(See Section 12.9.2.2.2). This is necessary for preserving the signs of displacement and force results.

The disadvantage of the second approach is that moving the center of mass affects the modal properties (mode shapes and frequencies) of the system, and this may complicate the formation of load combinations because such combinations must be made from separate analytical models. Some software available for dynamic analysis automatically adjusts the mass distribution to accommodate the required mass eccentricity and simplifies the load combinations among different runs. An advantage of the second approach is that torsional amplification need not be included ($A_x = 1.0$) (see Sections 12.9.1.5 and 12.9.2.2.2).

Procedure for Shifting Center of Mass

Supplement 2 of ASCE 7-16 restricts the use of mass offsets in modal response spectrum analysis to systems that do not have an extreme torsional irregularity. No such restrictions are placed on linear or nonlinear response history analysis. See the discussion at the end of this chapter for background on this issue.

When accidental torsion is allowed to be used by modeling a mass offset, it is necessary to shift the mass in the *X*-direction without affecting the *Y*-direction, and vice versa. The following procedure provides the required results by dividing the diaphragm into four regions and then determining mass modifiers that provide the desired offsets. The procedure is summarized as follows:

1. Divide each floor plate into four regions as shown in **Figure G20-5**.
2. Name these regions A, B, C, D as shown.
3. Compute the mass and center of mass for each region, where the center of mass is oriented relative to a given origin. One can alternately work with weights.

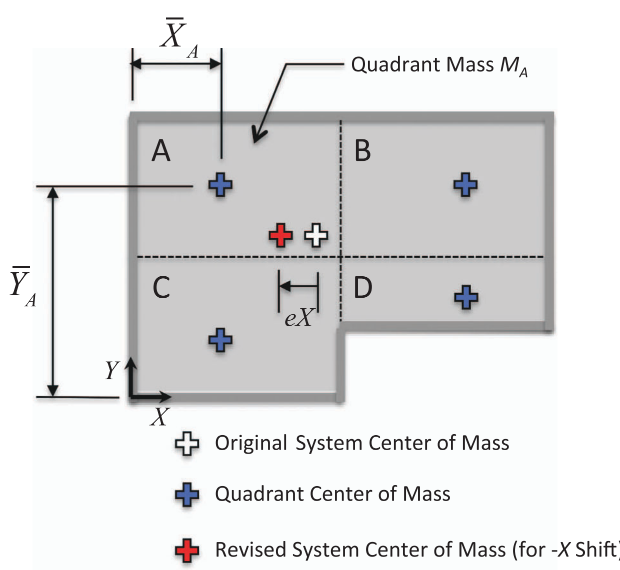

Figure G20-5. **Diaphragm mass divided into four quadrants.**

4. Using the same origin, compute the total mass and the center of mass for the full plate.
5. Establish coefficients α_A, α_B, α_C, α_D, such that the center of mass is moved in the desired direction.

The procedure utilizes the following equations:

$$\alpha_A M_A + \alpha_B M_B + \alpha_C M_C + \alpha_D M_D = M_{\text{total}} \quad \textbf{(G20-1a)}$$

$$\alpha_A M_A \bar{X}_A + \alpha_B M_B \bar{X}_B + \alpha_C M_C \bar{X}_X + \alpha_D M_D \bar{X}_D = (\bar{X}_{\text{total}} + eX) M_{\text{total}} \quad \textbf{(G20-1b)}$$

$$\alpha_A M_A \bar{Y}_A + \alpha_B M_B \bar{Y}_B + \alpha_C M_C \bar{Y}_Y + \alpha_D M_D \bar{Y}_D = (\bar{Y}_{\text{total}} + eY) M_{\text{total}} \quad \textbf{(G20-1c)}$$

In **Equation (G20-1a)** the total mass is unchanged, and in **Equations (G20-1b) and (G20-1c)** the desired mass offset is determined. If one is determining the desired offset eX, set eY to zero. If one is determining the desired eY, set eX to zero. Note however that there are three equations and four unknowns, which means that there are an infinite number of solutions. To resolve this issue, a fourth equation can be written that establishes a relationship between two of the alpha terms. The suggested rules for doing this are as follows:

1. If it is desired to move the mass in the + or − X-direction, set $\alpha_A = \alpha_C$, or $\alpha_B = \alpha_D$.
2. If it is desired to move the mass in the + or − Y-direction, set $\alpha_A = \alpha_B$, or $\alpha_C = \alpha_D$.

This produces the same result as if the floor plate was divided into only three segments, in which case a unique solution can be found.

The added equation produces a system of equations. For example, if it is desired to find the positive Y offset with $\alpha_A = \alpha_B$, **Equation (G20-2)** is applicable.

$$\begin{bmatrix} M_A & M_B & M_C & M_D \\ M_A \bar{X}_A & M_B \bar{X}_B & M_C \bar{X}_C & M_D \bar{X}_D \\ M_A \bar{Y}_A & M_B \bar{Y}_B & M_C \bar{Y}_C & M_D \bar{Y}_D \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \alpha_A \\ \alpha_B \\ \alpha_C \\ \alpha_D \end{Bmatrix} = \begin{Bmatrix} M_{\text{total}} \\ \bar{X}_{\text{total}} M_{\text{total}} \\ (\bar{Y}_{\text{total}} + eY) M_{\text{total}} \\ 0 \end{Bmatrix} \quad \textbf{(G20-2)}$$

Example

The floor plate shown in **Figure G20-6** has dimensions shown, and a uniform interior weight of 0.1 ksf. Perimeter-cladding weights parallel to the x-direction are 1.0 klf, and perimeter-cladding weights parallel to the y-direction are 1.5 klf. (Here we work with weight instead of masses simply for convenience). The structure is divided into four quadrants, A, B, C, and D. Weight and center of weight distributions are as shown in **Table G20-7**. The floor-plate weight modifiers were computed in Excel, with the results presented in **Figure G20-7** for the desired 5% mass offsets in the $+X$, $-X$, $+Y$ and $-Y$ directions. As may be observed the desired offsets are obtained in the given direction without affecting the center of weight in the perpendicular direction.

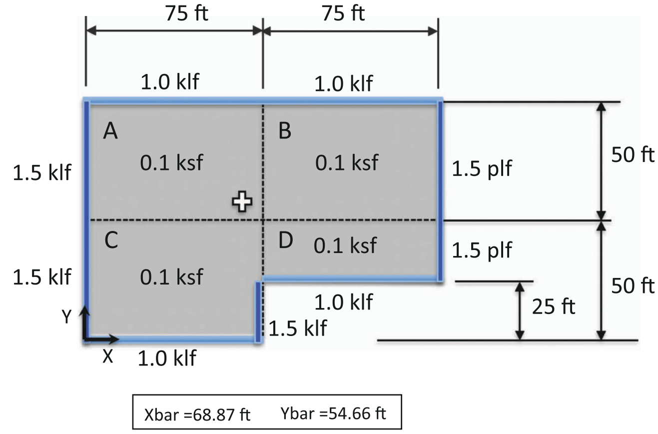

Figure G20-6. **Floor plate dimensions, weights, and subplate designations.**

Table G20-7. **Properties of Subplate Weights**

Subplate	*Weight (k)*	$\bar{X}$ *(ft.)*	$\bar{Y}$ *(ft.)*
A	525	34.143	78.571
B	525	117.857	78.571
C	562.50	35.000	20.833
D	300.00	110.937	34.757
Total	191.25	68.873	54.657

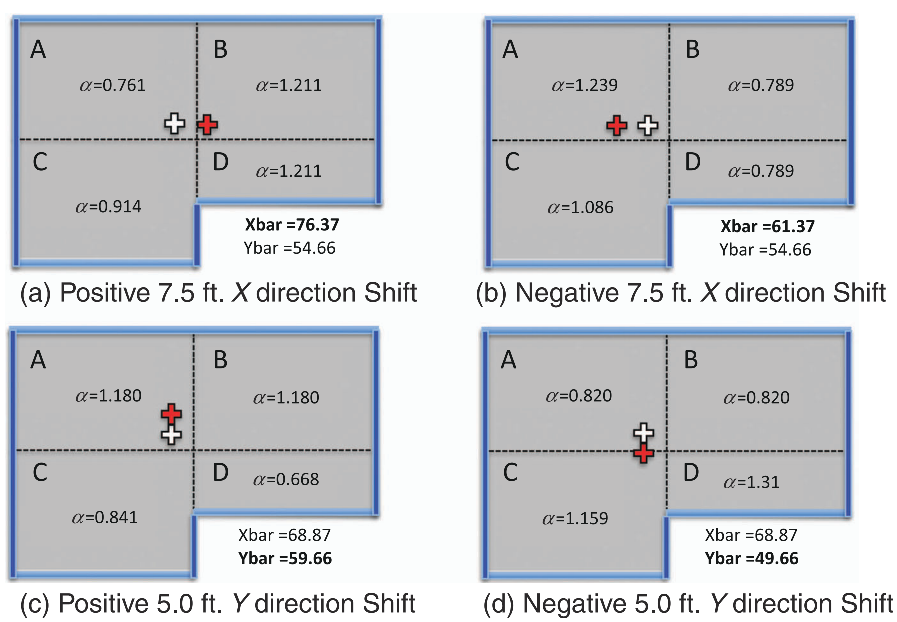

Figure G20-7. **Mass adjustment factors and revised centers of mass.**

A very significant mass redistribution is required to obtain a 5% *accidental* eccentricity. It is highly unlikely that this level of uncertainty in the mass distribution will occur in a real building. However, there are other uncertainties that contribute to the 5% mass offset used in the standard. These include uncertainties in locating the center of rigidity, the likelihood of nonuniform yielding in the lateral systems (which causes a migration of center of rigidity), and the possibility of torsional components of ground motion.

Background on ASCE 7- Supplement 2 Prohibition of Mass Offset in MRS Analysis

In the ATC 123 Project (FEMA 2018), the horizontal and vertical system irregularities discussed standard Section 12.3.2 were studied in some detail, and several recommendations were made for modifying the requirements that trigger the irregularity, or in some cases, eliminating the irregularity altogether. When studying torsional irregularities, it was found that designs using MRS analysis together with the mass offset method of applying accidental torsion could result in unsafe designs relative to designs that used ELF or MRS analysis with a statically applied accidental torsion.

Figure **G20-8**, taken directly from Appendix A of FEMA P-2012 (FEMA 2018), illustrates the issue. Here, systems were designed using MRS analysis with a static torsion applied with a force offset of $0.05A_x$ times the building dimension perpendicular to the loading (the baseline condition), and redesigned using MRS analysis with accidental torsion applied using a mass offset of 0.05 times the perpendicular building dimension. The vertical axis is the ratio of adjusted collapse margin ratios (ACMR) for the two systems, computed in accordance with FEMA P-695 (FEMA 2009). The horizontal axis is the torsional irregularity factor, which is equal to the displacement of the edge of the building relative to the displacement at the center of the building when lateral forces are applied at a 5% eccentricity. An extreme torsional irregularity occurs where the TIF is greater than about 1.42. In **Figure** **G20-8** colored curves are provided for each

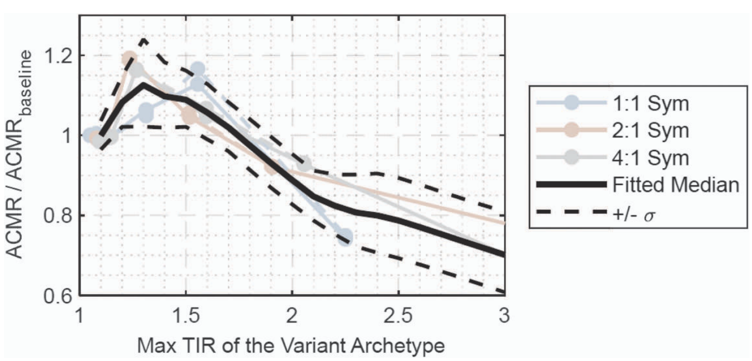

***Figure G20-8*. Results from the FEMA P-2012 report regarding use of mass offset versus static torsion.**

Source: FEMA (2018).

of three different plan aspect ratios in rectangular buildings. The lateral load resisting systems were placed symmetrically about the center of the building. If the ratio of the ACMRs is less than 1.0, the system designed using a mass offset is unsafe relative to the system designed using static torsion. Using the median response minus one standard deviation, this occurred only for systems that had extreme torsional irregularities, or worse. Interestingly, systems with TIFs less than 1.4 are safer using a 5% mass offset than they are using a static torsion. More detail on this issue is presented in DeBock et al. (2019).

21

Equivalent Lateral Force Analysis

Changes in ASCE 7-16 relative to ASCE 7-10 that affect this chapter:

- Ground motion parameters S_S, S_1, F_a, and F_v are different because of the development of new hazard maps and changes in the site factor coefficients.
- Accidental torsion need not be included in the analysis if the structure is in SDC D through F and there is no torsional irregularity.

In this chapter, the eight-story steel frame described in Appendix B is analyzed using the equivalent lateral force procedure. Included in the analysis are preliminary issues such as determination of the presence of structural regularities, diaphragm flexibility, and the redundancy factor. Story drifts are computed and compared to specified limits, with P-delta effects explicitly included in the analysis. Seismic forces for several elements are computed and tabulated for comparison with results obtained using the modal response spectrum and the linear response history methods of analysis. This comparison of results is presented in Chapter 24 of this guide.

A second example of a five-story building is provided to demonstrate the two-stage ELF analysis procedure that is provided in Section 12.3.2 of the standard. This structure is analyzed using both the two-stage procedure and the traditional ELF procedure, and the results are compared and discussed.

21.1 8-Story Building

The 8-story building analyzed in this chapter is described in detail in Appendix B of this guide. Analysis is performed using ETABS, developed by Computers and Structures (CSI 2018). A plan and elevation of the building is provided in **Figure G21-1**, and an image of the ETABS 3D model is shown in **Figure G21-2**.

Structural System

A detailed discussion of the evolution of the structural system is presented in Appendix B. For loads in the east–west direction, the system is a special steel moment-resisting frame (System C1 in Table 12.2-1). Two perimeter frames are

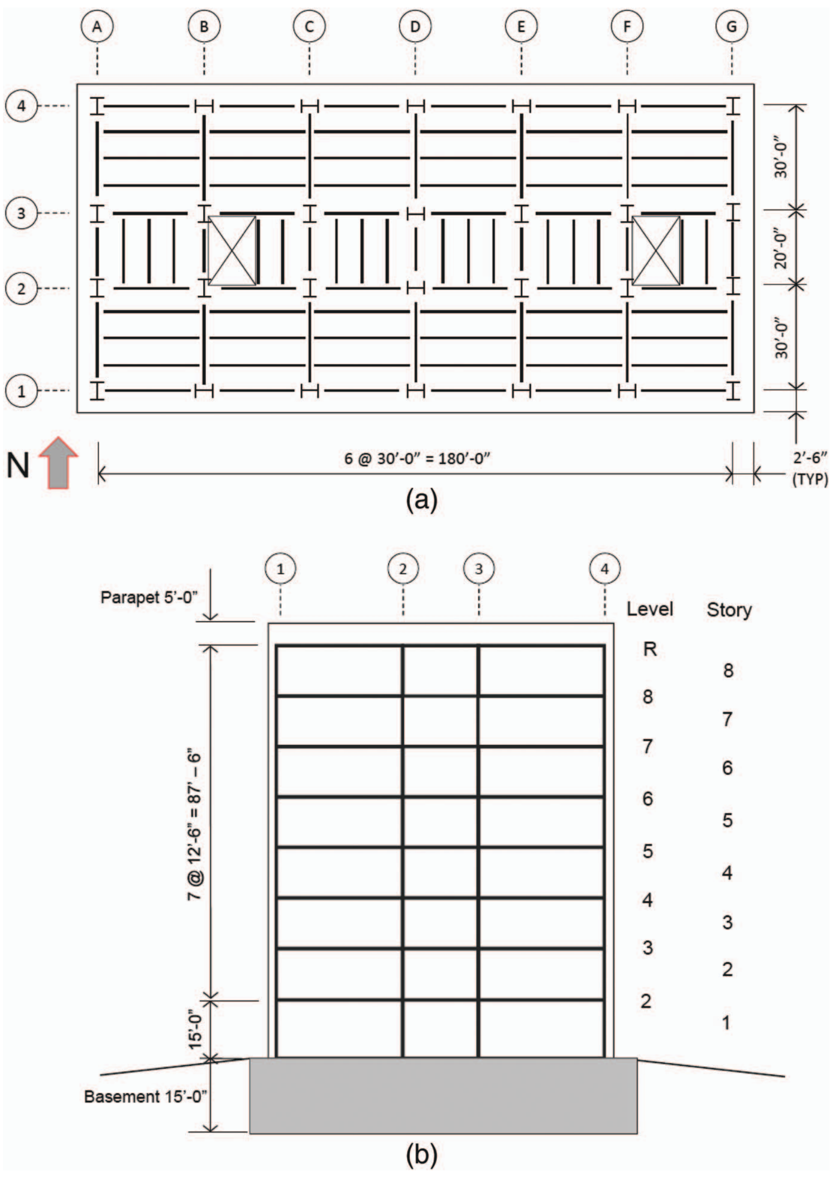

Figure G21-1. **Plan and elevation of the 8-story building.**

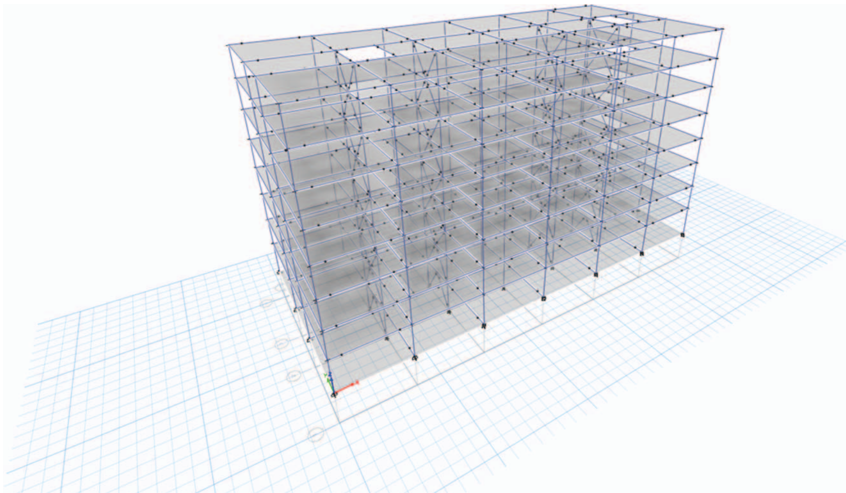

***Figure G21-2*. 3D ETABS model of the 8-story building.**

provided, and these frames utilize the four interior bays (Grid lines B through F) of frames on Grids 1 and 4. The base conditions of the columns can play an important role in modeling moment frame systems. In this case, because of the presence of a basement wall, it is assumed that the perimeter columns are fixed at the base as they are intended to be embedded into the wall (using pilasters if required), and this will help significantly with the stiffness of the system. Columns not on the perimeter that extend into the basement are assumed to have pinned bases. System parameters for the frames are as follows:

$$\text{Special steel moment frame:} \quad R = 8 \quad \Omega_0 = 3.0 \quad C_d = 5.5$$

In the north–south direction, the system is a dual special moment frame/special concentrically braced frame (system D2 in Table 12.2-1). The braced frames are located in the center bay of Grid lines B, C, E, and F. The moment-resisting frames utilize all three bays aligned along Grid lines A and G. System parameters are as follows:

$$\text{Dual system:} \quad R = 7 \quad \Omega_0 = 2.5 \quad C_d = 5.5$$

As noted in Appendix B, the special moment frames in the dual system, acting independently, have been proportioned to have the capacity to resist at least 25% of the base shear in the north–south direction. Both systems satisfy the height limitation requirements of Table 12.2-1. The member sizes used in the analysis are shown in **Figures GB-8 through GB-11**.

The structural system is essentially symmetric in each direction of response, with the only deviation resulting from the skewed arrangements of diaphragm openings.

Ground Motion Parameters

The ground motion parameters for the Site Class C location near Raleigh Hills, Oregon, are described in Appendix B of this guide and are summarized as follows:

$$S_S = 0.893g \quad F_a = 1.2 \quad S_{MS} = 1.072g \quad S_{DS} = 0.715g$$
$$S_1 = 0.405g \quad F_v = 1.5 \quad S_{M1} = 0.607g \quad S_{D1} = 0.405g$$
$$T_L = 16\,\text{s}$$

In addition, $T_S = S_{D1}/S_{DS} = 0.405/0.715 = 0.566$ s.

Seismic Design Category and Importance Factor

The building is designated as Risk Category III on the basis of current and future use. Because of this, it is determined that the seismic importance factor $I_e = 1.25$ (Table 1.5-2) and that the structure is in SCD D (Tables 11.6-1 and 11.6-2).

Story Weights

The story weights for the building are determined from the system geometry and loading described in Appendix B and are presented in **Table G21-1**. These weights include only dead load contributions from the structural weight, ceiling and mechanical systems, roofing, partitions, and cladding. The total system weight $W = 14{,}018$ kips. This represents a system density of approximately 8.7 pcf for the above-grade portion of the building.

Seismic Base Shear

The seismic base shear, V, is determined in accordance with Section 12.8. For loading in the east–west direction, the system is a special moment-resisting frame. Using from Table 12.8-2 $C_t = 0.028$, $x = 0.8$, and $h_n = 102.5$ ft, then

$$T_a = C_t h_n^x = 0.028(102.5)^{0.8} = 1.137\,\text{s}$$

In anticipation of the computed (eigenvalue) period's being greater than $C_u T_a$, the base shear is based on $T = C_u T_a$ as

$$\text{For } S_{D1} = 0.405g,\ C_u = 1.4, \text{and } T = C_u T_a = 1.4(1.137) = 1.592\,\text{s}$$

Table G21-1. **Story Weights for 8-Story Building**

Level	*Contribution from horizontal surfaces (kip)*	*Contribution from vertical surfaces (kip)*	*Total (kip)*
Roof and parapet	1,604	280	1,884
8	1,494	236	1,730
7	1,494	236	1,730
6	1,494	236	1,730
5	1,494	236	1,730
4	1,494	236	1,730
3	1,494	236	1,730
2	1,494	260	1,754
Total	12,062	1,956	14,018

Because this period is greater than T_s and less than T_L, the design base shear is computed using Equations (12.8-1) and (12.8-3)

$$V = C_s W$$

$$C_s = \frac{S_{D1}}{T(R/I_e)} = \frac{0.405}{1.592(8/1.25)} = 0.0398$$

but in accordance with Equation (12.8-5), C_s shall not be less than

$$C_s = 0.044 S_{DS} I_e = 0.044(0.715)(1.25) = 0.0393$$

Equation (12.8-3) controls, giving $V = 0.0398(14{,}018) = 558$ kips in the east–west (moment frame) direction.

For loading in the north–south direction, the system is a dual special moment-resisting frame/special concentrically braced frame. From Table 12.8-2, $C_t = 0.020$, $x = 0.75$, and $h_n = 102.5$ ft, then

$$T_a = C_t h_n^x = 0.020(102.5)^{0.75} = 0.644 \text{ s}$$

In anticipation of the computed (eigenvalue) period being greater than $C_u T_a$, the base shear is based on $T = C_u T_a$

For $S_{D1} = 0.405g$, $C_u = 1.4$, and $T = C_u T_a = 1.4(0.644) = 0.902$ s

Because this period is greater than T_s and less than T_L, the design base shear is based on Equations (12.8-1) and (12.8-3)

$$V = C_s W$$

$$C_s = \frac{S_{D1}}{T(R/I_e)} = \frac{0.405}{0.902(7/1.25)} = 0.0802$$

This value controls over Equation (12.8-5) ($C_s = 0.0393$), giving $V = 0.0802$ $(14{,}018) = 1{,}124$ kips in the north–south (dual system) direction. This design base shear is approximately 2.0 times that in the east–west direction, with the main difference being caused by the different periods of vibration in the two directions.

The base shears of 558 kips in the east–west direction and 1,124 kips in the north–south direction will be the basis for scaling forces determined from the modal response spectrum and linear response history analyses.

Preliminary Lateral Forces

Lateral forces and story torques are needed to check for the presence of several of the horizontal and vertical irregularities and to compute the redundancy factor. These forces are provided in **Tables G21-2 and G21-3** for forces in the east–west and the north–south directions, respectively. The torques are based on an accidental eccentricity of 0.05 times the building dimension perpendicular to the direction of loading.