Example 6-7

Pressure readings on the suction and discharge lines of a pump are, respectively: −1.264 m of water; 1.697 m of mercury. The fluid being pumped is oil of specific gravity 0.85. The suction manometer tube connecting to the pump contains only air. On the discharge side, the reference point of the mercury gage is 0.630 m below the pressure tap. The vertical distance between the two pressure taps is 2.000 m. What is the total pressure head imparted to this fluid system by the pump? The suction and discharge lines are of the same diameter.

Pressure at A and at B, and thence the total head, may be determined by the application of Equation (6.1): $p = \rho g h$

$$p_A = -1000(9.807)1.264$$

$$= -12\,396\ \text{N/m}^2 \qquad \frac{\text{kg}}{\text{m}^3}\cdot\frac{\text{m}}{\text{s}^2}\cdot\text{m} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\frac{1}{\text{m}^2} = \frac{\text{N}}{\text{m}^2} = \text{Pa}$$

$$p_A = -12\,396\ \text{Pa}$$

$$p_A = -12.4\ \text{kPa}$$

Similarly:

$$p_C = 13\,570(9.807)1.697$$

$$= 225\,838\ \text{N/m}^2 = 225\,838\ \text{Pa} = 225.84\ \text{kPa}$$

$$(p_B - p_C) = 850(9.807)0.630$$

$$= 5252\ \text{N/m}^2 = 5252\ \text{Pa} = 5.25\ \text{kPa}$$

from which:

$$p_B = 225.84 - 5.25 = 220.59\ \text{kPa}$$

$$(p_B - p_A) = 850(9.807)2.000$$

$$= 16\,672\ \text{N/m}^2 = 16\,672\ \text{Pa} = 16.67\ \text{kPa}$$

$$\text{Total Head}_{AB}\ \text{to fluid} = 220.59 + 16.67 - (-12.40)$$

$$= 249.66\ \text{kPa}$$

Alternately the pressures at A, B, and C and the respective differential pressures can be obtained (or checked) by the conversion factors cited in Section 6.2 as follows:

$p_A = -1.264$ m water $\quad p_A = -1.264(9.807) \quad p_A = -12.40$ kPa

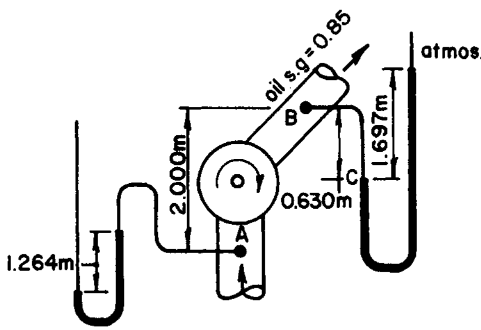
$p_C = 1.697$ m mercury $\quad p_C = 1.697(133.08) \quad p_C = 225.84$ kPa

$p_B - p_C = 0.630$ m oil $\quad p_B - p_C = 0.630(0.85)(9.807)$

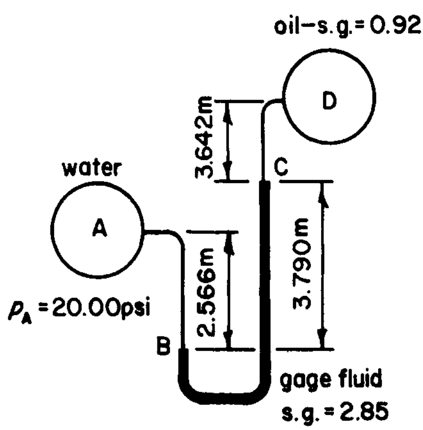
$p_B - p_C = 5.25$ kPa $\quad p_B = 225.84 - 5.25 = 220.59$ kPa

$z_B - z_A = 2.000$ m oil $\quad 2.000(0.85)(9.807) = 16.67$ kPa

Total Head $_{AB}$ to fluid $= 220.59 + 16.67 - (-12.40) = 249.66$ kPa



Example 6-7.



Example 6-8.

Example 6-8

A differential manometer using a gage fluid of specific gravity of 2.85, connects two pipelines A and D. Pipeline A contains water at a gage pressure of 20.00 lbf/in.^{2}. What is the pressure in kPa in the oil at D when the position of the gage and the gage fluid is as indicated?

$$s_1 = 1.000 \text{ (water)} \quad s_2 = 0.920 \text{ (oil)} \quad s_g = 2.850 \text{ (gage)}$$

$$l_1 = 2.566 \text{ m}; \quad l_2 = 3.642 \text{ m}; \quad h = 3.790 \text{ m}$$

Noting that:

$$p_B = p_A + (\rho g s_1)l_2,$$

and that:

$$p_B = p_D + (\rho g s_2)l_2 + (\rho g s_g)h$$

whence:

$$p_D = p_A + (\rho g s_1)l_1 - (\rho g s_2)l_2 - (\rho g s_g)h$$

in which for water:

$$\rho g = 9807 \text{ Pa/m} \qquad \frac{\text{kg}}{\text{m}^3}\cdot\frac{\text{m}}{\text{s}^2} = \frac{\text{N}}{\text{m}^3} = \frac{\text{Pa}}{\text{m}}$$

also:

$$p_A = 20.00(6.8948) = 137.90 \text{ kPa (given)}$$

Solving for the pressure at D in kPa

$$p_D = 137.90 + 9.807(2.566) - 9.807(0.92)(3.642) - 9.807(2.85)(3.790)$$

$$p_D = 24.27 \text{ kPa}$$

Alternately the solution may be viewed in these terms:

$$p_A = 20.00(6.8948) = 137.90 \text{ kPa}$$

$$p_B - p_A = 2.566(9.807) = 25.16 \text{ kPa}$$

$$p_B = 163.06 \text{ kPa}$$

$$p_C - p_B = 3.790(9.807)2.85 = -105.93 \text{ kPa}$$

$$p_C = 57.13 \text{ kPa}$$

$$p_D - p_C = 3.642(9.807)0.92 = -32.86 \text{ kPa}$$

$$p_D = 24.27 \text{ kPa}$$

6.8 Surface Tension

Manometers whose operation is based on the observation of liquid levels are subject to inaccuracy arising from meniscus effects, which are the consequence of surface tension. This phenomenon is defined as σ, the force per unit length and accordingly, the SI units, are N/m.

Example 6-9

A glass tube of 0.75 mm internal diameter is dipped vertically into water. How far will the water level rise in the tube?

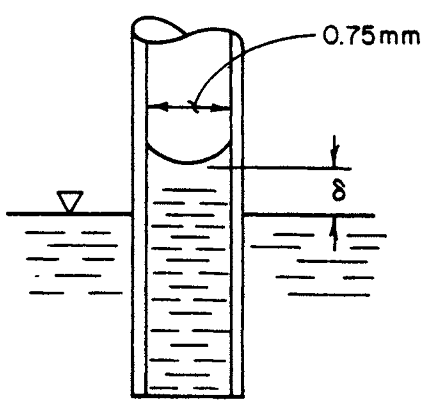
The water rises because of "capillarity," which is a consequence of surface tension. It may be shown that the rise δ is a function of the diameter of the tube, the density, and the surface tension of the liquid in accordance with the following formula:

(Eq. 6.10) $$\delta = \frac{4\sigma}{\rho g d} \qquad \frac{kg}{s^2} \cdot \frac{m^3}{kg} \cdot \frac{s^2}{m} \cdot \frac{1}{m} = m$$

for water σ = mN/m

$$\delta = \frac{4(73 \times 10^{-3})}{1000(9.8)(0.75 \times 10^{-3})}$$

$$\delta = 0.0397 \text{ m} = 39.7 \text{ mm}$$



Example 6-9.

6.9 Incompressible Fluid Flow

The study of fluids in motion utilizes the concepts of velocity and acceleration which were introduced earlier in respect to solid bodies or particles under the general heading of Dynamics. Although in most fluid flows there is a variation in velocity across the stream, resulting in what is termed a "velocity profile," the general approach for most purposes is to utilize the "average velocity," usually denoted by $\bar{V}$. In terms of SI units $\bar{V}$ in m/s multiplied by the cross-section area to which it is applicable, A, usually given in m², results in a volumetric flow rate Q in m³/s. Thus:

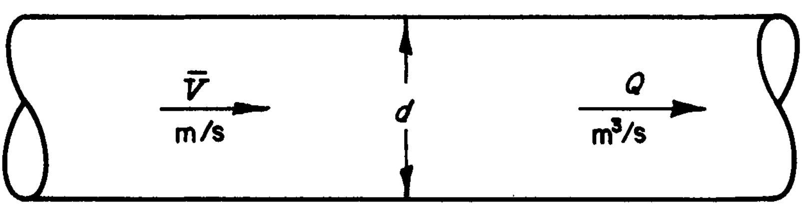
(Eq. 6.11) $$Q = A\bar{V} \qquad \text{m}^2 \cdot \frac{\text{m}}{\text{s}} = \text{m}^3/\text{s}$$

The volumetric rate-of-flow multiplied by density of the fluid gives a mass rate-of-flow:

(Eq. 6.12) $$\dot{m} = Q\rho \qquad \frac{\text{m}^3}{\text{s}} \cdot \frac{\text{kg}}{\text{m}^3} = \text{kg/s}$$

Example 6-10

A liquid of density 950 kg/m³ flows at a rate of 15 kg/s through a pipe of 140 mm diameter. What is the volumetric rate-of-flow and the average velocity?



Example 6-10.

$$A = 0.7854(0.140)^2 = 0.0154 \text{ m}^2$$

$$Q = \frac{15}{950} = 0.0158 \text{ m}^3/\text{s} \qquad \frac{\text{kg}}{\text{s}} \cdot \frac{\text{m}^3}{\text{kg}} = \text{m}^3/\text{s}$$

$$\bar{V} = Q/A = \frac{0.0158}{0.0154} = 1.03 \text{ m/s} \qquad \frac{\text{m}^3}{\text{s}} \cdot \frac{1}{\text{m}^2} = \text{m/s}$$

6.10 Measuring the Rate-of-Flow

The most accurate way of measuring a flow rate is to make related measurements of quantity (i.e. volume or mass) and time; e.g., by collecting the fluid in a vessel, but this usually is not practicable. Indirect methods, such as the use of orifice plates or venturi meters, and other special devices such as rotameters and propeller meters, are commonly employed. These devices are usually calibrated against standards, or are exact copies of devices that have been calibrated. As already indicated, preferred units for rates of flow in SI are m^3/s and kg/s, the use of the former being much more prevalent since mass flow meters are difficult to construct.

While the base units m^3/s are used to describe most large flows the magnitudes of resultant quantities frequently are considered somewhat inconvenient for expression at low flows. For instance, with a velocity of 2 m/s, which is quite common for pipe flow, the delivery of 1.0 m^3/s would involve a pipe approximately 800 mm in diameter.

There is, of course, nothing unusual in this situation and the unit prefixes are available to produce quantities of more convenient size. Therefore it is most common practice in respect to low flows to use the cubic decimeter which is 1/1000 of the cubic meter. The liter and the dm^3 formerly were not identical, although the difference was extremely small. But since 1964 the liter has been redefined as exactly equal to the cubic decimeter and the two names now are synonymous. Thus, many small flows are expressed in liters per second (L/s). Examples of this will follow.

6.11 Energy Equation

Many problems in fluid mechanics are solved by the application of Bernoulli's Theorem. It will be useful to consider various forms expressed in SI units.

Essentially this is an "energy" equation which states that neglecting losses, for steady flow with fixed boundaries, the sum of the kinetic energy and the two potential energy components is constant. As would be expected each term of the equation is expressed in joules. For a mass *m* of fluid, at an elevation *z* above a given datum the equation is:

(Eq. 6.13) $$\frac{m\bar{V}^2}{2} + mgz + \frac{mp}{\rho} = \text{constant} \qquad \text{J}$$

This expression in terms of SI units is:

$$(\mathrm{kg})\frac{\mathrm{m}^2}{\mathrm{s}^2} + (\mathrm{kg})\frac{\mathrm{m}}{\mathrm{s}^2}\cdot \mathrm{m} + (\mathrm{kg})\frac{\mathrm{kg\cdot m}}{\mathrm{s}^2}\cdot\frac{1}{\mathrm{m}^2}\cdot\frac{\mathrm{m}^3}{\mathrm{kg}} = \mathrm{N\cdot m} = \mathrm{J}$$

The equation frequently appears without the mass term m, implying energy *per unit mass*, the expression then being:

(Eq. 6.14) $$\frac{\bar{V}^2}{2} + gz + \frac{p}{\rho} = \text{constant/unit of mass} \qquad \mathrm{J/kg}$$

In terms of SI this is:

$$\frac{\mathrm{m}^2}{\mathrm{s}^2} + \frac{\mathrm{m}}{\mathrm{s}^2}\cdot \mathrm{m} + \frac{\mathrm{kg\cdot m}}{\mathrm{s}^2}\cdot\frac{1}{\mathrm{m}^2}\cdot\frac{\mathrm{m}^3}{\mathrm{kg}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

but

$$\frac{\mathrm{m}^2}{\mathrm{s}^2} = \frac{\mathrm{kg\cdot m}^2}{\mathrm{kg\cdot s}^2} = \frac{\mathrm{N\cdot m}}{\mathrm{kg}} = \frac{\mathrm{J}}{\mathrm{kg}} = \mathrm{J/kg}$$

If this equation is multiplied by the mass rate of flow in kg/s, or by the volumetric rate-of-flow and the density, $(\mathrm{m}^3/\mathrm{s})(\mathrm{kg/m}^3)$, the various terms in the equation then represent rates of energy transfer, and are, accordingly, in joules per second, or watts.

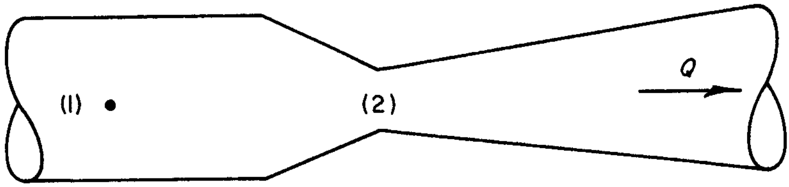
The equation can also be expressed in terms of head of fluid; to achieve this the unit mass form is divided by g, whence the pressure term then becomes $p/\rho g$, which is denoted by h, and the equation takes what is probably the most frequently used form:

(Eq. 6.15) $$\frac{\bar{V}^2}{2g} + z + h = \text{constant} \qquad \frac{\mathrm{m}^2}{\mathrm{s}^2}\cdot\frac{\mathrm{s}^2}{\mathrm{m}} + \mathrm{m} + \mathrm{m} = \mathrm{m}$$

in which the unit of each term is the meter. To obtain the rate of energy transfer from this equation it is necessary to multiply by g and the mass flow rate, kg/s.

Example 6-11

A horizontal pipeline, carrying sea water, changes gradually in diameter from 1.5 m to 0.7 m. The pressure at the larger end is 30 kPa and the discharge flow rate is 1.2 m^3/s. Neglecting losses, what will be the pressure at the smaller section? $\rho = 1030\ \mathrm{kg/m}^3$.



Example 6-11.

Applying the energy equation of Bernoulli in the form per unit of mass, namely, Equation (6.14):

$$\frac{\bar{V}^2}{2} + gz + \frac{p}{\rho} = \text{constant}$$

For horizontal pipe $z_1 = z_2$

$$A_1 = \frac{\pi}{4}(1.5)^2 = 1.77 \text{ m}^2$$

$$\bar{V}_1 = \frac{Q}{A_1} = \frac{1.2}{1.77} = 0.678 \text{ m/s}$$

$$\bar{V}_2 = (0.678)\frac{(1.5)^2}{(0.7)^2} = 3.11 \text{ m/s}$$

$$\frac{(0.678)^2}{2} + gz_1 + \frac{30 \times 10^3}{1030} = \frac{(3.11)^2}{2} + gz_2 + \frac{p_2}{1030}$$

whence

$$p_2 = 30\,000 + 1030[(0.678)^2 - (3.11)^2]$$

$$= 30\,000 - 4744$$

$$p_2 = 25\,260 \text{ Pa} = 25.3 \text{ kPa}$$

Observe that, as always, SI units can be used directly in the formula without modification.

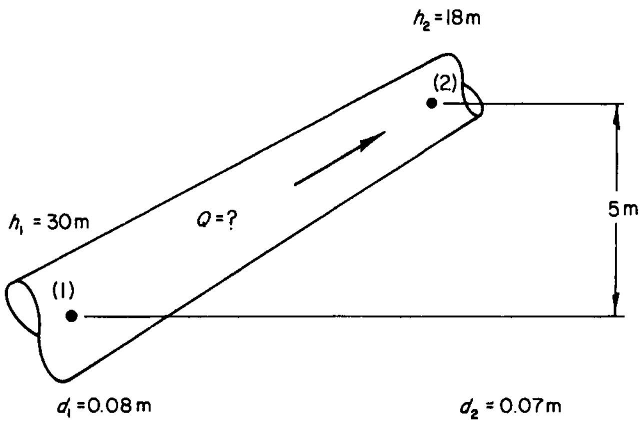
Example 6-12

Water flows through a pipe of gradually varying cross section. At a point where the diameter is 80 mm, the pressure head is 30 m; at another point, 5 m higher than the first, the diameter is 70 mm and the pressure head is 18 m. What is the volumetric rate-of-flow of water.

$$A_1 = \frac{\pi}{4}(0.08)^2 = 0.005026 \text{ m}^2$$

$$\bar{V}_1 = \frac{Q}{A_1} = \frac{Q}{0.005026} = 199Q$$

$$\bar{V}_2 = \bar{V}_1 \frac{(0.08)^2}{(0.07)^2} = 260Q$$



Example 6-12.

Using Bernoulli's energy equation expressed in terms of "head," in the form of Equation (6.15):

$$\frac{\bar{V}^2}{2g} + z + h = \text{constant}$$

$$\frac{(199Q)^2}{2(9.8)} + z_1 + 30 = \frac{260Q^2}{2(9.8)} + (z_1 + 5) + 18$$

whence

$$\frac{Q^2}{2(9.8)}\{(260)^2 - (199)^2\} = 7$$

$$Q = 0.07 \text{ m}^3/\text{s}$$

$$Q = 70 \text{ L/s}$$

Note that in this type of calculation, when using head consistently as a measure of pressure, the density does not appear.

6.12 Total Energy System

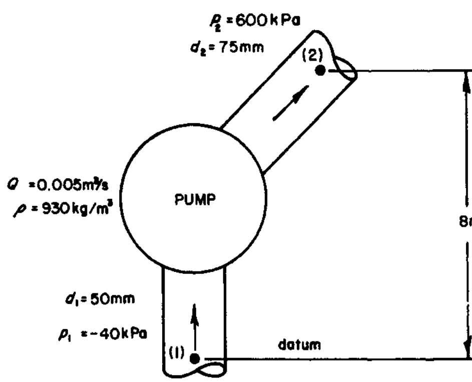
In most actual fluid flow systems there is a loss of energy, h_f, most frequently due to friction, or an input of energy, Δe_m, most often due to pumping, or some of each. The energy equations can be modified for use in such cases by algebraically adding, in the proper units, an energy gain and/or loss term; for example; Eq. 6.15 may be expanded as follows:

$$\frac{\bar{V}_1^{\,2}}{2g} + z_1 + h_1 - h_{f1-2} + \frac{\Delta e_m}{g} = \frac{\bar{V}_2^{\,2}}{2g} + z_2 + h_2 \qquad \text{(Eq. 6.16)}$$

in which h_{f1-2} = head loss due to friction between two points (1) and (2); while $\Delta e_m/g$ is energy introduced (or withdrawn; use negative sign) between the same two points, expressed also in terms of "head" of fluid.

Example 6-13

Oil of density 930 kg/m^3 is being pumped at a rate of 0.005 m^3/s. The diameters of the suction and discharge pipes of the pump are respectively 50 mm and 75 mm; the suction and discharge gages read −40 kPa and 600 kPa, respectively, with the vertical distance between the pressure tappings being 8 m. Calculate the power required at the pump, assuming no pipe losses and a pump efficiency of 70 percent.



Example 6-13.

$$\bar{V}_1 = \frac{Q}{A_1} = \frac{0.005}{\frac{\pi}{4}(0.05)^2} = 2.55 \text{ m/s}$$

$$\bar{V}_2 = 2.55\,\frac{(50)^2}{(75)^2} = 1.13 \text{ m/s}$$