# Chapter 9

# HOW TO OPTIMIZE TALL STEEL BUILDING FRAMEWORKS

Due to the complex nature of a modern tall building consisting of thousands of structural members, the traditional trial-and-error design method is generally highly iterative and time consuming. This c'apter presents an optimal sizing technique for the design of practical tall steel building frameworks. A computer-based technique is developed to formalize the numeric tasks of the analysis-design cycle and produce minimum cost design of tall steel frameworks of given topology under static gravity and lateral loadings. Most economical member sizes are automatically selected from databases of commercial standard steel sections, and service lateral drift, ultimate member strength, and discrete sizing constraints are simultaneously satisfied in accordance with building code and constructability requirements.

The design optimization approach is based on an Optimality Criteria method, which is well suited for large-scale tall building frameworks. By exploiting the fact that member force distributions are relatively insensitive to changes in member sizes for building frameworks, the design optimization technique generally converges rapidly in a few cycles. A full-scale 50-story building example is presented to illustrate the applicability, efficiency, and practicality of the automatic optimal sizing technique.

## INTRODUCTION

Modern tall steel buildings are complex large-scale structures. Their design is generally a complicated, laborious, and time-consuming

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task. The design process involves the coordinated application of a preliminary conceptual design procedure, and then repetitive application of structural analysis, design evaluation, and member resizing procedures. Once the structural layout and loadings for a building framework are established, most of the engineering effort is expended in an iterative analysis and design process. The design of tall buildings is primarily controlled by lateral stiffness criteria rather than member strength requirements (Council 1979). Modifications of member sizes to satisfy the stiffness constraints are rather difficult because all members in the structure have to be considered simultaneously under multiple loading conditions. In fact, for a large-scale tall building with thousands of structural members, a consideration of all design performance constraints including multiple stiffness and strength requirements is incomprehensible using the traditional trialand-error design method. Oftentimes in practice, an optimally sized tall building structure is not sought due to time and budget consideration and the absence of a practical and efficient optimization tool

Research on structural optimization was motivated in large part by the aerospace industry in the 1960s and 1970s. Very little practical optimization work has been carried out for the civil engineering building industry. An early attempt for optimal steel building framework design was reported by Brown and Ang (1966), who developed a program based on a mathematical programming approach using the Gradient Projection method. Although the work produced a structural synthesis program using the 1964 AISC specification, the efficiency of the computer program was very poor even for twodimensional frameworks. To improve efficiency, Tabak and Wright (1981) developed a program using the Optimality Criteria (OC) approach. However, their program was developed without accounting for design code specifications in the optimization procedure. Other researchers such as Cheng and Truman (1983), Khan (1984), and Sadek (1992) also adopted the OC approach for the design of steel frameworks. However, none of these studies had explicit concern for the lateral stiffness problem associated with practical tall steel frameworks and designs were finalized using continuous member sizes instead of discrete standard steel sections. The first commercial structural optimization software for practical building design is called SODA (Grierson and Cameron 1990), which is capable of handling both stiffness and member strength constraints in accordance with steel design standards. Although this software is remarkably useful, its capacity is still limited to relatively small skeleton steel frameworks.

In addition to university-based researchers, several design professionals, Velivasakis and DeScenza (1983), Baker (1989), Charney (1991), and Gilsanz and Carlson (1991) have also recently developed software for sizing members of tall steel building frameworks to satisfy lateral stiffness criteria. Their methods are essentially equivalent to the early OC method developed by Venkayya et al. (1969). Based on the concept of uniform strain energy density, an optimal design can be achieved indirectly by requiring every structural member of a framework to contribute the same amount of work per unit volume of material to resist the occurrence of a particular displacement of concern to the design. Although this technique is quite efficient, it is useful only for single displacement constraint problems and is valid only for statically determinate truss structures.

This chapter presents a practical optimization technique for the design of tall steel building frameworks. Specifically, a computer-based method is developed to formalize the numeric-intensive tasks of the analysis-design cycle and produce minimum weight designs for tall steel building frameworks subject to multiple interstory drift, strength, and sizing constraints in accordance with building code and fabrication requirements. The design optimization problem is first formulated for a general asymmetrical three-dimensional framework and then the details of the resizing technique are developed. Using the principle of virtual work and section properties' regression relationships, the original implicit design problem is expressed in an explicit continuous form that is readily solved numerically. A rigorously derived OC algorithm is developed to solve the explicit design problem until convergence of member sizes and satisfaction of constraints occur. To achieve a final optimal design using standard steel sections, a pseudo-discrete OC technique is applied to progressively assign standard steel sections to the members of the structure while maintaining the least change in structure weight. Finally, a full-scale 50story three-dimensional practical framework example is presented to illustrate the applicability, efficiency, and practicality of the automatic optimal design method.

## DESIGN PROBLEM FORMULATION

Consider a general three-dimensional steel building framework having i = 1, 2, ..., N members (or member fabrication groups), j = 1, 2, ..., M stories, k = 1, 2, ..., S column lines, under l = 1, 2, ..., L lateral loading conditions. The problem of finding the minimum structural weight of the framework can be posed as the following discrete optimization problem.

Minimize

$$W(A_i) = \sum_{i=1}^{N} w_i A_i$$
 (9-1a)

Subject to

$$d_{kjl} = (\delta_{kjl} - \delta_{kj-1l})/h_j \le d_j^{U} \qquad (k = 1, 2, ..., S);$$
  
(j = 1, 2, ..., M); (l = 1, 2, ..., L) (9-1b)

$$\sigma_p \leq \sigma_p^{U} \qquad (p = 1, 2, \ldots, P) \qquad (9-1c)$$

$$A_i \in \mathbf{A}_i \qquad (i = 1, 2, \dots, N) \tag{9-1d}$$

Equation (9-1a) defines the structure weight, where  $A_i$  is the axial cross-sectional area for member *i* and  $w_i$  is the corresponding weight coefficient (material density × member length); Eq. (9-1b) defines the multiple interstory drift constraints for the structure, where  $\delta_{kji}$  and  $\delta_{kj-1l}$  are the lateral deflections on a column line *k* at two adjacent story levels (*j* and *j* - 1) under lateral loading condition *l*,  $h_j$  is the corresponding story height, and  $d_j^{U}$  is the allowable *j*th story drift limit; Eq. (9-1c) defines *P* member strength constraints, where  $\sigma_p$  represents a stress state for a member and  $\sigma_p^{U}$  is the corresponding allowable member strength; Eq. (9-1d) requires each cross-sectional  $A_i$  to belong to the set of areas  $\mathbf{A}_i = \{A_1, A_2, \ldots\}_i$  prevailing for the standard steel section profile (e.g., W14-shape) specified for member *i*.

To facilitate computer solution of the design optimization problem Eqs. (9-1), and as discussed later, it is expedient to initially treat the sizing variables  $A_i$  as continuous variables that are restricted as

$$A_i^L \le A_i \le A_i^U \tag{9-1e}$$

where  $A_i^L$  and  $A_i^U$  are the corresponding lower and upper size bounds specified for member *i*. Moreover, it is also necessary to express the implicit drift and strength constraints Eqs. (9-1b) and (9-1c) as explicit functions of the sizing variables  $A_{ij}$  as described in the following.

#### **Drift Constraints**

Under the action of lateral loadings, asymmetrical building frameworks not only translate laterally but also rotate with torsional twisting. Unlike symmetric buildings where every point on a floor plane translates the same amount and a drift value can thus be expressed at a master node for each lateral loading condition, asymmetric buildings have different displacements at different points on the floor plane because of the additional torsional effect. Therefore control of drift must be applied on each individual column line of an asymmetric framework.

With the assumption that floor diaphragms are rigid in their planes, the primary lateral translation  $\delta_{kjl}$  of column line k at the *j*th floor under the *l*th lateral loading condition can be related to the corresponding master floor node displacements through a master-slave transformation as follows.

$$\delta_{kjl} = \begin{bmatrix} 1 & \Delta_{kjl} \end{bmatrix} \begin{bmatrix} \delta_{jl} \\ \theta_{jl} \end{bmatrix} = T_{kjl} \begin{bmatrix} \delta_{jl} \\ \theta_{jl} \end{bmatrix}$$
(9-2)

where  $\delta_{jl}$  and  $\theta_{jl}$  are the lateral translation and rotation at the *j*th floor master node;  $T_{kjl}$  denotes the transformation matrix relating the *k*th slave column node to the *j*th floor master node under the *l*th lateral loading condition. As illustrated in Fig. 9-1, if l = 1 represents the *X*direction loading, then  $\delta_{jl} = \delta_j^X$  and  $\Delta_{kjl} = -\Delta Y_{kj}$ , and similarly if l = 2represents the *Y*-direction loading, then  $\delta_{j2} = \delta_j^Y$  and  $\Delta_{kj2} = \Delta X_{kj}$ . The transformation matrix  $T_{kjl}$  is readily established since the geometric layout and the master floor nodes of the building framework are predefined.

To formulate explicit drift constraints for the slave column lines, one needs to first express explicitly the displacements at the master floor nodes. By the principle of virtual work, the lateral displacement at the *j*th floor master node under the *l*th lateral loading condition can be expressed as

$$\delta_{jl} = \sum_{i=1}^{N} \int_{0}^{L_{i}} \left( \frac{F_{Xl} f_{X}}{EA} + \frac{F_{Yl} f_{Y}}{GA_{Y}} + \frac{F_{Zl} f_{Z}}{GA_{Z}} + \frac{M_{Xl} m_{X}}{GI_{X}} + \frac{M_{Yl} m_{Y}}{EI_{Y}} + \frac{M_{Zl} m_{Z}}{EI_{Z}} \right)_{i} dx$$
(9-3)

where  $L_i$  is the length of member *i*; *E*, *G* are the axial and shear elastic material moduli; *A*, *A*<sub>Y</sub>, *A*<sub>Z</sub> are the axial and shear areas for the cross-section; *I*<sub>X</sub>, *I*<sub>Y</sub>, *I*<sub>Z</sub> are the torsional and flexural moments of inertia for the cross-section; *F*<sub>Xl</sub>, *F*<sub>Yl</sub>, *F*<sub>Zl</sub>, *M*<sub>Xl</sub>, *M*<sub>Yl</sub>, *M*<sub>Zl</sub> are the member forces and moments due to the actual *l*th lateral loading condition; *f*<sub>X</sub>, *f*<sub>Y</sub>, *f*<sub>Z</sub>, *m*<sub>X</sub>, *m*<sub>Y</sub>, *m*<sub>Z</sub> are the member forces and moments due to the framework at the location of and in the sense of displacement  $\delta_{jl}$ .



FIG. 9-1. Relationship Between Master and Slave Nodes on Rigid Floor Plane.

Now, for commercial standard steel sections the cross-section properties  $A_{Y}$ ,  $A_{Z}$ ,  $I_{X}$ ,  $I_{Y}$ , and  $I_{Z}$  may all be instantaneously expressed in terms of the axial area A as follows (Chan 1993).

$$1/A_Y = C_{AY}(1/A) + C'_{AY}$$
 (9-4a)

$$1/A_Z = C_{AZ}(1/A) + C'_{AZ}$$
 (9-4b)

$$1/I_X = C_{IX}(1/A) + C'_{IX}$$
 (9-4c)

$$1/I_Y = C_{IY}(1/A) + C'_{IY}$$
 (9-4d)

$$1/I_Z = C_{IZ}(1/A) + C'_{IZ}$$
 (9-4e)

where the coefficients *C* and *C'* are determined by linear regression analysis and have different values depending on the type and size of the section [e.g., see Tables 9-1 and 9-2 for W14 and W24 sections from the AISC-LRFD design manual (Manual 1986)]. Given a particular section type and depth for each member, the displacement  $\delta_{ji}$ from Eq. (9-3) can be concisely expressed solely in terms of the member cross-section areas  $A_i$  through Eqs. (9-4) as

$$\delta_{jl} = \sum_{i=1}^{N} \left( \frac{C_{ijl}}{A_i} + C'_{ijl} \right)$$
(9-5)

where, from Eqs. (9-3) and (9-4), the coefficients  $C_{ijl}$  and  $C'_{ijl}$  are given by

$$C_{ijl} = \int_{0}^{L_{i}} \left( \frac{F_{XI}f_{X} + M_{Yl}m_{Y}C_{IY} + M_{Zl}m_{Z}C_{IZ}}{E} + \frac{F_{Yl}f_{Y}C_{AY} + F_{Zl}f_{Z}C_{AZ} + M_{Xl}m_{X}C_{IX}}{G} \right)_{i} dx \quad (9-6a)$$

$$C'_{ijl} = \int_{0}^{L_{i}} \left( \frac{M_{Yl}m_{Y}C'_{IY} + M_{Zl}m_{Z}C'_{IZ}}{E} + \frac{F_{Yl}f_{Y}C'_{AY} + F_{Zl}f_{Z}C'_{AZ} + M_{Xl}m_{X}C'_{IX}}{G} \right)_{i} dx \quad (9-6b)$$

Applying the *L* actual lateral loadings, together with two virtual lateral loadings and one virtual torque corresponding to the three degrees of freedom per floor at the master node, an explicit lateral

Sections (No. of Sections)	$\frac{1}{A_Y} = \frac{C_{AY}}{A} + C'_{AY}$		Max. %	$\frac{1}{A_{\rm Z}} = \frac{C}{A_{\rm Z}}$	$\frac{1}{A_z} = \frac{C_{Az}}{A} + C'_{Az}$	
	C <sub>AY</sub>	$C'_{AY}$	error	$C_{AZ}$	C'AZ	error
W14X22-26 (2)	1.4354911	0.0952506	0.00	2.5618151	-0.0962252	0.00
W14X30-38 (3)	1.6353798	0.0843246	1.26	2.0862633	-0.0437900	1.16
W14X43-53 (3)	3.0048877	0.0010805	0.42	1.5728685	-0.0068556	0.04
W14X61-82 (4)	3.8144416	-0.0199141	1.14	1.3836832	0.0000917	0.49
W14X90-132 (5)	4.7186962	-0.0158902	1.10	1.2944072	-0.0004526	0.47
W14X145-176 (3)	4.9439546	-0.0162764	0.02	1.2641882	-0.0000348	0.15
W14X193-257 (4)	4.6603358	-0.0097951	0.70	1.2745534	-0.0003102	0.19
W14X283-426 (6)	4.4247100	-0.0068696	0.28	1.2777818	-0.0003280	0.41
W14X455-730 (6)	4.1089656	-0.0046189	0.25	1.2627338	-0.0001827	0.14
W24X55-62 (2)	1.3929148	0.0214267	0.00	3.0904030	-0.0494249	0.00
W24X68-84 (3)	1.4291581	0.0306909	0.55	2.5179705	-0.0301700	0.55
W24X94-103 (2)	1.8572561	0.0128252	0.00	2.0489677	-0.0109334	0.00
W24X104-131 (3)	2.3277754	0.0071301	0.20	1.7554814	-0.0050792	0.01
W24X146-192 (4)	2.4884181	0.0044622	0.44	1.6587269	-0.0030770	0.23
W24X207-306 (5)	2.8964781	-0.0030112	0.30	1.5187585	-0.0005579	0.20
W24X335-492 (5)	2.8209222	-0.0024542	0.68	1.5138177	-0.0005307	0.60

TABLE 9.1Relationships between Cross-Section Area A and Shear Areas  $A_Y$  and  $A_Z$  for AISC W14 and W24<br/>Sections

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Sections	$\frac{1}{I_z} = \frac{C_{IZ}}{A} + C'_{IZ}$		Max. %	$\frac{1}{l_{\gamma}} = \frac{C_{I\gamma}}{A} + C'_{I\gamma}$		Max. %	$\frac{1}{I_x} = \frac{C_{Ix}}{A} + C_{Ix}$		Max. %
(No. of Sections)	$C_{IZ}$	C' <sub>IZ</sub>	error	CIY	$C'_{IY}$	error	$C_{IX}$	$C'_{IX}$	error
W14X22-26 (2)	0.0392400	-0.0010211	0.00	1.2736421	-0.0533897	0.00	82.5200066	-7.9530423	0.00
W14X30-38 (3)	0.0353453	-0.0005698	0.79	0.5711482	-0.0137538	1.02	57.3168240	-3.8983881	4.30
W14X43-53 (3)	0.0319730	-0.0002026	0.15	0.3140174	-0.0028040	0.06	28.3914777	-1.3119968	2.43
W14X61-82 (4)	0.0298816	-0.0001096	0.38	0.1810829	-0.0007904	0.71	17.5919341	-0.5400378	3.65
W14X90-132 (5)	0.0290363	-0.0000971	0.48	0.0780857	-0.0001925	0.47	13.2299043	-0.2647232	6.62
W14X145-176 (3)	0.0285349	-0.0000839	0.15	0.0689388	-0.0001381	0.09	6.7408450	-0.0927814	2.05
W14X193-257 (4)	0.0280568	-0.0000771	0.13	0.0680315	-0.0001246	0.19	3.6149842	-0.0354938	2.74
W14X283-426 (6)	0.0271200	-0.0000652	0.35	0.0674557	-0.0001159	0.16	1.5839994	-0.0098414	6.73
W14X455-730 (6)	0.0244159	-0.0000440	0.66	0.0634434	-0.0000838	0.32	0.6200119	-0.0022544	9.57
W24X55-62 (2)	0.0140903	-0.0001290	0.00	0.7929359	-0.0145824	0.00	38.7216771	-1.5427694	0.00
W24X68-84 (3)	0.0134348	-0.0001225	0.23	0.3894727	-0.0052045	0.50	28.2704125	-0.8784636	2.73
W24X94-103 (2)	0.0119560	-0.0000613	0.00	0.2488717	0.0001898	0.00	15.9685482	-0.3863678	0.00
W24X104-131 (3)	0.0110073	-0.0000373	0.08	0.1371777	-0.0006215	0.02	15.7421923	-0.3050556	2.46
W24X146-192 (4)	0.0106682	-0.0000300	0.16	0.1206501	-0.0002557	0.72	7.5894514	-0.1037394	3.84
W24X207-306 (5)	0.0099410	-0.0000172	0.20	0.1200412	-0.0002494	0.18	3.1506929	-0.0271871	6.39
W24X335-492 (5)	0.0097758	-0.0000156	0.41	0.1149098	-0.0002011	0.91	1.2726781	-0.0067761	6.36

TABLE 9-2Relationships between Cross-Section Area A and Moment of Inertias  $I_Z$ ,  $I_Y$ , and  $I_X$  for AISC W14<br/>and W24 Sections

translation for the *k*th slave column node can be written in terms of sizing variables  $A_i$  using Eqs. (9-2) and (9-5), as follows.

$$\delta_{kjl} = T_{kjl} \left[ \sum_{i=1}^{N} \left( \frac{C_{ijl}}{A_i} + C'_{ijl} \right)_{8} \right] \quad (j = 1, 2, ..., M); \quad (l = 1, 2, ..., L)$$

$$(9-7)$$

Assuming that the slave column node locations with respect to the corresponding master node do not change from one floor to another (i.e.,  $\Delta_{kjl} = \Delta_{kj-1l}$  and consequently  $T_{kjl} = T_{kj-1l}$ ), the explicit interstory drift constraints for all k = 1, 2, ..., S critical column lines can be expressed as

$$d_{kjl} = \frac{\delta_{kjl} - \delta_{kj-1l}}{h_j} = \sum_{i=1}^N \left( \frac{e_{ikjl}}{A_i} + e'_{ikjl} \right) \le d_j^U \qquad (k = 1, 2, \dots, S);$$
  
(j = 1, 2, ..., M); (l = 1, 2, ..., L) (9-8)

where

$$e_{ikjl} = T_{kjl} \begin{bmatrix} \left(\frac{C_{ijl} - C_{ij-1l}}{h_j}\right)_{8} \\ \left(\frac{C_{ijl} - C_{ij-1l}}{h_j}\right)_{8} \end{bmatrix}; \qquad e'_{ikjl} = T_{kjl} \begin{bmatrix} \left(\frac{C'_{ijl} - C'_{ij-1l}}{h_j}\right)_{8} \\ \left(\frac{C'_{ijl} - C'_{ij-1l}}{h_j}\right)_{8} \end{bmatrix}$$
(9-9a,b)

### Strength Constraints

Although the serviceability drift requirement is of paramount importance for the design of tall buildings, the adequacy of the structural safety of these frameworks cannot be overlooked. One approach to account for the member strength requirements is to express them explicitly in terms of design variables in the same manner as the drift constraints. Though direct, this approach has a major obstacle in its implementation. As each member has at least several strength constraints for different load combinations, a practical tall steel building having several thousand members will result in an excessive number of strength constraints and will require enormous computer effort for solution.

As the design of a tall building framework is generally controlled by its lateral stiffness, member strength requirements can be treated as secondary constraints because most of them are usually far from