stress fields that satisfy the equilibrium equations and the stress boundary conditions, without violating the failure criterion anywhere in the medium. On the other hand, the kinematics of the problem is not examined and, therefore, compatibility of deformations is generally not satisfied. For convex materials, formulations of this type are inherently *safe* that is, they overestimate active pressures and underestimate the passive. The best known such solution is that of Rankine, the applicability of which is severely limited by the assumptions of horizontal backfill, vertical wall and smooth soil-wall interface. Owing to difficulties in deriving pertinent stress fields for all but the simplest geometries, the vast majority of limit-analysis solutions in geo-engineering are of the kinematic type [Chen, 1975]. With minor exception [Lancellotta 2007, Mylonakis et al 2007], no closed-form stress solutions have been derived for seismic earth pressures.

Notwithstanding the theoretical significance and practical appeal of the Coulomb and Mononobe-Okabe solutions, these formulations can be criticized on the following important aspects: (1) in the context of limit analysis their predictions are unsafe; (2) their accuracy (and safety) diminishes in the case of passive pressures on rough walls, (3) the mathematical expressions are complicated and difficult to verify, (4) the distribution of contact stresses on the wall are not predicted (typically assumed hydrostatic following Rankine's solution), (5) optimization of the failure mechanism is required in the presence of multiple loads, to determine a stationary (optimum) value of soil thrust, and (6) stress boundary conditions are not satisfied, as the yield surface does not generally emerge at the soil surface at angles $45^o \pm \varphi/2$.

In light of the above, it appears that the development of a closed-form solution of the stress type for assessing seismically-induced earth pressures would be desirable. It will be shown that the proposed solution is mathematically simpler than the existing kinematic solutions, offers satisfactory accuracy, yields results on the safe side, satisfies the stress boundary conditions, and predicts the elevation of soil thrust.

PROBLEM DEFINITION AND MODEL DEVELOPMENT

The problem under investigation is depicted in Figure 1: a slope of dry cohesionless soil retained by an inclined gravity wall, is subjected to plane deformations under the combined action of gravity (g) and seismic body forces (a_h x g) and (a_v x g) in the horizontal and vertical direction, respectively. The problem parameters are: the height (H) and inclination (ω) of the wall, the inclination (β) of the slope; the roughness (δ) of the wall-soil interface; the friction angle (φ) and unit weight (γ) of the soil material, and the surface surcharge (q). Since backfills typically consist of granular materials, cohesion in the soil and the soil-wall interface are not considered. Note that the retained soil is considered rigid* before yielding, so the seismic force is uniform within the backfill, so the resultant body force is acting at an angle ψ_e from vertical

$$\tan\psi_e = \frac{a_h}{1-a_v} \tag{1}$$

* This assuption is not essential from a limit analysis viewpoint. It is merely a convenient assumption regarding earthquake action in the backfill.

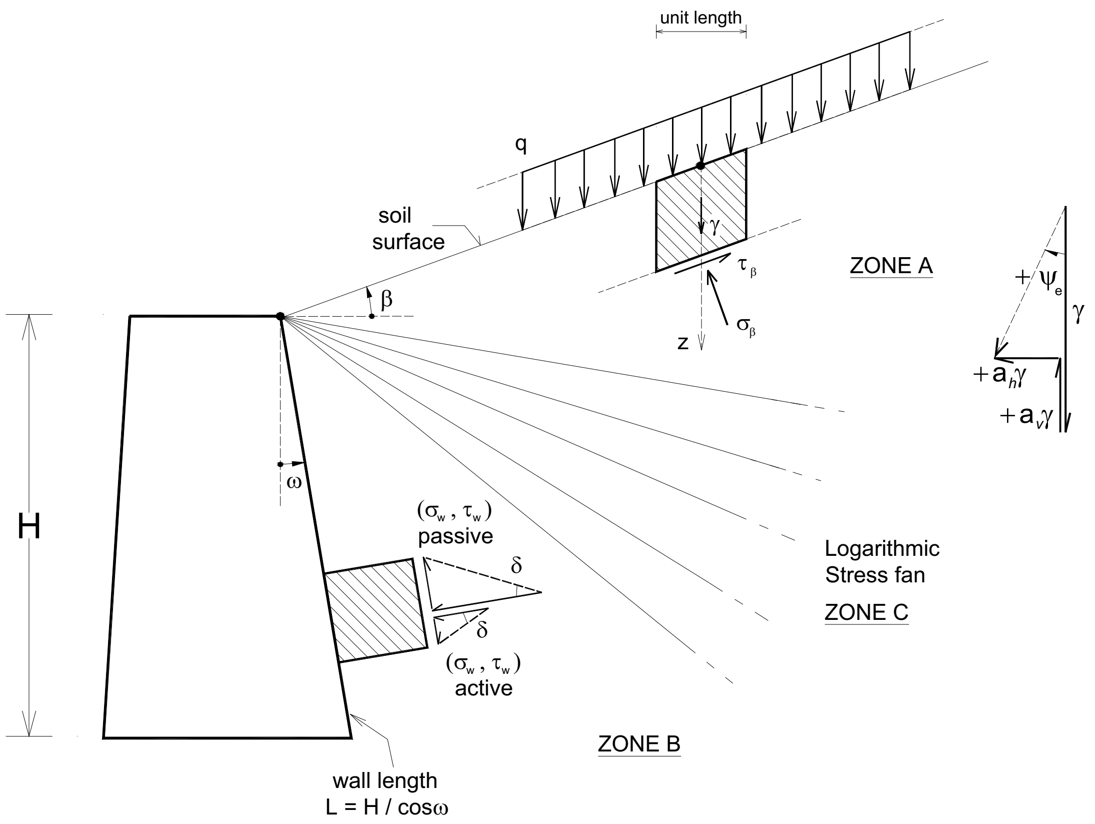

FIG. 1. Stress fields close to soil surface (Zone A), wall (Zone B) and transition zone (Zone C).

To analyze the problem, the backfill is divided into two main regions subjected to different stress fields: the first region (A) is located close to the soil surface, whereas the second (B) close to the wall. In both regions the soil is assumed to be in a condition of impeding yielding under the combined action of gravitational and seismic forces. The same assumption is adopted for the soil-wall interface, which is subjected exclusively to contact stresses. A transition zone between regions A and B is introduced below.

Fundamental to the proposed analysis is the assumption that stresses close to the soil surface can be well approximated by those in an *infinite slope* so the stress boundary conditions at the surface are satisfied. For points in region B, it is assumed that stresses are functions exclusively of the vertical coordinate and obey the strength criterion at the frictional soil-wall interface.

Considering the material to be in a condition of impeding yielding, the Mohr circles of stresses in regions A and B are depicted in Figure 2. It becomes evident that the orientation of principal planes (and thereby stress characteristics) in the two regions is different. In addition, the mean stresses S_A and S_B (Figure 2) generally do not coincide. To determine the separation of mean stresses S_A and S_B and ensure a smooth transition in the orientation of principal planes in the two zones, a logarithmic *stress fan* is adopted (Zone C), centered at the top of the wall (Figure 1).

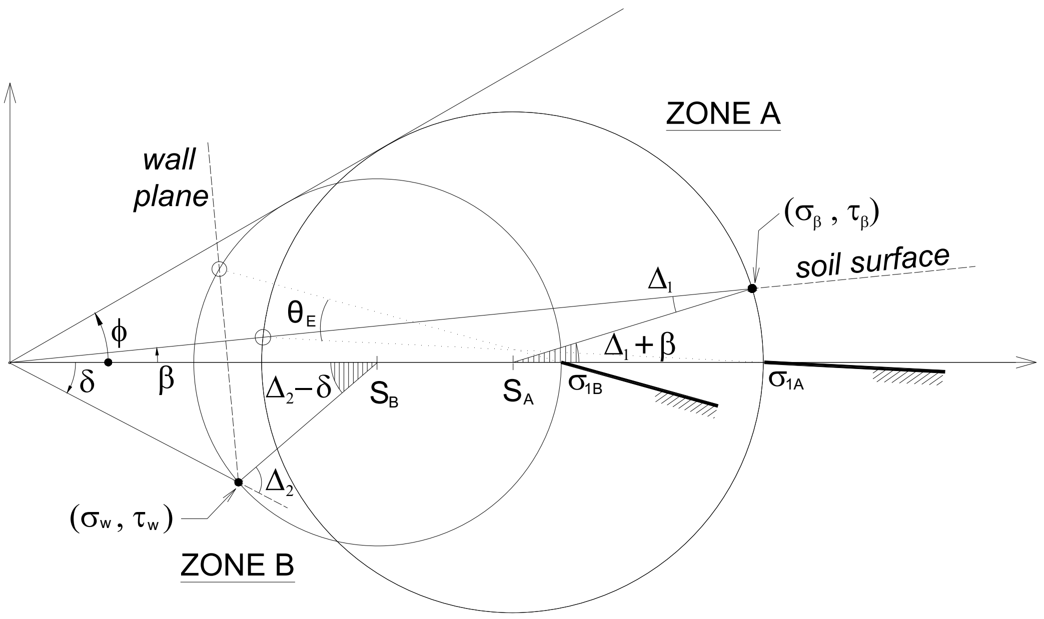

FIG. 2. Mohr circles of stresses and major principal planes in zones A and B.

In the interior of the fan, principal stresses are gradually rotated by the angle θ separating the major principal planes in the two regions, as shown in Fig 1. This additional condition is written as [Chen, 1975]:

$$S_B = S_A \exp\left(-2\theta \tan\phi\right) \tag{2}$$

The negative sign in the above equation pertains to the active case ($S_B < S_A$) and vice versa. Equation (2) is an exact solution of the governing Kötter equations for a weightless material and, thereby, it is only approximate for a fan with weight.

PSEUDO-DYNAMIC SOLUTION FOR EARTHQUAKE LOADING

Recognizing that earthquake action imposes a resultant thrust in the backfill inclined at a *constant* angle ψ_e from vertical (Fig 1), it becomes apparent that the seismic problem does not differ fundamentally from the static problem, as the former can be obtained from the latter through a rotation of the reference axes by the angle ψ_e, as shown in Fig 3. In other words, considering ψ_e does not add an extra physical parameter to the problem, but simply alters the values of the other variables. This property of *similarity* was apparently first employed by Briske and later by Terzaghi and Arango [Seed & Whitman, 1970; Ebeling et al, 1992] for the analysis of related problems. So, the solution to the seismic problem, can be derived from the statics of the gravitational problem. The limit thrust on the wall is given by the well-known expression:

$$P_E = K_{qE}(1-a_v)qH + \frac{1}{2}K_{\gamma E}(1-a_v)\gamma H^2 \tag{3}$$

where earth pressure coefficients $K_{\gamma E}$ and K_{qE} are given by (Mylonakis et al 2007)

$$K_{\gamma E} = \frac{\cos(\omega-\beta)\cos(\beta+\psi_e)}{\cos\psi_e\cos\delta\cos^2\omega}\left[\frac{1-\sin\phi\cos(\Delta_2-\delta)}{1+\sin\phi\cos\left[\Delta_1^*+\beta+\psi_e\right]}\right]\exp(-2\theta_E\tan\phi) \quad (4)$$

$$K_{qE} = \frac{\cos\omega}{\cos(\omega-\beta)}K_{\gamma E} \quad (5)$$

In the above equation,

$$2\theta_E = (\Delta_2-\delta)-(\Delta_1^*-\beta)-2\omega-\psi_e \quad (6)$$

is twice the revolution angle of principal stresses in the two regions and Δ^*_1 and Δ_2 denote the two Caquot angles [Caquot, 1934; Sokolovskii, 1965] given by

$$\sin\Delta_1 = \sin(\beta+\psi_e)/\sin\phi, \qquad \sin\Delta_2 = \sin\delta/\sin\phi \quad (7)$$

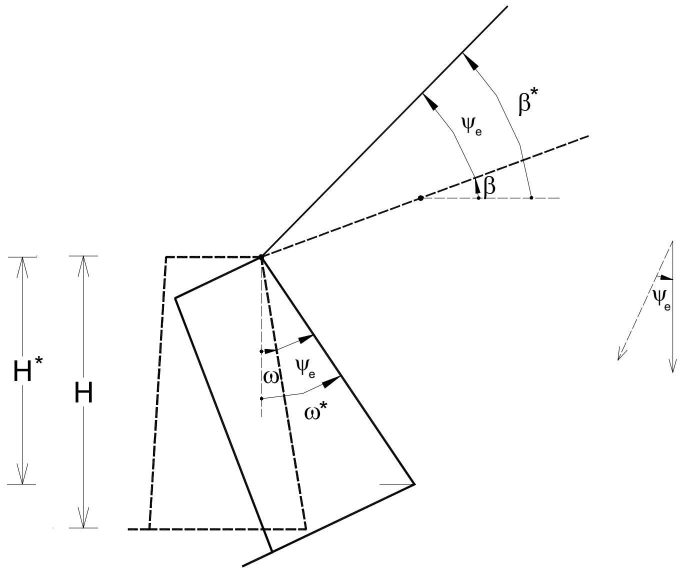

FIG. 3. Similarity transformation based on a rotation of the reference axes for analyzing the seismic case as a gravitational problem.

Results for active seismic earth pressures are given in Figure 4, referring to cases examined in the seminal study of Seed & Whitman [1970], for a reference friction angle of 35°. Naturally, active pressures increase with increasing levels of seismic acceleration and slope inclination and decrease with increasing friction angle and wall roughness. The conservative nature of the proposed analysis versus the Mononobe-Okabe (M-O) solution is evident in the graphs. The trend is more pronounced for high levels of horizontal seismic coefficient ($a_h > 0.25$), smooth walls, level backfills, and high friction angles. Conversely, the trend becomes weaker with steep backfills, rough walls, and low friction angles.

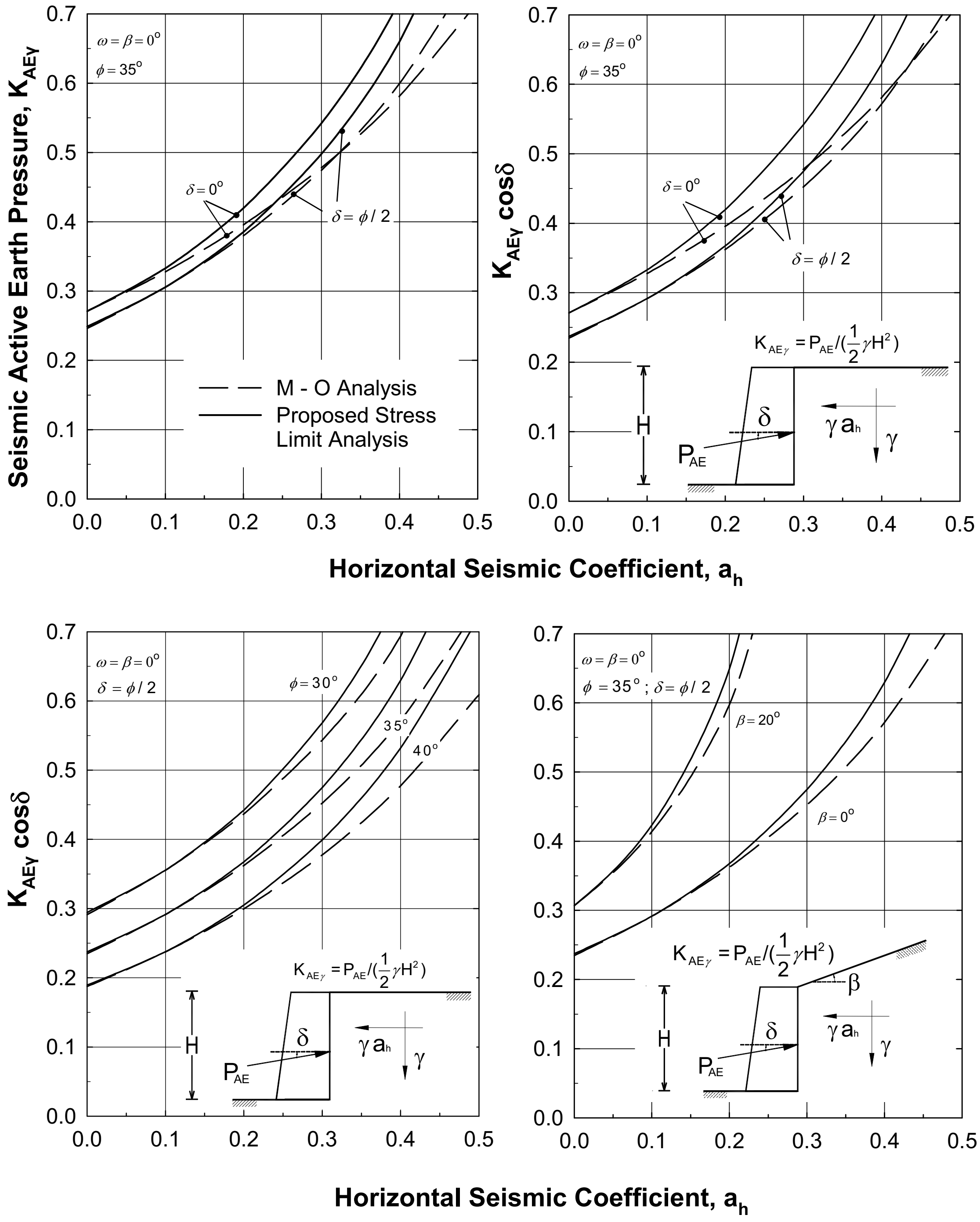

FIG.4. Comparison of active seismic earth pressures predicted by the proposed solution and from conventional M – O analysis, for different geometries, material properties and accelaration levels. (Modified from Seed & Whitman, 1970)

EARTH PRESSURE DISTRIBUTION: SIMPLE WAVE SOLUTION

The classical kinematic-type solutions by Coulomb and Mononobe–Okabe do not provide information on the distribution of stresses with height. They adopt a hydrostatic distribution, as a practical simplification for design purposes. The same limitation applies to more rigorous numerical kinematic solutions pioneered by Chen (1975).

On the other hand, in stress solutions (Terzaghi 1943) the hydrostatic distribution results naturally from the formulation itself. This is due to the linear variation of stresses with depth in the Rankine zone close to the soil surface, which is not altered in the stress fan and close to the wall.

It is experimentally known, however, that the actual distribution of stresses is not hydrostatic. Two major mechanisms are responsible for this. They both relate to the basic assumptions about the behavior of the retained soil and the kinematics of the problem. First, the soil mass responds dynamically and, thereby, the distribution of accelerations (and associated body forces) is not uniform with depth. These effects have been incorporated in elastodynamic solutions (Veletsos & Younan 1994) and some limit analysis solutions (Steedman & Zeng 1990). Secondly, the distribution of earth pressures changes for different kinematic constraints (e.g., rotation about wall base or top), which relate directly to arching in the backfill. The redistribution of stresses due to arching leads to changes in the magnitude and point of application of soil thrust. Only the effect of dynamic response of the backfill is addressed herein.

The proposed approach allows evaluation of dynamic limit thrust on gravity walls by means of the stress solution and the simple wave solution for the response of a homogeneous soil layer to vertically-propagating SH waves shown in Figure 5.

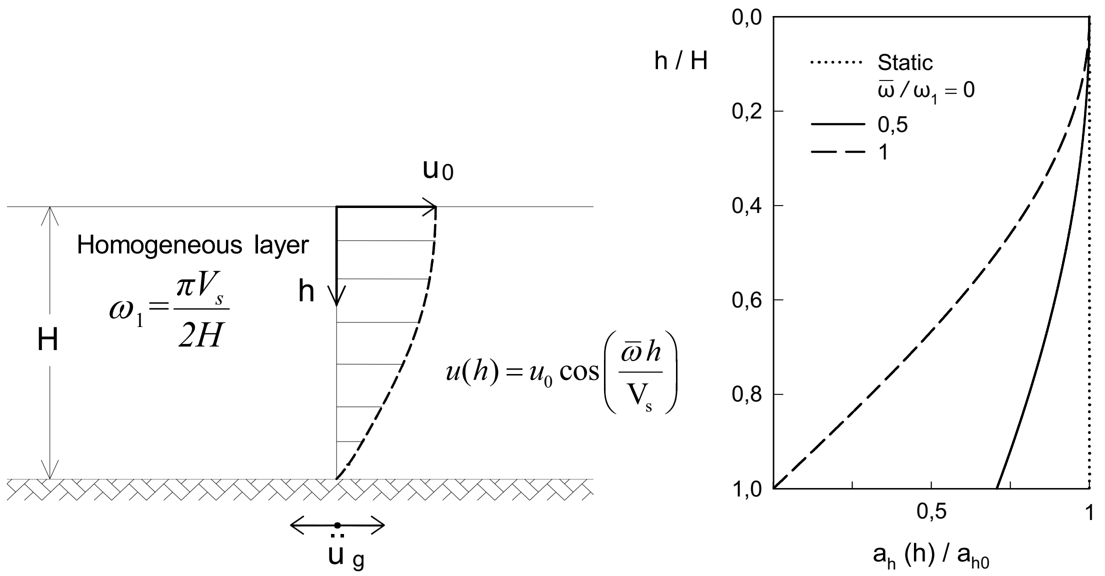

FIG.5. Dynamic response of backfill and distribution of inertial loads with height.

This dynamic response results to a non uniform distribution of inertial accelerations with height and, thereby, seismic angle ψ_e

$$\psi_e(h) = \tan^{-1} a_h(h) = \tan^{-1}\left[a_{h0} \cos\left(\frac{\pi}{2}\frac{\bar{\omega}}{\omega_1}\frac{h}{H}\right)\right] \quad (8)$$

where $\bar{\omega}$ denotes the cyclic excitation frequency. The use of the above elastodynamic function into the proposed plasticity solution is theoretically admissible, as the medium in a condition of impeding yielding, and, thereby, elasticity is valid. Based on the foregoing, the dynamic pressures on the retaining wall are obtained by the expression:

$$p(h) = \frac{\cos(\omega-\beta)\cos(\beta+\psi_e(h))}{\cos\psi_e(h)\cos\delta\cos^2\omega}\left[\frac{1-\sin\phi\cos(\Delta_2-\delta)}{1+\sin\phi\cos\left[\Delta_1^*+\beta+\psi_e(h)\right]}\right]\exp(-2\theta_E(h)\tan\phi) \quad (9)$$

where ψ_e varies with depth, h, according to equation (8).

Distributions of seismic earth pressures along the back of the wall are given in Figure 6. Evidently, the closer the excitation frequency to the fundamental natural frequency of the medium, the larger the deviation from the conventional triangular distribution.

It is important to mention that the dynamic effects lead invariably to a decrease in magnitude of total thrust, as seen in the left graph of Figure 7. It also leads to an increase in the elevation of point of application of active thrust (left graph of Fig 7). The maximum elevation is observed at resonance ($\bar{\omega}/\omega_1 = 1$), and does not exceed H/2 for the purely seismic component of the thrust.

Corresponding results are provided in Figure 8, plotted as function of horizontal ground acceleration a_{ho}. Naturally, total thrust increases with increasing horizontal ground acceleration. The elevation of point of application is affected to a lesser degree by the level of ground shaking.

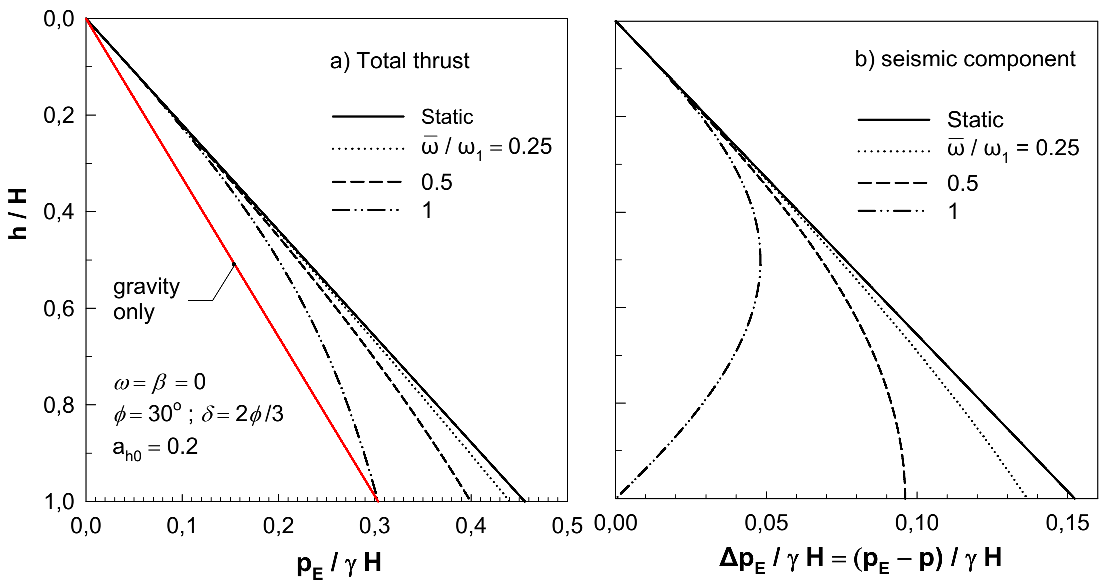

FIG.6. Distribution of earth pressures: a) Total thrust, b) Seismic component only

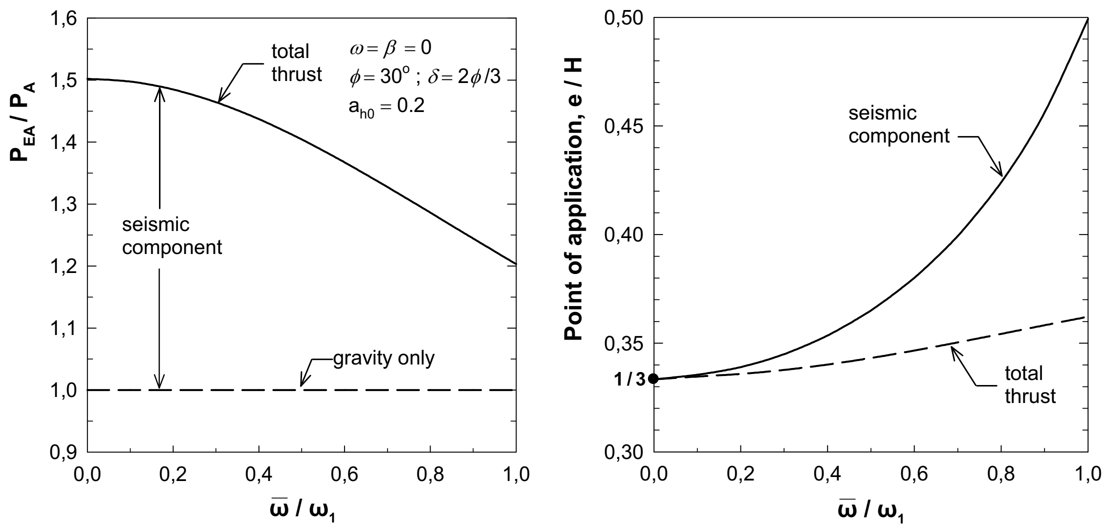

FIG.7. Variation of the magnitude and the point of application of the seismic thrust with respect to exciting frequency.

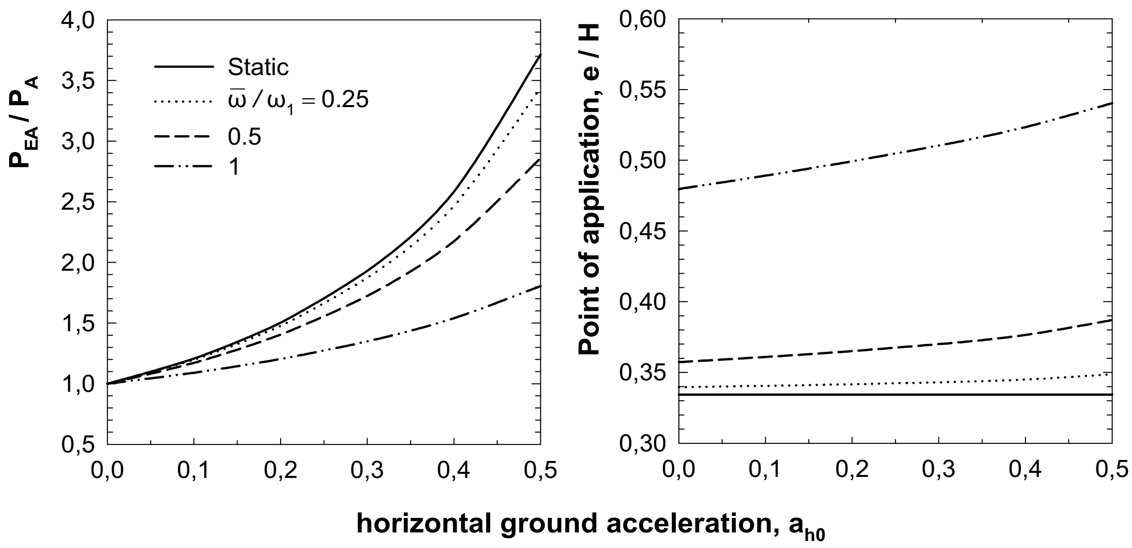

FIG.8. Effect of the peak ground acceleration (PGA) level on the magnitude and the point of application of the seismic thrust, for varying exciting frequency.

CONCLUSIONS

A stress plasticity solution was presented for gravitational and earthquake-induced earth pressures on gravity walls. The following are the main conclusions of the study:
(1) The proposed solution is simpler than the Mononobe-Okabe equations, and safe, as it over-predicts active pressures and under-predicts the passive.
(2) For active pressures, the accuracy of the solution is excellent. The largest deviations

occur for high seismic accelerations, high friction angles and steep backfills.
(3) The pseudo-dynamic seismic problem can be deduced from the gravitational problem through a revolution of the reference axes by the seismic angle ψ_e (Fig 3).
(4) Stress limit analysis is suitable for determining traction distributions on the wall. By incorporating the dynamic response of backfill, the distribution of pressures becomes parabolic and the elevation of the seismic component rises to above 50% of wall height. The deviation from hydrostatic distribution may be important for walls taller than approximately 5m.

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Dynamic Centrifuge Study of Seismically Induced Lateral Earth Pressures on Retaining Structures

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ABSTRACT: A set of two dynamic centrifuge experiments on stiff and flexible U-shaped retaining structures with dry medium dense sand backfill was performed in order to evaluate the validity of the various assumptions and analysis procedures currently used for the evaluation of seismically induced forces on retaining structures. The experimental results show that dynamic moments and dynamic earth pressures tend to be overestimated using the current analysis methods. More importantly, the dynamic earth pressure increases monotonically with depth and the maximum measured moments are not necessarily in with maximum measured earth pressure. These results are consistent with results obtained independently for gravity retaining structures (Nakamura, 2006) and call into question the validity of the basic assumptions in the currently used analysis and design procedures.

INTRODUCTION

The problem of seismically induced lateral earth pressures on retaining structures and basement walls has been the topic of considerable research over the last 80 years. The earliest and most widely used method for estimating the magnitude of seismic forces acting on a retaining wall is based on the experimental and analytical work of Okabe (1926) and Mononobe and Matsuo (1929) following the 1923 great Kanto earthquake in Japan. The Mononobe-Okabe (M-O) method, as it is commonly referred to, was originally developed for gravity walls retaining dry cohesionless backfill. It is an extension of Coulomb's static earth pressure theory to include the inertial forces due to the horizontal and vertical backfill accelerations. Since then, the method has been adapted for analysis of all types of retaining structures and many researchers have concluded that the M-O method gives adequate estimates of the magnitude of the dynamic earth pressures on retaining walls (e.g. Prakash and Basavanna 1969, Seed and Whitman 1970, Bolton and Steedman 1982, Sherif. et al. 1982, Ortiz et al. 1983, Ishibashi and Fang 1987, Stadler 1996). However, more recently, analytical studies have led some researchers to suggest that the M-O method underestimates the dynamic earth pressures (e.g. Morrison and Ebeling 1995, Green et al. 2003, Ostadan and White 1998, Ostadan 2004). As a result, there has been a trend toward more and more conservative design recommendations. However, documented failures of basement walls or underground structures in non-liquefiable deposits in recent major earthquakes are exceedingly rare (e.g. Sitar 1995, Al Atik and Sitar 2007) even in locations that were subjected to very strong ground motions and where the retaining structures were not particularly designed to consider seismic loading of that magnitude. In addition, a review of case history data suggests that retaining structures