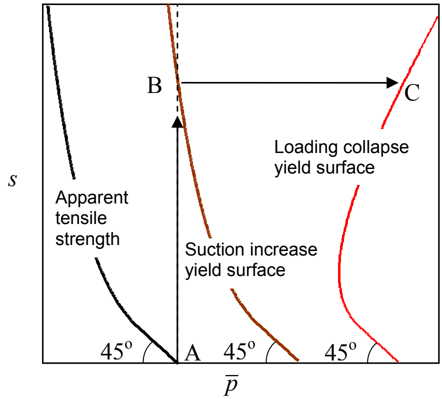

Figure 4. Yield stress variation with suction in the SFG model (Sheng et al. 2008a)

VOLUME CHANGE BEHAVIOUR

The volume change behaviour is one of the most fundamental properties for saturated and unsaturated soils. For unsaturated soils, it forms the base for the yield stress – suction relationship and the shear strength – suction relationship (Sheng et al. 2008a, Zhang and Lytton 2009). The volumetric model that defines the volume change caused by stress and suction changes should again be applicable to the entire range of possible pore pressures. A common starting point is the linear relationship between void ratio (e) and the logarithm effective mean stress ($\ln p'$) for a normally consolidated and saturated clay:

$$v = 1 + e = N - \lambda \ln p' = N - \lambda \ln\left(p - u_w\right) \tag{3}$$

where v is the specific volume, λ is the slope of the $v - \ln p'$ line, and N is the intercept on v axis when $\ln p'$=0.

We should note that equation (3) represents a line in the total stress space ($v - \ln p$) only if the pore water pressure is zero. If the pore water pressure were allowed to be kept at a negative value during the isotropic compression of the soil, equation (3) would predict a smooth curve in the $v - \ln p$ space, as shown by Figure 5. Indeed, these compression lines look very much like those for overconsolidated soils. However, the soil modelled by equation (3) is normally consolidated. The curvature of the normal compression lines is purely due to the feature of the logarithmic function. If the air entry suction for the soil in Figure 5 is larger than 100 kPa, the compression curves for s=10 and 100 kPa for the normally consolidated and saturated soil would take the shape as those in Figure 5. This is perhaps why the isotropic compression lines in Jennings and Burland (1962) and Cunningham et al. (2003) were curved for soils prepared from slurry states.

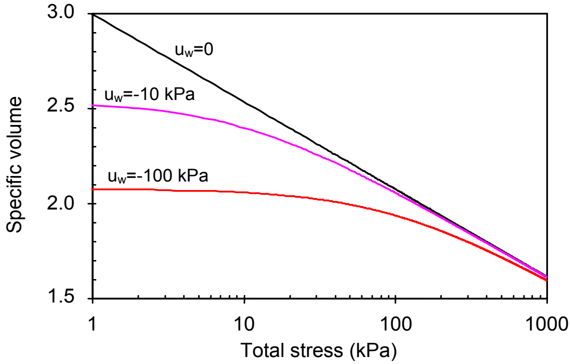

Figure 5. Normal compression lines for saturated clay under constant pore water pressures (λ=0.2, N=3)

Equation (3) can also be written in an incremental form:

$$\mathrm{d}v = -\lambda \frac{\mathrm{d}p}{p-u_\mathrm{w}} - \lambda \frac{\mathrm{d}(-u_\mathrm{w})}{p-u_\mathrm{w}} \tag{4}$$

The equation above clearly states that a change in pore water pressure has exactly the same effect on the volume of a saturated soil as an equal change in mean stress.

In the literature, equation (3) is extended to unsaturated states in one of the two approaches exclusively:

Approach A: Separate stress-suction (or the net stress and suction) approach.

Approach B: Combined stress-suction (or the effective stress) approach.

Approach A is perhaps first adopted by Coleman (1962) and is then used in many models such as Alonso et al. (1990), Wheeler and Sivakumar (1995), Cui and Delage (1995), and Vaunat et al. (2000). In this approach, the compressibility due to stress variation under constant suction is separated from the shrinkability due to suction variation under constant stress. A typical example is:

$$v = N - \lambda_\mathrm{vp} \ln \overline{p} - \lambda_\mathrm{vs} \ln\left(s + p_\mathrm{at}\right) \tag{5}$$

where λ_p is the slope of an assumed $v - \ln \overline{p}$ line or the compressibility due to stress change λ_s is the slope of an assumed $v - \ln s$ line or the shrinkability due to suction change, and p_at is the atmospheric pressure and used to avoid the singularity when s=0.

The main advantage of equation (5) is that the volume changes caused by stress and suction changes can be dealt with separately. Experimental data also indicate that the two compressibilities λ_vp and λ_vs can be very different (Toll 1990, Figure 6). It is usually true that the shrinkability (λ_vs) decreases with increasing suction or decreasing degree of saturation. On the other hand, the compressibility (λ_p) can increase with decreasing degree of saturation, due to the structural changes during drying processes (Toll 1990, Gallipoli 2003).

There are a few confusing points about equation (5). First, equation (5) does not recover equation (3) when the soil becomes saturated. Second, the atmospheric pressure (p_{at}) makes the change of suction insignificant when $s<p_{at}$. Third, the suction-compressibility is independent of stress. Fourth, the volume change becomes undefined at the transition suction between saturated and unsaturated states. The third point contradicts with experimental data (e.g. Delage and Graham 1996). The and first and last points were two of the main reasons that some researchers turned to the effective stress approach instead of the net stress approach (Sheng et al. 2003a, 2003b, Pereira et al. 2005, Santagiuliana and Schrefler 2006). A simple numerical example will illustrate the problem associated with equation (5). Let a soil be compressed at the transition suction (s_{sa}) from mean stress 1 kPa to 100 kPa. Let the air pressure remain atmospheric. In the saturated zone, the volume change according to equation (3) would be:

$$\Delta v\big|_{s_{sa}^{-}} = -\lambda_{vp} \ln\left[(100+s_{sa})/(1+s_{sa})\right]$$

In the unsaturated zone, the volume change according to equation (5) would be:

$$\Delta v\big|_{s_{sa}^{+}} = -\lambda_{vp} \ln 100$$

These two volume changes can be very different.

In Approach B, suction and stress are combined into one single variable (effective stress) to define their effects on soil volume. A general form of the effective stress is:

$$p' = \overline{p} + f(s) = \overline{p} + f(S_r, s) \tag{6}$$

where f is either a function of suction or a function of suction and degree of saturation. Obviously such a definition of effective stress is very general and covers most of existing definitions in the literature. With such an effective stress, equation (3) is assumed to be valid for unsaturated states via the following form:

$$v = N - \lambda \ln p' = N - \lambda\,(s) \ln\left(\overline{p} + f(S_r, s)\right) \tag{7}$$

In equation (7), the coefficient λ is generally assumed to be a function of suction. However, the parameter N should generally be independent of suction. Otherwise, the volume of the soil would change even if the effective stress (p') was kept constant,

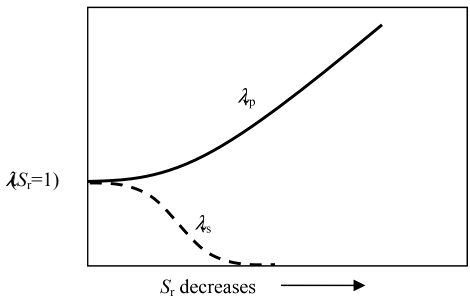

Figure 6. Variation of compressibilities with degree of saturation (after Toll 1990)

and an additional function would then be required to define the volume change caused by suction change under constant effective stress.

Equation (7) is widely used in the literature and in fact most models based on the effective stress approach adopt it as the volume change equation (Kohgo et al. 1993, Bolzon et al. 1996, Sheng et al. 2004, Li 2007, Sun et al. 2007a). If the function f is properly chosen, equation (7) should recover equation (3) when the soil becomes saturated. This is one of the greatest advantages of using an effective stress.

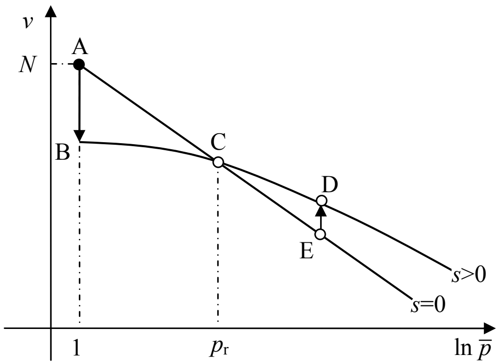

Figure 7. Normal compression lines according to the effective stress approach

However, there are a couple of issues with equation (7). The obvious issue is the difficulty in addressing the different compressibilities due to stress and suction changes, as shown in Figure 6. The second issue is related to a constraint on the compressibility λ Let a saturated slurry soil be dried from zero suction to an arbitrary suction under constant mean stress of 1 kPa, i.e. the stress path AB in Figure 6. As discussed above, this drying path is elastoplastic, not elastic. The volume of the soil then changes according to:

$$v_B = N - \lambda(s)\ln\left(1 + f(s)\right) \tag{8}$$

Now let the soil be compressed under constant suction, i.e. stress path BC in Figure 7. The compression line will be curved in the v - $\ln \bar{p}$ space, due to the term $f(s)$. If the suction at point B is above the air entry value, this compression line is anticipated to intersect with the initial compression line for the saturated soil, as shown in Figure 7. Let the intersection be point C. The volume at point C is then:

$$v_C = v_B - \lambda(s)\ln\left(\frac{p_r + f(s)}{1 + f(s)}\right) = N - \lambda(s)\ln\left(p_r + f(s)\right) \tag{9}$$

Alternatively alone path AC the volume changes according to:

$$v_C = N - \lambda(0)\ln\left(p_r + f(0)\right) = N - \lambda(0)\ln p_r \tag{10}$$

Therefore, we have

$$\frac{\lambda(s)}{\lambda(0)} = \frac{\ln p_{\mathrm{r}}}{\ln\left(p_{\mathrm{r}} + f(s)\right)} < 1 \tag{11}$$

We usually anticipate the effective stress increases with increasing suction, at least for low suction values. Therefore, we have: $\lambda(s) < \lambda(0)$, meaning that the slope of the compression line must decrease with increasing suction. Such a conclusion is however not supported by experimental data. In the data by Jennings and Burland (1962) for air-dry soils, the slope of the compression lines at different suctions (saturation) is more or less constant. In the data by Sivakumar and Wheeler (2000) for compacted soils, the slope of the compression lines increases somewhat with increasing suction. In addition, experimental data on collapse (e.g. Sun et al. 2007b) do not support an increasing collapse volume with increasing mean stress.

Another issue associated with equation (7) about the volume change during drying at high stresses. Let a saturated slurry soil be consolidated to a mean stress larger than $\bar{p}_{\mathrm{r}}$ (say point E) and then dry the soil to the suction at point B. Because the volume change according to equation (7) is stress-path independent, the stress point at the end of the drying path ED must be on the compression curve BCD. Since the drying path is elastoplastic (not purely elastic), as shown in Figure 2, it is only possible to have a volume increase to research point D during the drying path ED. Clearly, this is not reasonable. Therefore, further research is required if equation (7) is adopted for the volume change, particularly in terms of explaining the collapse volume. Realising that the degree of saturation can increase during isotropic compression under constant suction, we note that a possible solution to some of the issues mentioned above is to have the slope λ as a function of the degree of saturation instead of suction. As such, $\lambda(S_{\mathrm{r}})$ can increase as S_{r} increases, even though $\lambda(S_{\mathrm{r}}) \leq \lambda(S_{\mathrm{r}} = 1)$. Further exploration in this direction is worthwhile.

Sheng et al. (2008a) proposed a third way to model the volume change for unsaturated soils. The new model, referred to as the SFG model, represents a somewhat middle ground between Approach A and Approach B and is expressed in an incremental form as follows:

$$\mathrm{d}v = -\lambda_{\mathrm{vp}} \frac{\mathrm{d}\bar{p}}{\bar{p} + f(s)} - \lambda_{\mathrm{vs}} \frac{\mathrm{d}s}{\bar{p} + f(s)} \tag{12}$$

Like Approach A, equation (12) separates the compressibilities due to stress and suction changes. Like Approach B, it also recovers equation (3) when the soil becomes saturated. However, there is no constraint on parameter λ_{vp}. As a first approximation, λ_{vp} can be assumed to be independent of suction, as indicated by the data of Jennings and Burland (1962) for reconstituted soils. More realistically it should depend on suction and perhaps on degree of saturation as well. The data of Sivakumar and Wheeler (2000) shows λ_{vp} increases with increasing suction for compacted soils. Parameter λ_{vs} must equal λ_{vp} when the soil is fully saturated and it decreases with increasing suction. Sheng et al. (2008a) suggested the following function for λ_{vs}

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s \le s_{sa} \\ \lambda_{vp} \dfrac{s_{sa}+1}{s+1} & s > s_{sa} \end{cases} \tag{13}$$

where s_{sa} is the saturation suction which is the unique transition suction between saturated and unsaturated states (see Sheng et al. 2008a). We note that the number '1' in equation (13) is used to avoid the singularity when s_{sa}=0 and is not needed if s_{sa} is larger than zero:

$$\lambda_{vs} = \begin{cases} \lambda_{vp} & s \le s_{sa} \\ \lambda_{vp} \dfrac{s_{sa}}{s} & s > s_{sa} \end{cases} \tag{14}$$

The difference between equation (13) and (14) is minimal, but equation (14) is preferred to equation (13). Equation (14) can be applied as long as the transition suction is not absolutely zero. Both λ_{vp} and λ_{vs} vary with stress path and can take different values on a loading and unloading path respectively.

The function $f(s)$ in equation (12) can also take different forms. Sheng et al. (2008a) initially used the simplest form possible:

$$f(s) = s \tag{15}$$

Even with this simplest form, Zhou and Sheng (2009) showed that the SFG model is able to predict a good set of experimental data on volume change and shear strength, both for reconstituted soils prepared from slurry and for compacted soils.

An alternative form of $f(s)$ could be:

$$f(s) = S_r s \tag{16}$$

This equation would also guarantee the continuity between saturated and unsaturated states. More interestingly, both equations (15) and (16) lead to the same yield stress – suction and shear strength – suction relationships in the SFG model. However, the validity of equation (16) is yet to be tested against experimental data.

SHEAR STREHGTH

The shear strength is related to the apparent tensile strength function, which is related to the volume change equation (Sheng et al. 2008b). However, this relationship has been overlooked in most existing models for unsaturated soils. If the slope of the critical state line is assumed to be independent of suction, such as supported by experimental data by Toll (1990), the shear strength – suction relationship can indeed be derived from the volume change equation. In the case that the slope of the critical state line depends on suction, two equations are needed to define the shear strength – suction relationship: the volume change equation and the $M(s)$ function, with M being the slope of the critical state line in the deviator – mean stress space.

Fredlund et al. (1978) proposed the following relationship which conveniently separates the shear strength due to stress from that due to suction:

$$\tau=\left[c'+(\sigma_{\mathrm{n}}-u_{\mathrm{a}})\tan\phi'\right]+\left[(u_{\mathrm{a}}-u_{\mathrm{w}})\tan\phi^{\mathrm{b}}\right]=\overline{c}+(\sigma_{\mathrm{n}}-u_{\mathrm{a}})\tan\phi' \quad (17)$$

where τ is the shear strength, c' is the effective cohesion, σ_{n} is the normal stress on the failure plane, ϕ' is the effective friction angle of the soil, ϕ^{b} is the frictional angle due to suction, and $\overline{c}$ is the apparent cohesion due to suction.

In the case that ϕ' is independent of suction, the shear strength – suction relationship for unsaturated states is solely determined by the apparent tensile strength function $\overline{p}_0$. For example, in the SFG model, the apparent cohesion is

$$\overline{c}=-\overline{p}_0\tan\phi'=\begin{cases} s\tan\phi' & s<s_{\mathrm{sa}},c'=0 \\ \tan\phi'\left(s_{\mathrm{sa}}+(s_{\mathrm{sa}}+1)\ln\dfrac{s+1}{s_{\mathrm{sa}}+1}\right) & s\geq s_{\mathrm{sa}},c'=0 \end{cases} \quad (18)$$

This apparent cohesion can then be used to derive the friction angle ϕ^{b} due to suction.

CONCLUSIONS

This paper presents a critical review of some common concepts used in constitutive modelling of unsaturated soils. These concepts include net stress, matric suction, yield stress, apparent consolidation, apparent tensile strength, loading-collapse yield surface, volume change model, shear strength and effective stress. They are often used in the literature without clear definitions and can sometimes cause confusion. Realizing the fact that all soils can be partially saturated with water and partial saturation is a state of any soil, we can define these concepts in a more consistent way. For example, the matric suction can be understood as negative pore water pressure when the air pressure is atmospheric. The loading-collapse yield surface and the apparent tensile strength surface can be understood as the extension of the bounds of the elastic zone to the negative pore water pressures (or the extension of the bounding surfaces to the negative pore water pressures in a bounding surface model). A constitutive model for a soil should therefore cover all possible ranges of stress and pore pressures variations.

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Elastoplastic Modelling of Hydraulic and Mechanical Behaviour of Unsaturated Expansive Soils

W. Sun[1], D. Sun[2], and J. Li[3]

[1] Department of Civil Engineering, Shanghai University, 149 Yanchang Road, Shanghai 200072, China, e-mail: wjsun@shu.edu.cn
[2] Department of Civil Engineering, Shanghai University, 149 Yanchang Road, Shanghai 200072, China, e-mail: sundean@shu.edu.cn
[3] School of Civil, Environmental and Chemical Engineering, RMIT University, GPO Box 2476V, Melbourne, VIC 3001 Australia, e-mail: jie.li@rmit.edu.au

ABSTRACT: The Barcelona Expansive Model (BExM) for unsaturated expansive soils is complicated and the micro parameters and the coupling function from micro-structural strain to macro-structural strain are difficult to be determined. This paper presents an elastoplastic constitutive model from the macroscopic observation for predicting the hydraulic and mechanical behaviour of unsaturated expansive soils. The model predictions are performed on the results of triaxial tests on compacted unsaturated expansive clay available in the literature. The comparisons between measured and predicted results indicate that the model offers great potential for quantitatively predicting the hydraulic and mechanical behaviour of unsaturated expansive soils.

INTRODUCTION

Since the Barcelona Basic Model (BBM) was published by Alonso et al. (1990), a number of elastoplastic constitutive models for unsaturated soil have been proposed. Although these models can describe most basic mechanical response of non-expansive unsaturated soil including collapse phenomenon, they cannot predict the mechanical behaviour of unsaturated expansive soil. Gens and Alonso (1992) presented a framework for describing the mechanical behaviour of unsaturated expansive soil. Alonso et al. (1999) presented the Barcelona Expansive Model (BExM) in which two levels of structure were considered. The behaviour of the macrostructure follows the BBM, and that of the microstructure is adopted from the framework proposed by Gens and Alonso (1992). In the BExM, the micro parameters and the coupling function from micro-structural strain to macro-structural strain are difficult to be determined. Moreover, the models can only predict the strength and stress-strain behaviour without incorporating the water retention behaviour.