

$$\text{Order } p_{cr} = N_q \gamma D + N_c c \tag{12}$$

$$\text{Where } N_q = \frac{2(1-\mu) \sin \varphi (\cot \varphi + \varphi - \frac{\pi}{2}) + \pi (\sin \varphi + 2\mu - 1)}{2(1-\mu) \sin \varphi (\cot \varphi + \varphi - \frac{\pi}{2})} \tag{13}$$

$$N_c = \frac{\pi \cot \varphi}{\cot \varphi + \varphi - \frac{\pi}{2}} \tag{14}$$

If the depth of the plastic zone allowed to carry out in the foundation  $z_{max} = B/4$  (B is the width of the base), we substitute it into equation (15), similarly, we can obtain the simplified expression of the corresponding critical load  $p_{1/4}$

$$p_{1/4} = N_r \gamma B + N_q \gamma D + N_c c \tag{15}$$

$$\text{Where } N_r = \frac{\pi (\sin \varphi + 2\mu - 1)}{8(1-\mu) \sin \varphi (\cot \varphi + \varphi - \frac{\pi}{2})} \tag{16}$$

Other symbols are the same in the equations (13) , (14).

$N_q$   $N_r$   $N_c$  are called the capacity factors which are only related to the internal friction angle and Poisson’s ratio of soil. Under the different Poisson’s ratios of soil, the values of  $N_q$   $N_r$   $N_c$  change with the internal friction angle as shown follows.

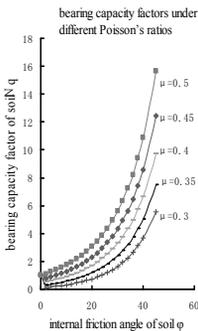


Fig. 1  $N_q$

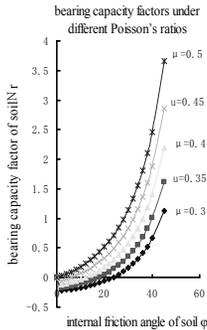


Fig. 2  $N_r$

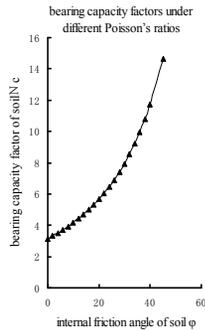


Fig. 3  $N_c$

Fig. 1, Fig. 2, Fig.3 stand for the variation curve of the values of  $N_q$ ,  $N_r$  and  $N_c$  with the change of the angle of internal friction under the different Poisson’s ratios of soil.

Fig. 1 shows that the bearing capacity factor  $N_q$  increased with the Poisson’s ratio  $\mu$  increasing , and got the maximum at  $\mu=0.5$ , but the trend remained constant.

Fig.2 shows that the bearing capacity factor  $N_r$  increased with the Poisson’s ratio  $\mu$  increasing, and got the maximum at  $\mu=0.5$ , but the trend remained constant.

Fig.3 shows that the bearing capacity factor  $N_c$  was only related with the internal friction angle  $\phi$ , and  $N_c$  would be larger with the increased internal friction angle  $\phi$ .

Description: In the application of the equation (12), (15), we need to pay attention to the following points:

- 1) All formulas are derived based on the strip foundation, and the solution to the strip foundation involves the problem of solving the elastic plane strain. It is safer to apply the presented results to the rectangular, square, circular foundation.
- 2) All formulas are derived based on concentrated load or uniformly distributed load based on the strip foundation. Therefore, if there is a significantly eccentric or inclined load in the actual project, we need to modify or re-derived the formula above.
- 3) Poisson's ratio  $\mu$  is kept constant on computing the lateral pressure coefficient of soil  $K_0$ . Nevertheless, the actual soil is anisotropic in the vertical, and Poisson's ratio  $\mu$  has changed since the soil transitioned from the elastic zone to the plastic zone. Therefore, it has some differences to consider that Poisson's ratio  $\mu$  is a constant in calculating with the actual situation.

## CONCLUSIONS

This paper considered the different Poisson's ratio and derived the expression of the critical edge load, edge load and the depth the plastic zone reached, which are much closer to the actual situation. In addition, we also obtained the bearing capacity coefficient changing with the Poisson's ratio of soil, but its value was only related with the internal friction angle and Poisson's ratio. When the internal friction angle is constant, with the increase of Poisson's ratio, the bearing capacity factor increases gradually, so does the theoretical bearing capacity of foundation. Therefore, the consideration of appropriate Poisson's ratio in the actual engineering projects to some extent solves the over-conservative situation of calculating the bearing capacity of subsoil.

## ACKNOWLEDGEMENTS

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## HEAT TRANSFER CAPACITY OF HEAT EXCHANGER PILES IN SOFT CLAY

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**ABSTRACT:** The heat exchanger pile (HEP) is a new type of pile system that combines conventional piles with ground source heat pumps. It has its own characteristics, advantages, and application status, as presented in this paper with its design model and layout pattern explored in detail and its heat transfer capacity calculation discussed with a focus on soft clay ground in coastal regions. The calculations shown in this paper were applied to a six-story residential building in Shanghai. In general, HEP is significantly more cost-effective and environmentally-friendly than a traditional foundation pile.

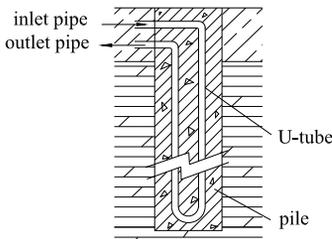
### INTRODUCTION

Geothermal energy is abundant, clean, and renewable; and has a great potential to use with broad application prospects (HARSH, 2007). However, with current available technology, exploring deep geothermal resources is difficult and cost prohibitive, but the exploration and use of shallow geothermal resources are not that difficult as compared to the deep geothermal resource exploration and have become an available method of using ground heat energy (SANNER, 2001).

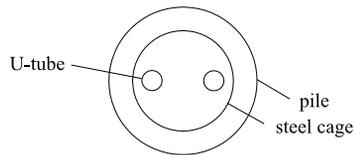
The heat exchanger pile (HEP) technology is a pile system that combines the conventional foundation piles and ground source heat pumps. It can both provide the mechanical function of the conventional foundation and achieve the heat exchange of shallow geothermal resources through the interaction of piles and the ground. Therefore, it can play the dual role as both the foundation piles and ground source heat pumps, with a cost effective use of construction land.

**HEP CONCEPT**

The pile foundation is widely used in coastal regions with soft clay of the Quaternary sedimentary soil. High water content and low bearing capacity make local residential buildings generally adopt the pile foundation. While connecting with the pipe surface, HEP achieves the geothermal energy exchange with upper structures through a tube heat exchanger embedded inside, which is filled with an energy exchanger fluid, such as water or water mixed with antifreeze, etc. According to the different patterns of the heat exchanger buried under the pile, the heat exchanger can be divided into the U-tube, tube, and single tube (HAMADA, 2007). The U-tube system can also be sub-divided into single U-tube, double U-tube, and multiple U-tube heat exchanger piles, respectively. Figure 1 and Figure 2 show the diagram and cross-section of a single U-tube heat exchanger pile. Currently in China, the engineering cases that adopt the heat exchanger pile system mostly use the single U-tube heat exchanger pile. Experimental studies showed that the heat capacity of the single-tube within single pile per unit length was increased by 20% -25% more than the double-tube within single pile (Gong, 2002).



**Figure 1. Diagram of HEP**



**Figure 2. Cross-section of HEP**

**HEAT TRANSFER CAPACITY OF HEP**

Although the HEP design can be compared to other forms of ground-source heat pump pipes, it has its own special characteristics. First of all, once the pile size and length are selected, the number of piles for pipes is also determined. That is, we cannot increase the number of piles according to the change of the hot and cold load. When the hot and cold load provided from the heat exchanger piles can not meet the necessary requirements, we have to consider other forms of pipes or increase the cooling towers and boilers, and other supportive measures.

We also need to take the heat balance issues of the ground heat exchanger into account when designing a HEP system. For the larger gap in the case of hot and cold load, we can select a smaller load value as the basis for determining the length of pipe. But for a relatively large load, we could choose support measures to solve them. Being similar to other forms of heat pump systems, soil's geothermal characteristics are equally important in HEP as in other systems. We need to measure the parameters of the soil composition, soil temperature, thermal conductivity, heat capacity, water quality characteristics, and the flow direction and speed. Each of these parameters has an important impact on the optimization of the entire system.

HEP is a closed loop combined ground source heat pump system that uses underground heat in soils. When winter heating, the fluid in the heat exchanger of HEP crosses the inside and outside layers of thermal resistance of the pile, followed by crossing the soil, pile wall, reinforced concrete of pile, U-tube wall, etc., collects heat from the underground and takes the heat to the interior through the system. While summer cooling, the system reverses the operation and takes the heat away from the room, again crossing U-tube wall, reinforced concrete of pile, pile wall, soil mass, and other related entities, transferring the heat to the earth. Therefore, the heat exchanger piles achieve the heat transferring through the U-tube's heating hot and cold process.

It is generally assumed that when the distance between piles is more than 6 times the length of a pile diameter, the impact of pile interaction can be ignored. We can also often neglect the impact of inter-piles when calculating the heat exchange capacity. For the winter and summer conditions, the engineering technical specifications of ground source heat pump system (GB50366, 2005) shows the heating capacity and cooling capacity of U-tube  $Q_c$  and  $Q_c'$  as:

$$Q_c = \frac{L_c(t_{max} - t_{\infty})}{R_f + R_{pe} + R_b + R_s \times F_C + R_{sp}(1 - F_C)} \quad (1)$$

$$Q_c' = \frac{L_c(t_{\infty} - t_{min})}{R_f + R_{pe} + R_b + R_s \times F_C + R_{sp}(1 - F_C)} \quad (2)$$

$$F_C = T_{c1} / T_{c2} \quad (3)$$

where:  $F_C$  is the heating (cooling) run share;  $T_{c1}$  is the operating hours of water source heat pump in a cooling season per month and determined in the hottest month's operating;  $T_{c2}$  is the number of hours in the heating season per month and determined in the coldest month's hours;  $L_c$  is the length of the U-tube;  $t_{max}$  is the design average temperature of heat transfer medium in HEP heat exchanger in the winter;  $t_{min}$  is the design average temperature of heat transfer medium in HEP heat exchanger in the summer;  $t_{\infty}$  is the initial temperature of the regional rock and soil.  $R_f$  is the heat transfer resistance between the heat transfer medium and the U-tube wall;  $R_{pe}$  is the thermal resistance of U-tube wall;  $R_b$  is the thermal resistance of pile's reinforced concrete of the heat exchange piles; and  $R_s$  is the thermal resistance considering heat exchange from the pile wall to infinity, also called thermal resistance of the stratum.  $R_{sp}$  is the additional thermal resistance caused by continuous pulse load.

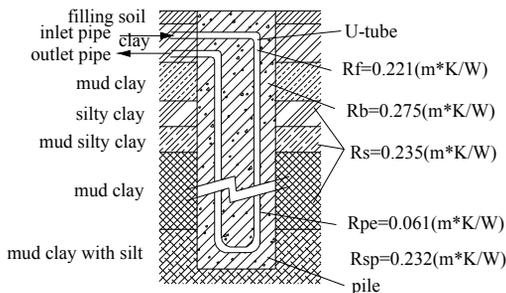
FU (2002) made a more detailed description on the values of the five major resistances of  $R_f$ ,  $R_{pe}$ ,  $R_b$ ,  $R_s$ ,  $R_{sp}$ , and analyzed the HEP heat transferring. The author suggested that, in the situation of actual heat transferring, the thermal conductivity of reinforced concrete and soil can be treated as same and the additional thermal resistance caused by the continuous pulse load can often be ignored.

The heat exchange tubes should be designed to ensure the water balance in the pipe system. Any local failure in the system should not affect the normal operation of the whole system.

**CASE STUDY OF HEP**

A 6-story residential building with a total construction area of 2025 m<sup>2</sup> is located in Shanghai. Its cooling load indicator is 60W/m<sup>2</sup> for the residential living space (Zhu, 1995). The bathroom and kitchen areas are not air-conditioned, so the air-conditioned area is equivalent to the living area. The living space of unit A is 87m<sup>2</sup>, with an air-conditioned load of 5220W; unit B's living space is 79m<sup>2</sup>, with an air-conditioned load of 4740W; unit C's living space is 78m<sup>2</sup>, with an air-conditioned load of 4680W. The heat exchanger of HEP for the residential building should bear a load of 14640 × 6 = 87840W. The buried pipes are designed with PVC (polyvinyl chloride). The calculation shows that the heat exchange capacity of the pipe pile is about 35.2 W / m.

The shallow soil in Shanghai is the Quaternary deposits with the geological condition from the surface down in the order of filling soil, clay, mud clay, silty clay, mud silty clay, mud clay, mud clay with silt, and other folders. However in this paper, the values adopted in the heat transfer calculation refer to the ones of the soft clay with the corresponding parameters similar to the soil in Shanghai. The main calculated parameters are shown in Figure 3:



**Figure 3.** Thermal resistances' values of heat exchanger piles in ground

The meaning of parameters in the figure can be explained as follows.  $t_{max}$  is the design average temperature of heat transfer medium in the heat exchanger of HEP in the winter, usually 37°C;  $t_{min}$  is the average temperature of heat transfer medium in the heat exchanger of HEP in the summer, usually -2 to 5°C, here 2°C;  $t_{\infty}$  is the initial temperature of the regional rock and soil of HEP, usually 15°C; and the inner diameter of U-tube  $d_i$  here is 40mm. According to  $K=Nu_p \lambda_p / l$ , we can conclude convective heat transfer coefficient  $K = 36.14 \text{ W/m}^2 \cdot \text{K}$ . Here,  $Nu_p$  is the Nusselt coefficient and  $l$  is the unit length of the tube. As a single U-tube, thermal conductivity  $\lambda_p$  is taken as 0.4 [W/(m\*K)]; the U-tube's diameter  $d_0$  and U-tube's equivalent diameter  $d_e$  are taken 50mm and 70mm respectively;  $\lambda_b$  is the thermal conductivity of reinforced concrete which can be up to 1.54[W/(m\*K)]; and  $d_b$  is the diameter of the pile which is taken as 1m. The average thermal conductivity of soil  $\lambda_s$ , and thermal diffusivity of soil are taken as 1.8[W/(m\*K)] and  $7 \times 10^{-7} \text{ m}^2/\text{s}$ , respectively. The HEP' radius  $r_b$  is taken as 0.5m and running time  $\tau$  and  $r_p$  are taken as 8h.

The winter heating and summer cooling capacity obtained by the heat exchange capacity formula are 55.1W/m and 32.5W/m, respectively, which is close to the summer heat exchange capacity of pipe piles 35.2 W/m obtained from the literature (ZHU,1995). The result is consistent with experimental data obtained from the same literature. It can be seen that in this case the calculation results are consistent with the observed status and the calculation process can be used in the future as useful reference for engineering.

## CONCLUSIONS

The HEP system combines conventional piles and ground source heat pumps, providing an innovative method that is economical, environment-friendly, and energy-saving. The calculation theory of heat transfer for HEP described in this article focuses on the soft clay's feature in Shanghai, which analyzes the heat exchange capacity of a residential building and verifies the application feasibility of HEP. It concludes that the related theoretically calculated results fit quite well with the data from real constructions, from which it has a guiding role in promoting the use of HEP in coastal soft clay regions. Since most researches studying HEP are still in the theoretical stage, the construction and application of HEP have to be further verified.

## ACKNOWLEDGEMENTS

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## AN EMPIRICAL STUDY ON THE ESTIMATION OF SOIL PROPERTIES OF LOESS GROUND AFTER DYNAMIC COMPACTION

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**ABSTRACT:** Using dimensional analysis, the dimensional equations for the estimation of soil properties after dynamic compaction (DC) in loess ground are established. Based on these equations and field data obtained from previous practices of DC, the equations are fitted relating soil properties after DC with construction parameters, soil properties before tamping, and even with the surficial deformation during tamping. Finally, the equations are verified by several project examples, and the results indicate that the calculated results agree well with the field measured results. These equations provide a practical way for the prediction of the soil properties after DC for loess.

### INTRODUCTION

Estimating soil properties after dynamic compaction (DC) represents an important research topic in the application of this ground improvement technique. Some progress has been made in the past years. Mayne et al. (1984) have drawn scatter diagrams to show the relationship between energy of tamping on a unit area and some soil parameters, such as the pressuremeter limit pressure and pressuremeter modulus. By applying the results of BEM, Qian et al. (1986, 1987) obtained a diagram relating the penetration per blow and tamping energy per unit area of hammer bottom. Lukas (1980, 1981) presented the limit values of some soil properties which can be reached by DC. Lo et al. (1990) also made contribution by presenting equations to calculate the average ground pressuremeter modulus ratio and the average limit pressure ratio. Based on the Biot theory, Zhou (1998) further proposed a method to compute the modulus of compressibility and seepage coefficient from the data obtained from the trial tamping. Luo (2008) worked out a method for estimating the improvement in sands by taking advantage of previous model tests and the quasi-static method. Kong et al. (1999) proposed a single-valued back analysis method by which the modulus of deformation of tamped ground can be deduced using a curve relating the calculated settlement and modulus of deformation. However, these research findings are not widely recognized, nor are they incorporated into design specifications. The current design of DC can only estimate the improvement depth quantitatively, yet there has been no generally accepted

equation to empirically estimate the soil properties after tamping. It is important, therefore, to improve the design and calculation method for DC.

In this paper, using dimensional analysis, the dimensional equations for estimating of soil properties of loess after DC will be established. Based on the field data obtained from previous engineering practices of DC, the fitted equations are established. The equation can be used to predict the soil properties of loess after tamping through construction parameters, soil properties before tamping, the surficial deformation during tamping, and provide a practical way for DC design in loess.

## EMPIRICAL EQUATIONS RELATING SOIL PROPERTIES BEFORE AND AFTER DC

### The Dimensional Equations

The soil properties after tamping are the dry bulk density  $\gamma'_d$ , modulus of compressibility  $E'_s$ , ground bearing capacity  $f'_s$ , and the soil properties before tamping are dry bulk density  $\gamma_d$ , modulus of compressibility  $E_s$ , ground bearing capacity  $f_s$ , water content  $\omega$ , void ratio  $e$ . The construction parameters are average tamping energy  $E_{av}$ , tamping pass  $P$ , diameter of hammer  $D$ , the empirical equations which relate the soil properties after tamping with the construction parameters and the soil properties before tamping could be established as follows:

$$F(E_{av}, P, D, \gamma_d, \omega, \gamma'_d) = 0 \quad (1)$$

$$F(E_{av}, P, D, \gamma_d, E_s, E'_s) = 0 \quad (2)$$

$$F(E_{av}, P, D, \gamma_d, E_s, f_s, f'_s) = 0 \quad (3)$$

Through dimensional analysis, the empirical Equations 1, 2 and 3 can be transformed into:

$$F(\pi_1, \pi_7, \pi_8, \pi_9) = 0 \quad (4)$$

$$F(\pi_2, \pi_8, \pi_9, \pi_{10}) = 0 \quad (5)$$

$$F(\pi_3, \pi_8, \pi_9, \pi_{10}, \pi_{11}) = 0 \quad (6)$$

where:

$$\pi_1 = \frac{\gamma'_d D^2}{E_{av}} \quad \pi_2 = \frac{E'_s D}{E_{av}} \quad \pi_3 = \frac{f'_s D}{E_{av}} \quad \pi_7 = \omega \quad \pi_8 = P \quad \pi_9 = \frac{\gamma_d D^2}{E_{av}} \quad \pi_{10} = \frac{E_s D}{E_{av}} \quad \pi_{11} = \frac{f_s D}{E_{av}}$$

**Empirical Equations for Estimating Soil properties After Tamping**

Using data collected from 23 sets of previous engineering practices (see the references, when soil parameters are used, the weighted average of the parameters within effective reinforcement depth is adopted), the values of dimensionless quantities in the Equations 4, 5 and 6 are obtained. After the multiple regression analysis by means of the MATLAB software, the fitted equations have been established, which can be applied to predict soil properties after tamping with reference to soil properties before tamping and construction parameters.

$$\pi_1 = -0.0004 - 0.0325\pi_7 + 0.001\pi_8 + 1.2063\pi_9 \tag{7}$$

$$\pi_2 = -0.001 - 0.0005\pi_8 + 0.3371\pi_9 + 0.6394\pi_{10} \tag{8}$$

$$\pi_3 = 0.0069 - 0.0028\pi_8 + 3.2615\pi_9 + 3.4172\pi_{10} + 0.6503\pi_{11} \tag{9}$$

Substituting the dimensionless quantities into the previous equations, Equations 7, 8, and 9 can be transformed into:

$$\gamma'_d = \left[ 0.0004 - 0.0325\omega + 0.001P + 1.2036 \frac{\gamma_d D^2}{E_{av}} \right] \frac{E_{av}}{D^2} \tag{10}$$

$$E'_s = \left[ -0.001 - 0.0005P + 0.3371 \frac{\gamma_d D^2}{E_{av}} + 0.6394 \frac{E_s D}{E_{av}} \right] \frac{E_{av}}{D} \tag{11}$$

$$f'_s = \left[ 0.0069 - 0.0028P + 3.2615 \frac{\gamma_d D^2}{E_{av}} + 3.4172 \frac{E_s D}{E_{av}} + 0.6503 \frac{f_s D}{E_{av}} \right] \frac{E_{av}}{D} \tag{12}$$

**EMPIRICAL EQUATIONS RELATING SOIL PROPERTIES AFTER DC WITH CONSTRUCTION PARAMETERS, SOIL PROPERTIES BEFORE TAMPING AND SURFICIAL DEFORMATION DURING TAMPING**

**The Dimensional Equations**

Suppose the dry bulk density, modulus of compressibility, and ground bearing capacity after tamping are  $\gamma'_d$ ,  $E'_s$  and  $f'_s$  respectively, and the soil properties before tamping are dry bulk density  $\gamma_d$ , modulus of compressibility  $E_s$ , ground bearing capacity  $f_s$ , water content  $\omega$ , void ratio  $e$ , and the construction parameters are average tamping energy  $E_{av}$ , tamping pass  $P$ , diameter of hammer  $D$ , and the weight of hammer  $W$ . The equations, which relate the soil properties after tamping with soil properties before tamping, construction parameters, as well as the surficial deformation during tamping, could be written as:

$$F(E_{av}, S_e, D, W, P, \gamma_d, \gamma'_d) = 0 \tag{13}$$