

$$\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \chi [(S)(\tan \phi')] \quad [7]$$

The χ parameter proposed by Bishop is replaced by the degree of saturation, S , in Eq. [7]. The author's state that the χ -factor proposed by Bishop reflects the fraction of the pore area that is occupied by water (i.e., A_w/A_t) which is approximately equal to the degree of saturation, S . Equation [7] has not been experimentally verified on soils through the measurement of the shear strength and the degree of saturation at the point of failure in the specimens.

Equation [7] suggests that there is a one to one relationship between the degree of saturation, S , and the area of water contact along the shear plane in the soil. Analysis of experimental results presented and shown in the figures of this paper using Equation [7] are referred as **Procedure 3**.

Khallili and Khabbaz (1998) have extended Bishop's equation (1959) (i.e., Equation 1) for predicting the shear strength of an unsaturated soil. An empirical constant has been suggested for the parameter, χ in Equation 1 as given below:

$$\chi = \left\{ \frac{(u_a - u_w)_f}{(u_a - u_w)_b} \right\}^{-0.55} \quad [8]$$

where:

$(u_a - u_w)_f$ = matric suction in the specimens at failure conditions,
 η = a value equal -0.55 was suggested based on 13 soils data published in the literature

The required parameters for predicting the shear strength of unsaturated soils based on Khallili and Khabbaz (1998) approach is the air-entry value, $(u_a - u_w)_b$ of the soil and the saturated shear strength parameters. Analysis of experimental results presented and shown in the figures of this paper using Khallili and Khabbaz (1998) procedure is referred as **Procedure 4**.

Practicing engineers are most interested in the shear strength behavior in the transition zone (Figure 1). The transition zone lies between the air-entry value and the residual zone of saturation (Vanapalli et al. 1996a). The variation of the soil-water characteristic curve behavior in the transition zone is linear on a semi-logarithmic plot (i.e., variation of degree of saturation, S , or volumetric water content, θ , or gravimetric water content, w versus logarithm of soil suction). Bao et al. (1998) suggest another equation for predicting the unsaturated shear strength taking into account of the linear variation of the soil-water characteristic curve in the transition zone as below:

$$\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \left[\xi - \zeta \log(u_a - u_w) \right] \tan \phi' \quad [9]$$

where:

$$\xi = \frac{\log(u_a - u_w)}{\log(u_a - u_w)_r - \log(u_a - u_w)_h}$$

$$\zeta = \frac{1}{\log(u_a - u_w)_r - \log(u_a - u_w)_h}$$

$(u_a - u_w)_r$ = soil suction at residual water content conditions

The parameter, ξ , represents the intercept (i.e., on the abscissa) and the parameter, ζ , represents the slope of the linear part of the soil-water characteristic curve respectively. Bao et al. (1998) suggests use of the expression given below for fitting the soil-water characteristic curve data.

$$\frac{(\theta_w - \theta_s)}{(\theta_s - \theta_r)} = \xi - \zeta \log(u_a - u_w) \tag{10}$$

The form and philosophy of the Equation [9] for predicting the shear strength of unsaturated soils is similar to Procedure 2 outlined in this paper.

EXPERIMENTAL DATA USED IN THE ANALYSIS

Escario and Juca (1989) measured the shear strength of three different soils with varying percentages of clay and plasticity indices, I_p , using modified direct shear equipment. Statically compacted specimens were used in the measurement of shear strength under consolidated drained conditions with different net normal stresses. The shear strength for two soils, namely, Madrid gray clay and Red silty clay was measured for a suction range between 0 to 15,000 kPa. The shear strength for Madrid clay sand was measured for a suction range between 0 to 5,000 kPa. The properties of three soils used in their testing program are summarized in Table 1.

Table 1. Summary of soil properties tested by Escario and Juca (1989).

	Madrid gray clay	Red silty clay	Madrid clay sand
Sand (%)	1	17	87
Silt (%)	27	48	7
Clay (%)	72	35	6
Liquid Limit, w_L (%)	71	33	28
Plasticity Index, I_p (%)	35	13.6	8
Specific Gravity, G_s	2.64	2.66	2.71
Void ratio, e	1.03	0.48	0.38

Escario and Juca (1989) have also measured the soil-water characteristic curves for the three soils for a suction range between 0 to 15,000 kPa. The soil-water characteristic curves were measured with an applied stress of 20 kPa using calibrated springs such that there is a good contact between the specimen and the ceramic stone of the pressure plate apparatus. Figure 2 presents the fitted soil-water characteristic curve data for the entire

suction range from 0 to 1,000,000 kPa using Equation 5 (Fredlund and Xing, 1994). It should be noted that the soil-water characteristic curve data in the high suction range were not measured.

The soil-water characteristic curve results suggest that the air-entry and the residual suction values of the soil increases with an increase in the percentage of fines present in the soil. Table 2 summarizes the estimates of air-entry value and the residual suction value based on the construction procedure defined in Figure 1.

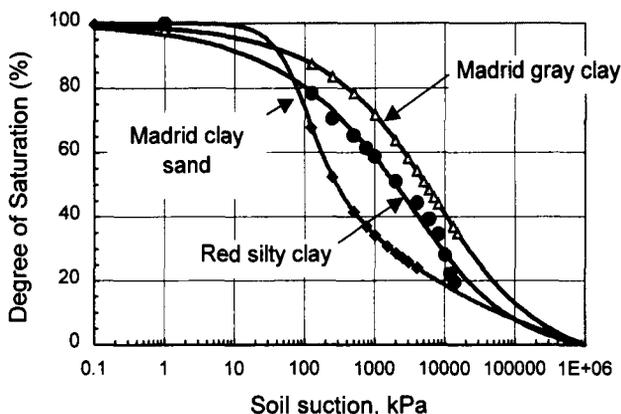


Figure 2. Soil-water characteristic curves for three soils tested by Escario and Juca (1989).

Table 2. Summary of the saturated shear strength parameters and the soil-water characteristic curve data

	Madrid gray clay	Red silty clay	Madrid clay sand
Air-entry value, $(u_a - u_w)_b$, kPa	110	40	30
Residual suction, ψ_r , kPa	40,000	33,000	12,000
Residual degree of saturation, (%)	23	16	17.6
Fitting parameter, κ (Procedure 1 using Equation 3)	2.8	1.9	1.65
Effective cohesion, c' , kPa	30	20	40
Effective angle of internal friction, ϕ' (degrees)	25.3	34	39.5
Net normal stress, $(\sigma - u_a)$, kPa	300	120	120
Saturated shear strength, kPa	173.3	96.3	138.9

The comparisons between the predicted and measured shear strength values are presented using the four different procedures discussed in earlier sections (i.e., Figures 3 to 8). These comparisons are provided both for a limited suction range and as well as a large suction range. The key information required for predicting the shear strength of unsaturated

soils using the procedures, along with the saturated shear strength parameters is summarized in Table 2.

ANALYSIS OF RESULTS

Several parameters such as initial compaction water content and the stress state influence the soil-water characteristic curve behavior (Vanapalli et al. 1999). It would appear that the shear strength contribution due to suction can be more accurately estimated using the soil-water characteristic curve that has been determined taking into account the influence of the stress state and the initial water content conditions. The soil-water characteristic curves are conventionally determined in the laboratory only with the application of a nominal normal stress or without the application of any applied vertical stress.

Vanapalli et al. (1998b) studies have shown that the shear strength of Indian Head till specimens can be predicted with a reasonable degree of accuracy using the soil-water characteristic curves that have been measured without considering the influence of stress state. Procedures 1 and 2 presented in this paper were used for analyzing the results. The Atterberg limits of the Indian Head till were as follows: liquid limit, w_L of 35.5% and plasticity index, I_p of 18.7%. The fractions of sand, silt, and clay were 28%, 42% and 30% respectively.

The predicted and measured shear strength values are compared for the three soils in this paper are; namely, Madrid gray clay, Red silty clay and Madrid clay sand. The shear strength of these soils were tested with different net normal stresses (Table 2). The soil-water characteristic curves measured with 20 kPa stress were used in the analyses (Figure 1).

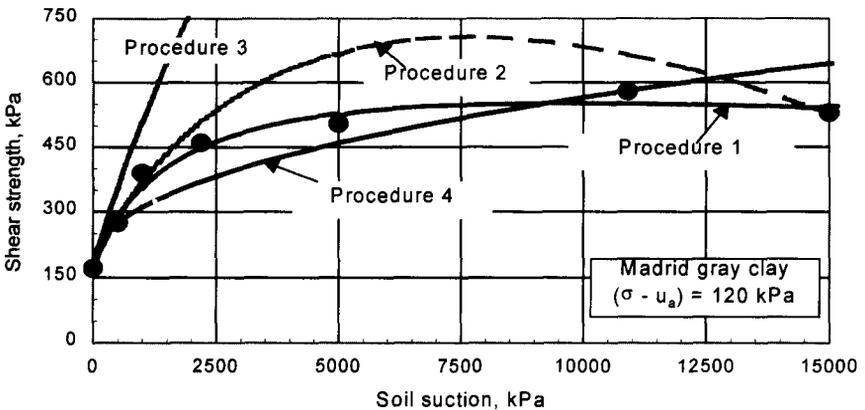


Figure 3. Comparison of predicted and measured shear strength values for Madrid gray clay using different procedures for a large suction range.

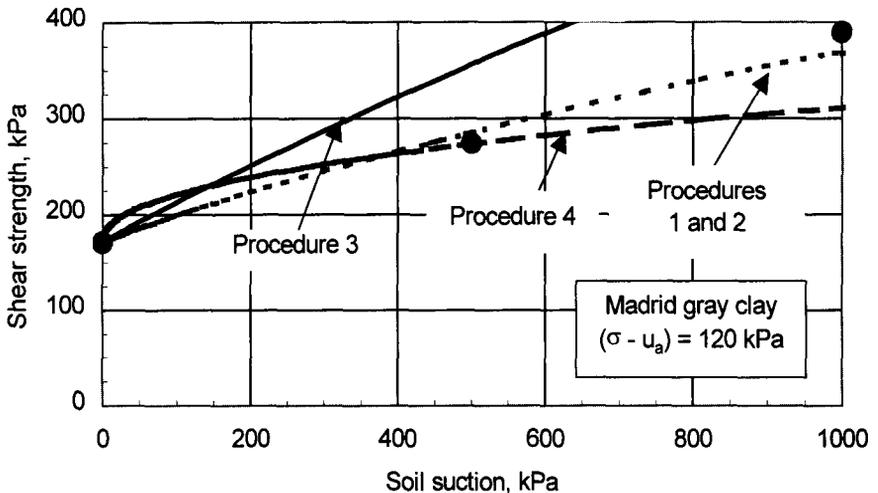


Figure 4. Comparison of predicted and measured shear strength values for Madrid gray clay using different procedures for a large suction range.

Figure 3 shows comparisons between the predicted and measured shear strength values for Madrid gray clay using the four different procedures discussed in the paper for a suction range between 0 to 15,000 kPa. The continuous lines in the figure represent the predicted shear strength values and the symbols are the measured shear strength values. There is a good comparison between the predicted and measured values of the shear strength for the suction range of 0 to 15,000 kPa using Procedure 1 (Figure 3). A fitting parameter value of κ equal to 2.8 was used for obtaining these predictions using Equation [3].

Figure 4 shows comparisons between predicted and measured shear strength values for a limited suction range of 0 to 1,000 kPa. Procedures 2 and 4 along with Procedure 1 provide good comparisons between the measured and predicted values of shear strength.

Figure 5 shows the comparison between the measured and predicted shear strength values for Red silty clay for the suction range 0 to 15,000 kPa. Procedure 1 provides good comparisons for the suction range of 0 to 15,000 kPa. Procedures 2, 3 and 4 do not provide good comparisons. These observations are consistent with the earlier observations for Madrid gray clay.

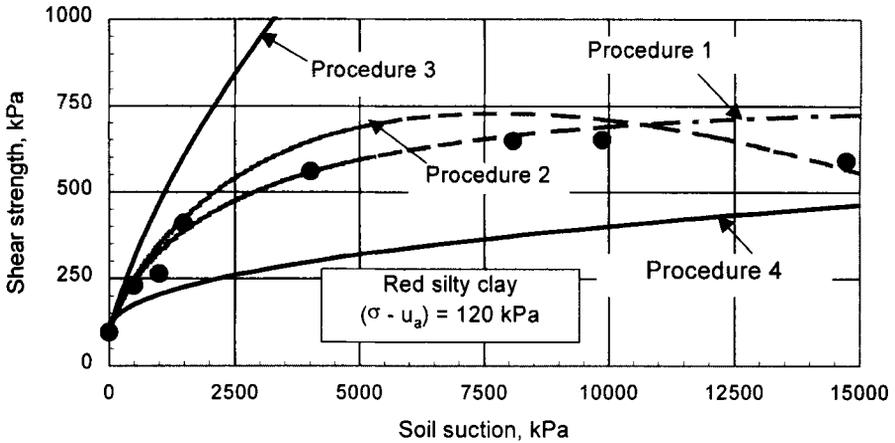


Figure 5. Comparison of predicted and measured shear strength values for Red silty clay using different procedures for a large suction range.

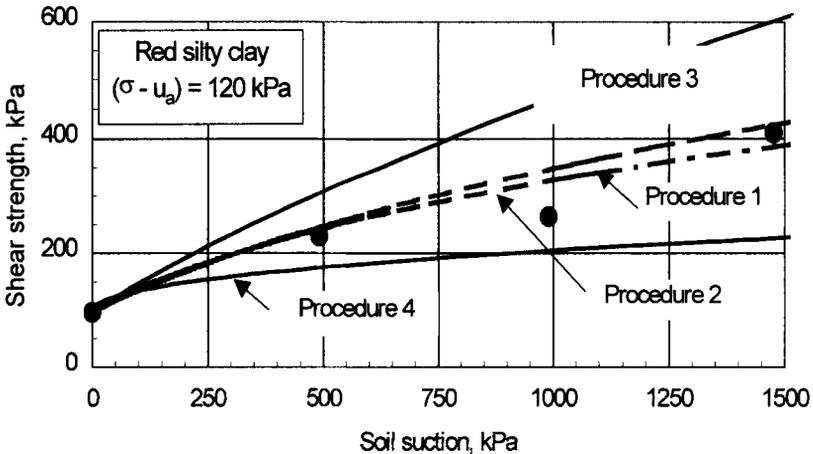


Figure 6. Comparison of predicted and measured shear strength values for Red silty clay using different procedures for a suction range between 0 to 1,500 kPa.

Figure 6 shows the comparison between predicted and measured shear strength values for Red silty clay for a limited suction range between 0 to 1,500 kPa. Procedures 3 and 4 do not provide good comparisons. Procedure 2 provides reasonably good comparisons for a suction range between 0 to 1,500 kPa.

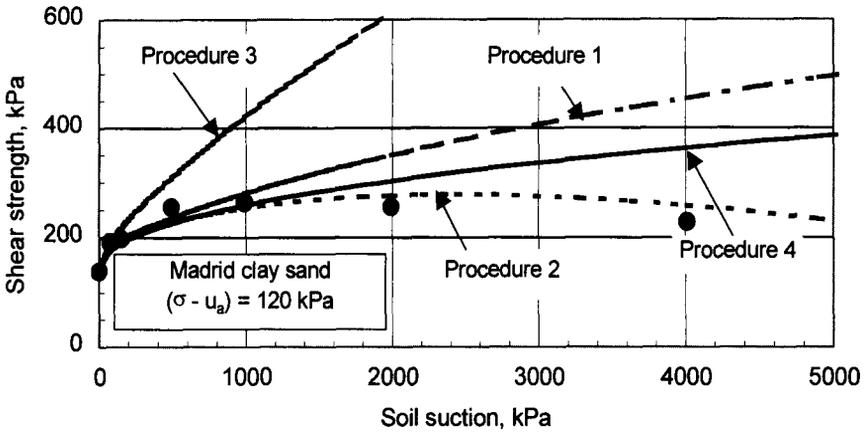


Figure 7. Comparison of predicted and measured shear strength values for Madrid clay sand using different procedures for a large suction range.

Figure 7 provides comparison between the measured and predicted shear strength values for Madrid clay sand for the suction range 0 to 5,000 kPa. Procedure 2 provides good comparison for the suction range of 0 to 5,000 kPa.

Figure 8 shows the comparison between predicted and measured shear strength values for Madrid clay sand for a limited suction range between 0 to 1,000 kPa. Procedure 1, 2, and 4 provide reasonably good comparisons.

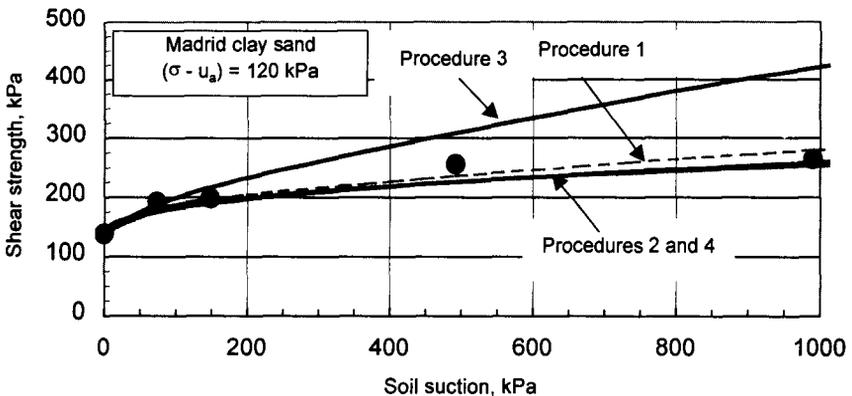


Figure 8. Comparison of predicted and measured shear strength values for Madrid clay sand using different procedures for a suction range between 0 to 1,000 kPa.

Summary and Conclusions

Comparisons between the measured shear strength and predicted shear strength values for three different soils are presented. Four different procedures available in the literature for predicting the shear strength are used both for limited and large suction ranges.

Procedure 1 provides good comparisons between the measured and predicted shear strength values for limited suction ranges between 0 to 1,500 kPa for all the three soils presented in this paper. Unsaturated soil behavior is of practical interest in this suction range. Good comparisons were also observed for two of the three soils for even for a large suction range. This procedure provides better predictions in comparison to the other procedures.

Figure 9 shows the relationship between the fitting parameter, κ , and plasticity index, I_p . Three points of κ versus I_p relationship were obtained from the study presented in this paper (Table 2). The other two points were based on the studies presented by Vanapalli et al. (1996) using Indian Head and Wulfsohn et al. (1996) using a non-plastic soil. A value of κ equal 2.3 provided good correlation between the measured and predicted values of shear strength for Indian Head till with a plasticity index, I_p , of 18.7. A value of κ equal 1 provided good correlation between predicted and measured values of shear strength for the non-plastic soil. The fitting parameter κ may be influenced by other parameters such as soil structure, nature of soil (i.e., slurry consolidated, dynamically compacted or statically compacted, natural) etc. More studies are necessary to establish the uniqueness of this relationship. Correlation such as shown in Figure 9 may be possible with other soil constants.

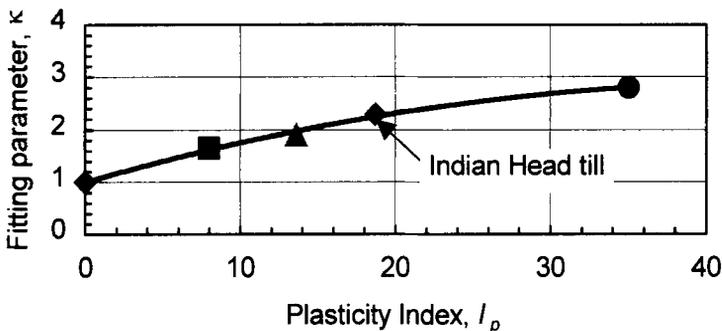


Figure 9. The relationship between the fitting parameter, κ , and plasticity index, I_p

Procedure 2 provides good comparisons between measured and predicted values for the three soils analyzed in this paper for a limited suction range between 0 to 1,500 kPa. However, the comparisons were not good for predicting the shear strength over a large suction range for two soils. This may be attributed to the possible errors in the estimation of

residual suction value from the construction procedure using limited soil-water characteristic curve data.

Procedure 3 does not provide good relations both for limited and as well large suctions ranges for the comparisons undertaken in the study presented in this paper. Figure 8 shows reasonable comparisons for a limited suction range between 0 to 200 kPa for Madrid clay sand.

Procedure 4 provides reasonable estimates for predicting the shear strength of two of the three soils analyzed in the low suction range but does not provide good comparisons for large suction ranges.

The studies presented in this paper show promise of using the procedures predicting the shear strength of an unsaturated soil using the soil-water characteristic curve and the saturated shear strength parameters. More experimental studies are necessary on different types of soils to better understand the shear strength behavior of unsaturated soils and develop better prediction procedures.

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