A Time-Domain Covariance-Based Parameter Estimation Method for Torsional Shear Buildings

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ABSTRACT

A time-domain parameter estimation algorithm for direct estimation of the stiffness properties of a torsional shear building has been developed. The algorithm decouples the banded global stiffness matrix into story stiffness matrices, and therefore is capable of identifying the lateral and torsional stiffness parameters of each story independent of others. Covariance matrices of the structural response to unknown sources of excitation are used in order to eliminate the external forces from the equations of motion. The method can be applied to ambient vibration data gathered before and after a severe event for damage identification and localization up to the story level. The proposed method is verified using analytical data from Phase I of the IASC-ASCE benchmark problem.

BACKGROUND & INTRODUCTION

Structural Health Monitoring (SHM) is typically deemed as the process of examining the present condition of a structure in order to identify and/or quantify damage to its components. Damage can be defined as deterioration in the structural properties, which presumably leads to deviations in the vibration properties of the monitored structure from a baseline condition that is considered to correspond to its undamaged state.

To date, countless system identification (SysID) and damage detection techniques have been proposed for tracking changes in the structural response, and for identifying structural properties (e.g., stiffness, damping, etc.) by analyzing vibration data. Such vibration-based SHM techniques can be powerful tools for monitoring the overall behavior of structures and may help identify damage even when it is confined to inaccessible parts of a structure such as joints embedded behind walls.

The acquisition of vibration data can be made either during a forced-vibration test or during the operational conditions of a structure. A significant advantage of forced-vibration techniques is the high signal-to-noise ratio of the measurements. However, it is usually very difficult and/or expensive to vibrate a civil structure such that the level of artificial vibrations exceeds that of natural vibrations induced by the operational conditions or environmental agents such as wind. Consequently, several system identification techniques have emerged that are referred to as operational SHM techniques. As discussed by Farrar & James (1997), the source of ambient vibrations of a structure can be modeled as uncorrelated whitenoise excitations. The most common problem associated with this approach is that the whitenoise assumption might be violated due to some dominant frequency components in the natural excitations of a structure. Peeters & Guido (2001) have compiled a literature survey on stochastic system identification methods for operational modal analysis. A, now well-established, method based on the assumption of stationary input has been proposed by James *et al*. (1993), dubbed the Natural Excitation Technique (NExT). It has been theoretically shown that under the assumptions of the ambient excitation being uncorrelated whitenoise, classical normal modes, and the responses being weak stationary processes, the cross-correlation function between two response time-histories can be represented as the sum of decaying sinusoids, which is the same as free vibration response of the same structure. Other researchers have employed this method in conjunction with system identification techniques that were originally applied to systems with known input/output, in order to develop output-only methods [Lin *et al*., 2005; Dyke *et al*., 2000]. Nayeri *et al.* (2008) incorporated the NExT method into the method proposed by Masri *et al.* (1982) for chain-like structures. The method is based on the assumption that "the restoring force at each element is only dependent on the relative displacement and velocity across the terminals of that element."

The method developed in the current paper can be considered as a variation of the method proposed by Nayeri *et al*. (2008), which was developed independently and is applicable to shear buildings with significant torsional motion and coupling between the two lateral directions. The algorithm is based on the time-domain equations of motion and utilizes the banded structure of the global stiffness matrix in order to decompose it into story stiffness matrices, and therefore is capable of identifying the stiffness parameters of each story independent of others. The use of covariance matrices of ambient vibration data makes it possible to diminish the external force terms from the equations of motion. This approach to eliminate the forcing terms is analogous to the NExT method [James *et al*., 1993], whereby the correlation and cross-correlation functions between given data channel and a reference channel are considered as free-vibration responses.

In what follows, we first present the formulation details of our technique, and then its verification using data from a benchmark study that has been developed by the IASC-ASCE task group [Johnson *et al.*, 2004].

FORMULATION

Assuming that the floor slabs are rigid diaphragms, lumping-mass at floor levels and neglecting vertical motions, the equations of motion of an n-story shear building can be defined as follows

$$\begin{aligned}
\mathbf{m}_n\ddot{\mathbf{x}}_n + \mathbf{c}_n\dot{\mathbf{x}}_n - \mathbf{c}_n\dot{\mathbf{x}}_{n-1} + \mathbf{k}_n\mathbf{x}_n - \mathbf{k}_n\mathbf{x}_{n-1} &= \mathbf{f}_n \\
\mathbf{m}_{n-1}\ddot{\mathbf{x}}_{n-1} - \mathbf{c}_n\dot{\mathbf{x}}_n + (\mathbf{c}_n + \mathbf{c}_{n-1})\dot{\mathbf{x}}_{n-1} - \mathbf{c}_{n-1}\dot{\mathbf{x}}_{n-2} - \mathbf{k}_n\mathbf{x}_n + (\mathbf{k}_n + \mathbf{k}_{n-1})\mathbf{x}_{n-1} - \mathbf{k}_{n-1}\mathbf{x}_{n-2} &= \mathbf{f}_{n-1} \\
&\vdots \\
\mathbf{m}_1\ddot{\mathbf{x}}_1 - \mathbf{c}_2\dot{\mathbf{x}}_2 + (\mathbf{c}_1 + \mathbf{c}_2)\dot{\mathbf{x}}_1 - \mathbf{k}_2\mathbf{x}_2 + (\mathbf{k}_1 + \mathbf{k}_2)\mathbf{x}_1 &= \mathbf{f}_1
\end{aligned} \quad \textbf{(1)}$$

where $\mathbf{m}_n$, $\mathbf{c}_n$, and $\mathbf{k}_n$ correspond to the n^{th} story, 3×3 mass, damping, and stiffness matrices respectively. $\mathbf{x}_n$ is the n^{th} story displacement vector including two lateral and one torsional displacement components. The vectors $\dot{\mathbf{x}}_n$ and $\ddot{\mathbf{x}}_n$ denote the first and second time-derivatives of $\mathbf{x}_n$. Finally, $\mathbf{f}_n$ is the vector of external lateral and torsional excitations applied at the n^{th} story. The global equation of motion can be set as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \quad \textbf{(2)}$$

with **M**, **C**, and **K** corresponding to the global mass, damping and stiffness matrices respectively. **X** and **F** denote the global displacement and external excitation vectors.

Assuming that no coupling exists between the lateral load resisting systems of non-adjacent stories (i.e., no bracing element is connecting non-adjacent floors), the global stiffness matrix would have a block tri-diagonal structure. Utilizing this banded assembly, equations of motion can be rearranged into a new set of equations such that in each equation the stiffness and damping matrices of only one story are involved. Therefore, it would be possible to solve for each story's stiffness and damping matrix independent of others, given the mass properties of the structure and response time-histories of all floors. As such, the global stiffness matrix is written as the product of three matrices as follows

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_1+\mathbf{k}_2 & -\mathbf{k}_2 & \mathbf{0} & \cdots & \mathbf{0} \\ & \ddots & \ddots & \ddots & \vdots \\ & & \mathbf{k}_{n-1}+\mathbf{k}_{n-2} & -\mathbf{k}_{n-1} & \mathbf{0} \\ & \mathit{Symm.} & & \mathbf{k}_n+\mathbf{k}_{n-1} & -\mathbf{k}_n \\ & & & & \mathbf{k}_n \end{bmatrix} = \mathbf{D}\underbrace{\begin{bmatrix} \mathbf{k}_1 & & 0 \\ & \ddots & \\ \mathit{Symm.} & & \mathbf{k}_n \end{bmatrix}}_{\hat{\mathbf{K}}}\mathbf{D}^T \tag{3}$$

where **D** can be considered as a differentiation operator with 3×3 identity blocks on its diagonal and negative 3×3 identity block on its upper diagonal.

The same procedure can be applied to the global damping matrix due to its similarly banded structure. Substituting for **K** and **C** in (2) and defining the state vector as $\mathbf{S} = \begin{bmatrix}\mathbf{D}^{\mathrm{T}}\mathbf{X} & \mathbf{D}^{\mathrm{T}}\dot{\mathbf{X}}\end{bmatrix}^T$ would yield

$$\begin{bmatrix}\hat{\mathbf{K}} & \hat{\mathbf{C}}\end{bmatrix}\mathbf{S} = \mathbf{D}^{-1}\mathbf{F}_{ext} - \mathbf{D}^{-1}\mathbf{M}\ddot{\mathbf{X}} \tag{4}$$

where $\hat{\mathbf{K}}$ and $\hat{\mathbf{C}}$ are block-diagonal matrices with the n^{th} diagonal blocks corresponding to the n^{th} story stiffness and damping matrices respectively. Expanding both sides of (4) yields

$$\begin{aligned} \mathbf{k}_1\mathbf{u}_1 + \mathbf{c}_1\dot{\mathbf{u}}_1 &= \mathbf{f}_1 + \cdots + \mathbf{f}_n - \left(\mathbf{m}_1\ddot{\mathbf{x}}_1 + \cdots + \mathbf{m}_n\ddot{\mathbf{x}}_n\right) \\ \mathbf{k}_2\mathbf{u}_2 + \mathbf{c}_2\dot{\mathbf{u}}_2 &= \mathbf{f}_2 + \cdots + \mathbf{f}_n - \left(\mathbf{m}_2\ddot{\mathbf{x}}_2 + \cdots + \mathbf{m}_n\ddot{\mathbf{x}}_n\right) \\ &\vdots \\ \mathbf{k}_n\mathbf{u}_n + \mathbf{c}_n\dot{\mathbf{u}}_n &= \mathbf{f}_n - \mathbf{m}_n\ddot{\mathbf{x}}_n \end{aligned} \tag{5}$$

or

$$\begin{bmatrix}\mathbf{k}_i & \mathbf{c}_i\end{bmatrix}\mathbf{S}_i = \sum_{j=i}^{n}\left(\mathbf{f}_{ext}\right)_j - \sum_{j=i}^{n}\mathbf{m}_j\ddot{\mathbf{x}}_j \quad i = 1, \cdots, n \tag{6}$$

where $\mathbf{u}_i$ is the relative displacement between the i^{th} and the $(i\text{-}1)^{\text{th}}$ floors and $\mathbf{S}_i = \begin{bmatrix}\mathbf{u}_i & \dot{\mathbf{u}}_i\end{bmatrix}^T$.

In the method proposed by Masri *et al*. (1982) for chain-like structures, identification of the restoring forces at each link is dependent on the restoring forces identified for the previous link, thus the error propagates through the chain. In contrast, in the current formulation, the identification of each story's stiffness and damping matrices is carried out independently.

In order to eliminate the external force from the equations of motion, the ambient excitation is assumed to be an uncorrelated whitenoise stochastic process. Therefore, small or no correlation exists between non-simultaneous external force and response. Post multiplying both sides of (4) by a lagged version of the state vector, **S**, and taking the expected values of both sides gives

$$\begin{bmatrix}\hat{\mathbf{K}} & \hat{\mathbf{C}}\end{bmatrix}\mathrm{E}\left\{\mathbf{S}(t)\mathbf{S}^{\mathrm{T}}(t-\tau)\right\}=\mathbf{D}^{-1}\left(\mathrm{E}\left\{\mathbf{F}_{ext}(t)\mathbf{S}^{\mathrm{T}}(t-\tau)\right\}-\mathrm{E}\left\{\mathbf{M}\ddot{\mathbf{X}}(t)\mathbf{S}^{\mathrm{T}}(t-\tau)\right\}\right). \quad \textbf{(7)}$$

The first and second terms on the right-hand side of (7) correspond to the cross-correlation matrix between the external force at time t and the state vector at time t-τ and the cross-correlation matrix between the inertial force at time t and the state vector at time t-τ respectively. As mentioned earlier, if the source of ambient vibration is whitenoise, the response at time t-τ is uncorrelated with the excitation at time t. This implies that the elements of the first cross-correlation matrix on the right-hand side are much smaller than the elements of the second cross-correlation matrix on the right-hand side of (7). By choosing a suitable lag (one which would maximize the inertial force-response correlation), the first term, which is unknown in the case of ambient vibrations, can be neglected, i.e.,

$$\begin{bmatrix}\hat{\mathbf{K}} & \hat{\mathbf{C}}\end{bmatrix}\mathrm{E}\left\{\mathbf{S}(t)\mathbf{S}^{\mathrm{T}}(t-\tau)\right\}=\mathbf{D}^{-1}\left(-\mathrm{E}\left\{\mathbf{M}\ddot{\mathbf{X}}(t)\mathbf{S}^{\mathrm{T}}(t-\tau)\right\}\right). \quad \textbf{(8)}$$

Combining (6) and (8) and applying the Ergodic Principle [Balakrishnan, 2005] for evaluating the expected values, the final formulation can be obtained as

$$\begin{bmatrix}\mathbf{k}_i & \mathbf{c}_i\end{bmatrix}\sum_{r=1}^{T}\mathbf{S}_i(t_r)\mathbf{S}_i^{T}(t_r-\tau)=-\sum_{r=1}^{T}\left[\sum_{j=i}^{n}\mathbf{m}_j\ddot{\mathbf{x}}_j(t_r)\right]\mathbf{S}_i^{\mathrm{T}}(t_r-\tau) \quad i=1,\ \cdots,\ n \quad \textbf{(9)}$$

where T is the total number of time-samples used for covariance calculations.

APPLICATION TO THE IASC-ASCE BENCHMARK PROBLEM

A series of benchmark studies [Johnson *et al.*, 2004] has been developed by the IASC-ASCE task group in order to provide a platform for comparing various structural health monitoring and damage detection techniques.

Introduction to IASC-ASCE Benchmark Problem

The benchmark structure is a four-story two-bay steel-frame quarter-scale model sited in the Earthquake Engineering Research Laboratory at the University of British Columbia. Lateral load resistance is provided through the lateral stiffness of columns and two diagonal braces at each face of the structure at each floor level.

The first phase of benchmark studies consists of *only analytical/simulated data* generated by finite element models of the existing model structure. In this Phase I, analytical data can be generated by using either a 12 or a 120 degree-of-freedom (DOF) finite element model, whereas the identification model is restricted to be a 12-DOF model. Using the 120-DOF model to generate simulated measurement data but using a 12-DOF model for identification incorporates the modeling errors into the benchmark study, which is present in most real-life model identification studies. Since the identification model is restricted to be a 12-DOF model, the 120-DOF model should be reduced such that it is possible to compare the identified 12-DOF model with a reference structure. In Jonhson *et al.* (2004), two approaches are proposed for reducing the model order, one of which is based on the comparison of stiffness matrices of the two (full and reduced) models. In the other approach, the model-order is reduced such that the flexibility matrix of the reduced model is close to the flexibility matrix of the full model. We shall refer to the former and the latter as the “first”, and the “second” approach equivalent models, respectively.

Excitation (modeled as whitenoise) is applied either at the middle of the north face of the structure in the y (weak) direction at all stories, or at the top of the central column in the diagonal direction. Due to the geometry of the structure and symmetry of the loading, no coupling exists between the x and y directions. In order to induce torsional motion in the structural response, asymmetry can be introduced to the structure by changing the mass of the north-east roof slab from its initial value, which is initially identical to the mass of the other three slabs. Depending on the different options available for the structure's geometry, excitation, and the analytical model, several cases can be considered as listed in Table 1.

Description		**Cases**				
		1	**2**	**3**	**4**	**5**
		(1D)	**(+ model error)**	**(roof excit.)**	**(3D)**	**(+ model error)**
Data Gen. Model	1. Floors rigid (12DOF)	×		×	×	
	2. Floors rigid in-plane (120DOF)		×			×
Mass Distribution	1. Symmetric	×	×	×		
	2. Asymmetric				×	×
Excitation	1."Ambient"	×	×			
	2.Shaker diagonal on roof			×	×	×
ID model	Linear 12DOF shear building	×	×	×	×	×

TABLE 1 – IASC-ASCE BENCHMARK PROBLEM IDENTIFICATION CASES

In addition to the undamaged (reference) structure several damage patterns, listed in Table 2, are available; and further details may be found in the work by Johnson *et al.* (2004).

Damage patterns: Remove the following	
(*i*). All braces in 1st story	(*iv*). One brace in each of 1st and 3rd stories
(*ii*). All braces in 1st and 3rd stories	(*v*). As *iv* , and loosen floor beam at 1st level
(*iii*). One brace in 1st story	(*vi*). 2/3 stiffness in one brace in 1st story

TABLE 2 - IASC-ASCE BENCHMARK PROBLEM AVAILABLE DAMAGE PATTERNS

A Review of Previous Research

Prior to moving on to the results obtained through the proposed method, we describe previous relevant work that made use of the data from the IASC-ASCE benchmark study first.

Lus *et al.* (2000) proposed a two-step method for system identification and damage detection of linear structures. In the first step, a first-order state space model is identified using known input and output data through Observer Kalman Identification Algorithm (OKID) and Eigensystem Realization Algorithm with data correlations (ERA/DC). In the second step, the first-order model is transformed into a second-order model, and the system mass, damping and stiffness matrices are evaluated. This method was applied to Case 3 considering damage pattern *iii* with known input and measurement noise. Bernal *et al.* (2000) proposed a three-module method. In the first module, the modal parameters are identified using OKID/ERA for the known input case, and a subspace method for the unknown input case. In the second module, the flexibility matrix is computed at sensor locations and the subset of damaged elements is identified subsequently. In the third module, an updating strategy is employed to quantify damage in the damaged element. This method was applied to Cases 1 through 3 considering damage patterns *i* and *ii*. Measurement noise was taken into account. Caicedo *et al.* (2004) proposed a technique based on ERA in conjunction with Natural Excitation Technique (NExT) to identify modal parameters. The stiffness parameters are estimated through a least-squares optimization. The method was applied to Cases 1 through 5, considering all available damage patterns. The authors employed the method in an iterative scheme with a view to applying it to the case of incomplete sensor array as well. In a preceding paper, Dyke *et al.* (2000) had identified the first four mode shapes of the benchmark structure successfully by employing an ERA/NExT method. Lin *et al.* (2005) proposed a damage detection technique based on Hilbert Huang Transform (HHT), in conjunction with the NExT method to obtain modal properties as well as story damping and stiffness matrices. This method was applied to Cases 1 and 2 considering damage patterns *i* and *ii*. Input is considered as unknown and measurement noise is taken into account. Beck *et al.* (2001) proposed "two-step" and "one-step" probabilistic approaches for structural damage detection. In the "two-step" approach, modal parameters and their uncertainties are identified in the first step. The prior probability density functions of the stiffness parameters are updated using the modal properties identified in the first step. In the "one-step" approach the modal properties are considered as functions of the stiffness parameters. Thus, the stiffness parameters and their uncertainties are updated directly from the measured time-histories through the update of the probability density function (PDF) of the modal properties. The method was applied to Cases 1-3 considering damage patterns *i* and *ii*. The damage detection technique was applied to both known and unknown input cases. Corbin *et al.* (2000) proposed a damage detection approach using wavelet analysis. This method can detect the instant when damage occurs from the location of spikes in the wavelet transforms of acceleration records as well as the location of the damaged region due to the spatial distribution of spikes. The method was applied to case1 considering damage pattern *ii*. Chase *et al.* (2005) proposed damage detection techniques employing adaptive recursive least-squares (RLS) filters. The methods use known input/output data to directly estimate the amount of changes in the stiffness matrix. These methods were applied to Cases 1, 3, and 4 considering damage patterns *i*-*iv*.

We applied our method to five different cases of the benchmark problem considering various damage patterns. Since Cases 4 & 5 are more general compared to the first three cases, only the results for these two cases are presented in what follows.

Results using the Proposed Method for Case 4

The maximum relative errors between the exact and identified model parameters in the weak and strong directions for Case 4 were 0.49% and 0.79%, respectively. The maximum relative error of the torsional stiffness was 3.22%. Plots shown in Figure 1 through Figure 3 imply that the algorithm is slightly overestimating the amount of relative torsional stiffness reductions in the floors adjacent to the damaged floors.

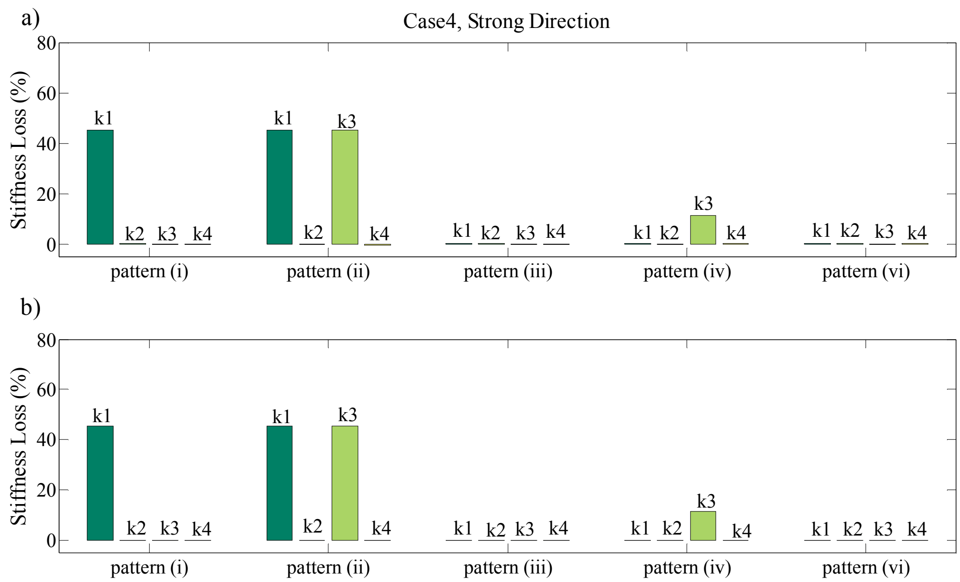

FIGURE 1 - CASE 4 RELATIVE STIFFNESS REDUCTION IN THE STRONG DIRECTION: A) IDENTIFIED, B) EXACT.

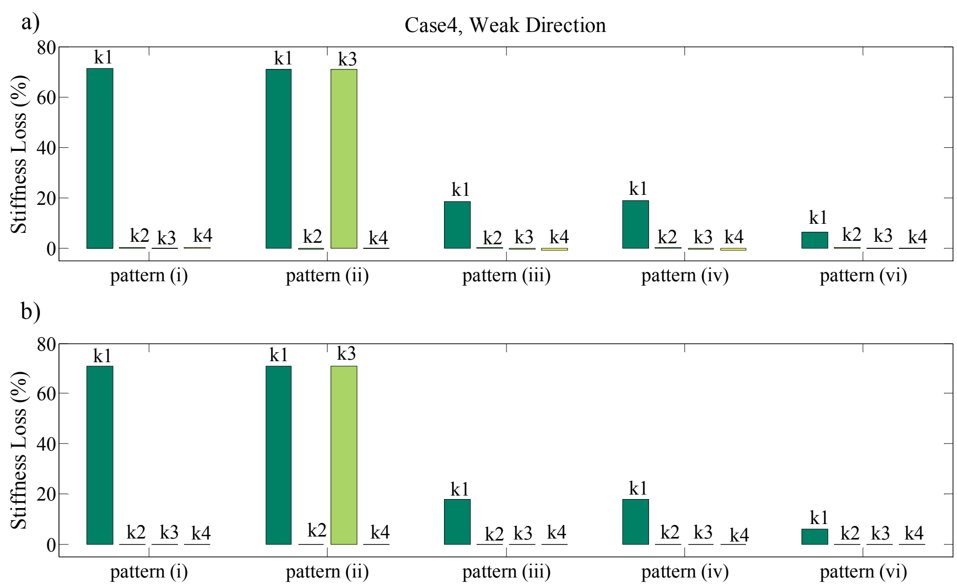

FIGURE 2 - CASE 4 RELATIVE STIFFNESS REDUCTION IN THE WEAK DIRECTION: A) IDENTIFIED, B) EXACT.

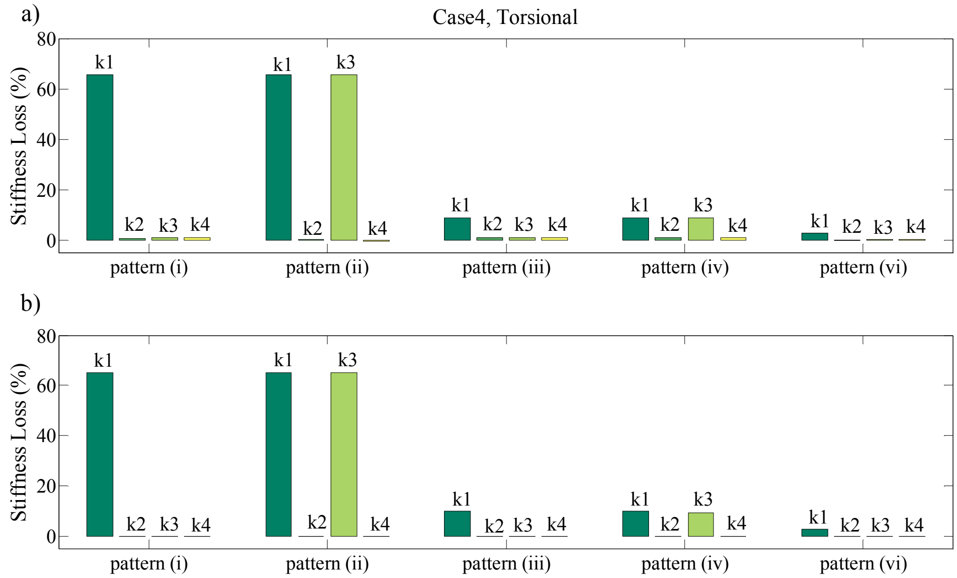

FIGURE 3 - CASE 4 RELATIVE TORSIONAL STIFFNESS REDUCTION: A) IDENTIFIED, B) EXACT.

Results using the Proposed Method for Case 5

The results for Case 5 are shown in Figure 4 through Figure 6. This case is the same as Case 4 with modeling error incorporated. The relative amounts of stiffness loss are bounded by the 1st and 2nd approach equivalent values. In damage pattern *v*, loosening a floor beam at the 1st floor would cause slight additional stiffness reduction in the weak direction compared to damage pattern *iv*, and it appears that the proposed method is capable of discriminating between the two patterns.

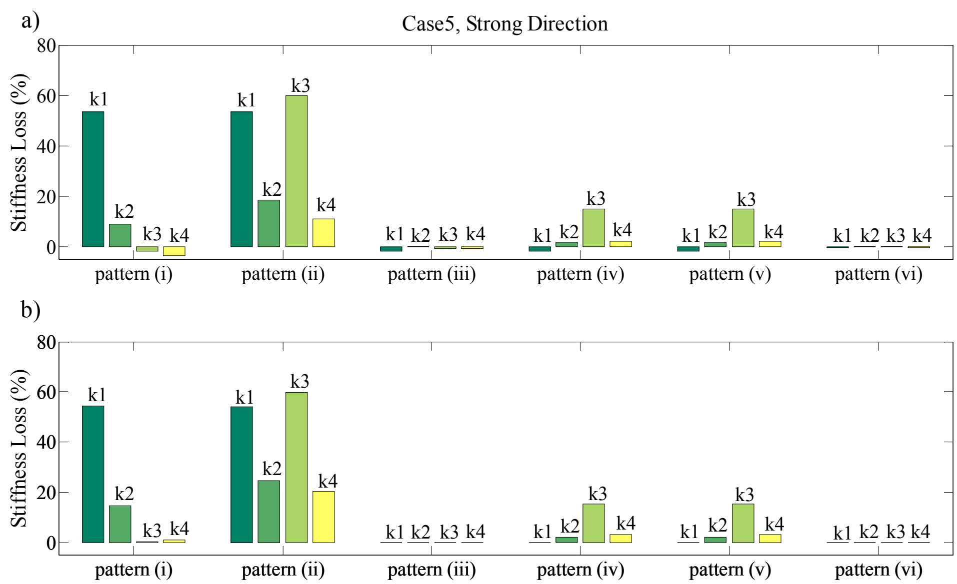

FIGURE 4 - CASE 5 RELATIVE STIFFNESS REDUCTION IN THE STRONG DIRECTION: A) IDENTIFIED, B) EXACT.

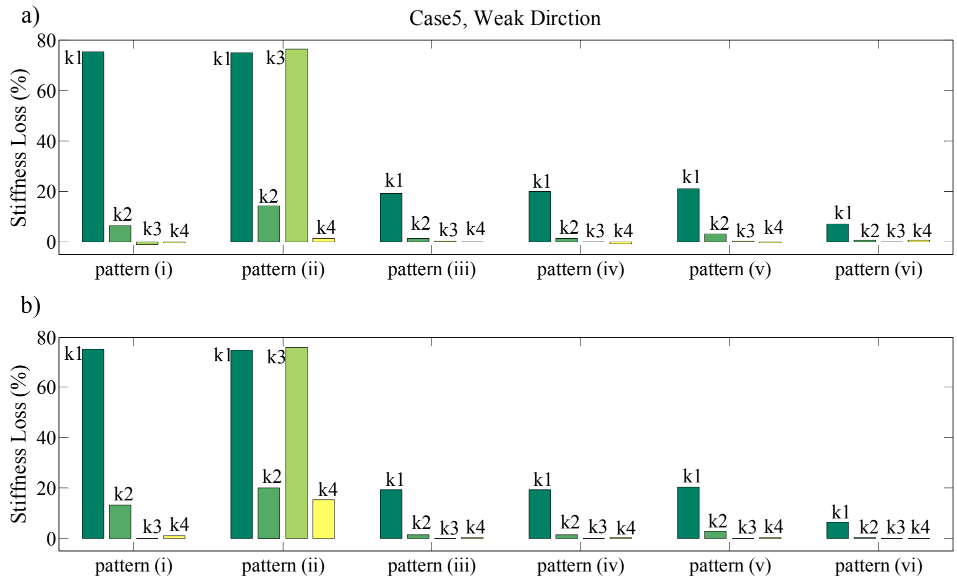

FIGURE 5 - CASE 5 RELATIVE STIFFNESS REDUCTION IN THE WEAK DIRECTION: A) IDENTIFIED, B) EXACT.

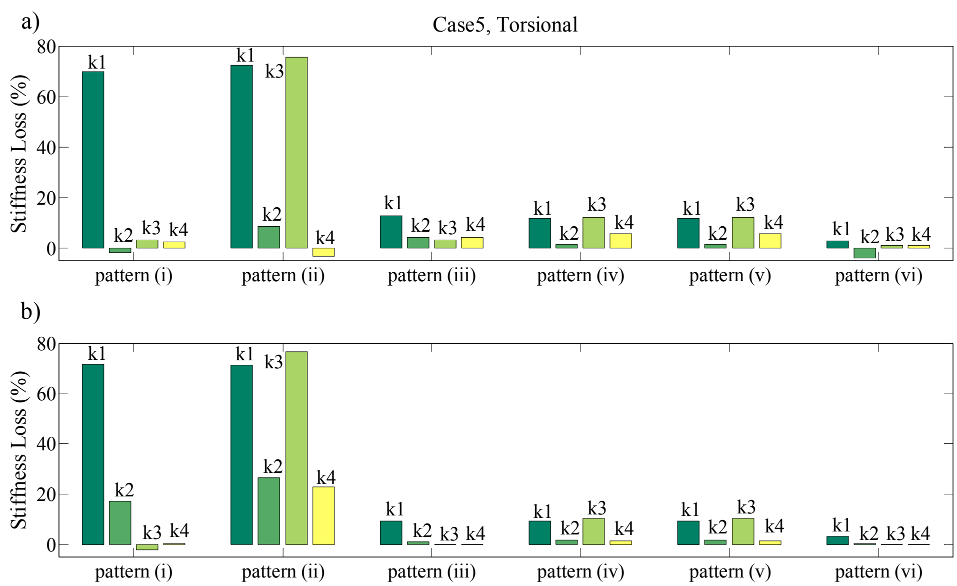

FIGURE 6 - CASE 5 - RELATIVE TORSIONAL STIFFNESS REDUCTION: A) IDENTIFIED, B) EXACT.

SUMMARY OF RESULTS

The proposed method is successful in identifying the stiffness parameters when there is no modeling error. In the presence of modeling error, the estimated relative stiffness reduction values are within a reasonable interval, and are bounded by the reduction values from the two different "equivalent" approaches. As such, it can be concluded that—although the identified structure is a simple shear building—the proposed algorithm is capable of capturing the various patterns and magnitudes of damage. A majority of other methods chronicled above predict that the amount of stiffness reduction in floors other than the 1st and 3rd floors is negligible altogether. Although such identification results restrict the damage to the floors where braces have been removed, they are dependent on the behavior of the model used in the damage detection process. The present algorithm also identifies the less severe damage patterns (*iii-vi*), successfully.

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