MPa and $K_s$=4882MPa. The void bulk modulus of porous asphalt mixture under the dry state is $K_\phi$=1058MPa and that of under the saturation state is $K_\phi$=1058MPa. The void bulk modulus reduces by 8.4% and 7.8% after the air voids reduction under the dry state and the saturation state, respectively. The results indicate that the influence of the dry state is more significant than that of the saturation state on the void bulk modulus.

For $0<\phi<5\%$, the void bulk modulus under the dry state ranges between 1058MPa and 1155MPa. On the other hand, the void bulk modulus under the saturation state ranges between 1058MPa and 1147MPa. The scopes of voids bulk moduli, which change little, indicate that the voids reduction of porous asphalt mixture tends to be stable gradually under the repeated loading. For $5\%<\phi<20\%$, the voids bulk modulus reduces further.

## CONCLUSIONS

(1) The reduction of the void bulk moduli appears inflection points when the percentage of air voids is 10%~20% and then the reduction trends to be steady under the dry state. But the curves have no inflection points under the saturation state.

(2) The void bulk moduli were almost the same as the initial designed air voids under the dry and saturation states indicating that the air voids reduction is in the initial stage.

(3) The reduction trends of air voids are steady when the percentage of air voids is less than 15% and the reduction under the dry state is larger than that of under the saturation state.

Asphalt mixture is a typical viscoelasticity material. However, this paper is based on the linear elasticity and the influence of temperature on viscoelastic materials should not be ignored. Therefore, the influences of viscoelasticity and temperature should be considered in further researches.

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## REFERENCES

Cowin, S.C. and Nunziato, J.W. (1983). “Linear elastic materials with voids.” *Journal of Elasticity*, Vol. 13 (2): 125-147.

Gassmann, F. (1951). “Elastic waves through a packing of spheres.” *Geophysics*, Vol. 16: 673-685.

Li, C.G. and Wang, S.L. (2007). “Relationship between bulk modulus and porosity of porous medium.” *Rock and Soil Mechanics*, Vol. 28 (2): 293-296.

Mavko, G. and Mukerji, T. (1995). “Seismic pore space compressibility and Gassmann's relation.” *Geophysics*, Vol. 60 (6): 1743-1749.

Murali Krishnan, J. and Rengaraju, V.R. (1999). “Air voids reduction phenomena of asphalt concrete–A continuum approach.” *Int. J. of Fracture*, Vol. 97(1-4): 337-354.

Passman, S.L. (1984). “Stress relaxation, creep and hysteresis in linear elastic materials with voids.” *Journal of Elasticity*, Vol. 14 (2): 201-212.

Pei, J.Z., Chang, M.F., Chen, S.F. (2009). “Prediction model of bulk modulus reduction for porous asphalt pavement.” *J. of Traffic and Trans. Eng.*, Vol. 9(6): 7-10.

Rivadulla, F., Bãobre-López, M., Quintela, C.X., and Pieiro, A. (2009). “Reduction of the bulk modulus at high pressure in CrN.” *Nature Materials*, Vol. 8(12): 947-951.

Shi, G., Yang, D.Q. and Yang, H.Z. (2000). “The Research of Rock Poroelasticity.” *Acta Scientiarum Naturalium Universitatis Pekinensis*, Vol. 36 (2): 214-219.

Wang, Z.W., Tait, K., Zhao, Y.S., and Schiferl, D. (2004). “Size-induced reduction of transition pressure and enhancement of bulk modulus of AlN nanocrystals.” *Journal of Physical Chemistry B*, Vol. 108 (31): 11506-11508.

Xu S.F. (1994). “Asphalt surface course with permeability.” *J. of China & Foreign Highway*, Vol. (5): 39-42.

# Microstructure Index of Hot Mix Asphalt

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ABSTRACT: Digital image processing technique is used to quantify the impact of aggregate gradation and compaction method on air voids distribution and aggregate orientation and distribution. The air voids distribution was found to be sensitive to water permeability than traditional volume indexes. A more orizontal orientation was found for the aggregate particles compacted in the Superpave gyratory compactor. On the other hand, aggregate particles appear to have more of a random distribution in vibratory compactor samples. Aggregate distribution was found to be sensitive to the aggregate gradation and type of compaction.

Base on digital image process technique, analyze the microstructure of hot mix asphalt, and present a series of microstructure parameters including the air voids distribution, aggregate distribution and major axis angle distribution. The result illustrate that the microstructure indexes can evaluate the internal structure difference of hot mix asphalt.

## INTRODUCTION

The microstructure of hot-mix asphalt mixture (HMA) plays a significant role in mixtures' performance to rutting, fatigue and thermal cracking. The microstructure character is influenced by many factors, including gradation, aggregate shape, asphalt content and even the compaction type.

In recent years, some studies have used the digital image technique to analyze the microstructure of engineering materials. Because of their difference in colors, asphalt, air voids and aggregate can be distinguished and identified by image processing and analysis. Some researches (1, 2) have used 3-dimension computed tomography (CT) and 2-dimension image processing techniques to analyze air voids and asphalt mixtures' microstructure. In this research, the 2-D digital image processing and analysis technique was used to analyze the image that was captured from the surface of a hot-mix asphalt (HMA) core in an attempt to present a series of microstructure parameters for quantitative analysis of mixtures.

## DIGITAL IMAGE PROCESS AND ANALYSIS

The first step in any image processing is to calibrate the image in terms of physical measurement unit. This step ensures clear representation of physical measurement units in terms of pixels in an image. The next step is to perform segmentation, identify the image to background and the features of interest. Figure 1 shows the 2-D scanned image of a HMA. The difference in the intensity of the gray color allows for the determination of air voids, aggregate and asphalt mastic. For the purpose of this study, an image processing approach was developed and implemented in Matlab to determine air voids and aggregate with a size larger than 2.36 mm (3).

First air voids are identified by converting the scanned image to a binary image using a given pixel threshold as shown in Figure 2. Then the coarse aggregate and asphalt mastic are identified by converting the scanned image to a binary image using a different pixel threshold as shown in Figure 3.

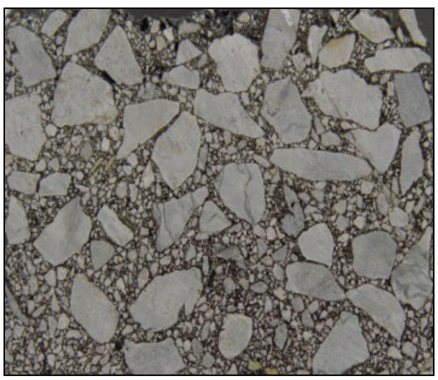

**FIG.1. Scanned image of HMA**

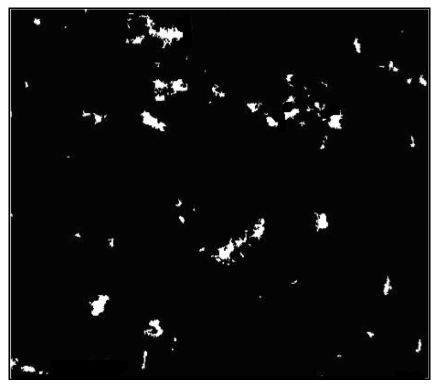

**FIG.2. Binary image of air voids**

**FIG.3. Binary image of aggregate and asphalt mastic**