distribution, the mean route travel time t_k^w, and the route travel time standard deviation $\sigma_{t,k}^w$ can be expressed as:

$$T_k^w \sim N(t_k^w, (\sigma_{t,k}^w)^2) \quad \forall w \in W,\ k \in R_w \tag{7}$$

$$t_k^w = \Sigma_a E(T_a)\delta_{a,k}^w \qquad (\sigma_{t,k}^w)^2 = \Sigma_a \delta_{a,k}^w Var(T_a) \qquad \forall w \in W,\ k \in R_w \tag{8}$$

Combining (4) and (5) give the expressions for t_k^w and $\sigma_{t,k}^w$ as the following:

$$t_k^w = \sum_a \delta_{a,k}^w \left[t_a^0 + \alpha t_a^0 x_a^\beta \frac{(1-\theta_a^{1-\beta})}{\bar{c}_a^\beta (1-\theta_a)(1-\beta)} \right] \quad \forall w \in W\,,\ \ k \in R_w \tag{9}$$

$$\sigma_{t,k}^w = \sqrt{\sum_a \delta_{a,k}^w \alpha^2 (t_a^0)^2 x_a^{2\beta} \left\{ \frac{1-\theta_a^{1-2\beta}}{\bar{c}_a^{2\beta}(1-\theta_a)(1-2\beta)} - \left[\frac{1-\theta_a^{1-\beta}}{\bar{c}_a^\beta (1-\theta_a)(1-\beta)} \right]^2 \right\}} \quad \forall w, k \tag{10}$$

2.2 Model of travel time budget

As mentioned above, the travel time budget is the summation of the mean route travel time and the safety margin. It can also be expressed as:

$$c_k^w = t_k^w + m_k^w \quad \forall w \in W,\ k \in R_w \tag{11}$$

where c_k^w is the travel time budget associated with route k between OD pair w and m_k^w is the travel time safety margin determined by the following chance-constrained model:

$$\min\ c_k^w \tag{12}$$

$$\text{s.t.} \quad \Pr[T_k^w \le c_k^w] \ge \rho \quad \forall w \in W,\ k \in R_w \tag{13}$$

where ρ is the travel time reliability requirement.

Since T_k^w follows a normal distribution, m_k^w can be obtained by directly solving constraint (13). As a result, m_k^w and c_k^w can be expressed as:

$$m_k^w = \sigma_{t,k}^w \Phi^{-1}(\rho) \qquad c_k^w = t_k^w + \sigma_{t,k}^w \Phi^{-1}(\rho) \qquad \forall w \in W,\ k \in R_w \tag{14}$$

where $\Phi(x)$ is the standard normal cumulative function of random variable x.

3 SUE condition and variational inequality model formulation

In general, since different travelers have different attitudes toward risk, it is unreasonable to assume that the travel time reliability requirement ρ is identical to all travelers in a real road network. Therefore, in this paper, the OD traffic demand q_w between OD pair w is divided into I classes. The travel time reliability requirement ρ is identical in each user class i ($\forall i \in I$) and the travelers belongs to different user classes will have different travel time reliability requirements.

In a real road network, due to the complexity of network structure and a high degree of uncertainty in traffic conditions, travelers usually have partial information about the congestion status and the actual route travel time of the network. Therefore, it is necessary for a traveler to consider the perception error when choosing a route. In this paper, a perceived travel time budget is introduced as follows:

$$C_{k,i}^{w} = c_{k,i}^{w} + \xi_{k,i}^{w} \qquad \forall w \in W,\ k \in R_w,\ i \in I \tag{15}$$

$$c_{k,i}^{w} = t_k^{w} + \sigma_{t,k}^{w} \Phi^{-1}(\rho_i) \qquad \forall w \in W,\ k \in R_w,\ i \in I \tag{16}$$

where $C_{k,i}^{w}$, $c_{k,i}^{w}$, $\xi_{k,i}^{w}$, and ρ_i are, respectively, the perceived travel time budget, the actual travel time budget, the perception error term of user class i on route k between OD pair w, and the travel time reliability requirement of user class i. It is assumed here that $E[\xi_{k,i}^{w}] = 0$ and therefore, $E[C_{k,i}^{w}] = c_{k,i}^{w}$.

In reality, the OD traffic demand may be influenced by level of service on the network (Sheffi 1985). To take this phenomenon into account, for each user class i, the OD traffic demand $q_{w,i}$ is assumed to be a strictly monotonic decreasing function with respect to the expected minimal perceived travel time budget. In other words,

$$q_{w,i} = D_{w,i}(C_{w,i}) \qquad \forall w \in W,\ i \in I \tag{17}$$

where $D_{w,i}()$ and $C_{w,i}$ are, respectively, the OD traffic demand function and the expected minimal perceived travel time budget of user class i between OD pair w. $C_{w,i}$ can be computed by using the following equation.

$$C_{w,i}(\mathbf{c_w}) = -\ln \Sigma_{k \in R_w} \exp(-\theta_i c_{k,i}^{w}) / \theta_i \qquad \forall w \in W,\ i \in I \tag{18}$$

where parameter θ_i is a constant related to the perception error of travelers.

We assume that the perception error terms on a traveler's routes between OD pair w are independently and identically Gumble distributed. Therefore, the probability $p_{k,i}^{w}$ for travelers in user class i to choose route k from the set of routes between OD pair w is given by:

$$p_{k,i}^{w} = \exp(-\theta_i c_{k,i}^{w}) / \Sigma_r \exp(-\theta_i c_{r,i}^{w}) \qquad \forall w \in W,\ k \in R_w,\ i \in I \tag{19}$$

For travelers in each user class i ($\forall i \in I$), the stochastic user equilibrium condition can be written as:

$$f_{k,i}^{w} = q_{w,i}\, p_{k,i}^{w} \qquad \forall\ k \in R_w,\ w \in W,\ i \in I \tag{20}$$

where $f_{k,i}^{w}$ denotes the mean route flow of user class i on route k between OD pair w.

We formulate the travel time budget-based stochastic user equilibrium traffic assignment model with multiple user classes and elastic demand as an equivalent variational inequality problem as follows.

Find a mean route flow vector $\mathbf{f}^*$ and an OD demand vector $\mathbf{q}^* \in \Psi$, such that:

$$\sum_w \sum_k \sum_i \left(c_{k,i}^{w^*} + \ln\left(f_{k,i}^{w^*} / q_{w,i}^{*}\right) / \theta_i\right)\left(f_{k,i}^{w} - f_{k,i}^{w^*}\right) - \sum_w \sum_i D_{w,i}^{-1}\left(q_{w,i}^{*}\right)\left(q_{w,i} - q_{w,i}^{*}\right) \geq 0$$
$$\forall f_{k,i}^{w}, q_{w,i} \in \Psi \tag{21}$$

where the superscript "*" is used to designate the solution of the variational inequality problem; $D_{w,i}^{-1}()$ denotes the inverse function of the OD traffic demand function; and Ψ is the feasible set for the mean route flows and OD demands determined by the following constraints.

$$\Sigma_k f_{k,i}^{\ w} = q_{w,i} \qquad \forall w \in W,\ i \in I \tag{22}$$

$$f_{k,i}^{\ w} \geq 0 \qquad \forall\ k \in R_w,\ w \in W,\ i \in I \tag{23}$$

$$q_{w,i} \geq 0 \qquad \forall w \in W,\ i \in I \tag{24}$$

$$\Sigma_w \Sigma_k \Sigma_i\, f_{k,i}^{\ w} \delta_{a,k}^{\ w} = x_a \qquad \forall a \in \mathrm{A} \tag{25}$$

The proposed variational inequality problem can be solved by a number of route-based algorithms such as sequential quadratic programming or gradient projection methods etc.

4 A numerical example

The example network shown in Figure 1 includes six nodes, seven links, and one OD pair (from node 1 to node 6). Associated with each link are four numbers: the index, the free-flow travel time (h), the design capacity (pcu/h), and the worst degraded coefficient θ_a. The link performance function is given by Eq. (1) with α = 0.15 and β = 4. The following linear demand function is adopted.

$$q_i(C_i) = q_{i,max} - 500C_i \tag{26}$$

where $q_{i,max}$ is the maximum (or potential) OD demand. For ease of exposition, it is assumed that there are only two user classes and their potential OD demand are set to be $q_{1,max}$ = 2500 pcu/h and $q_{2,max}$ = 2000 pcu/h.

In this example, we first assume that the dispersion parameters for the two user classes are θ_1 = 10; θ_2 = 2. Then, we solve the proposed SUE model by using a route-based gradient projection algorithm under different combinations of the two user classes' travel time reliability requirements ρ_1 and ρ_2. Table 1 depicts the corresponding results, which include the traffic flow on each route, the OD demand of each user class, and the travel time budget of each route for each user class at the equilibrium point.

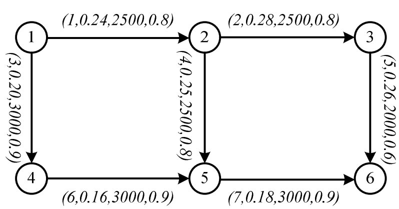

Figure 1. A six-node network for testing the proposed model

Table 1 Equilibrium flow patterns of the travel time budget-based SUE model

ρ_i	Route	Link sequence	Route flow	c_1(h)	c_2(h)	q_1	q_2
ρ_1=0.50 ρ_2=0.95	1	1-2-5	777	0.80	0.81	2211	1914
	2	1-4-7	1040	0.74	0.75		
	3	3-6-7	2308	0.61	0.62		
ρ_1=0.90 ρ_2=0.90	1	1-2-5	780	0.81	0.81	2207	1915
	2	1-4-7	1035	0.75	0.75		
	3	3-6-7	2308	0.62	0.62		

From Table 1, we can find that if the travel time reliability requirement increases, the OD demand will decrease and the corresponding route travel time budget will increase accordingly.

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Calibration of Lowry Model Using Immune Genetic Algorithm

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Abstract: In order to investigate the suitability and potential benefits of applying land-use forecasting models in China, this paper presents an immune genetic algorithm for the calibration of Lowry model based on a maximum likelihood approach. The calibration procedure comprises three stages. In the first stage, an immune genetic algorithm is employed to calibrate the population and employment potentials, together with a coefficient associated with the travel impedance function in the study area. The second stage investigates the relationship between the calibrated potentials and various land-use variables, using a multivariate stepwise regression analysis. The third stage is model validation. A case study of Hu Zhou city, in Zhe Jiang province of China, was employed to demonstrate the performance of the proposed methodology. The results indicate that the calibrated Lowry model is acceptable for forecasting the future population and employment distribution in China.
Key words: *Land-use and transportation; Lowry model; Calibration; Immune genetic algorithm*

1 Introduction

Land use and transportation interaction is a dynamic process that involves changes over spatial and temporal dimensions between the two systems. Since the 1960s, many theories and models have been used to study land use and transportation interaction (Waddell et al., 2006). The Lowry model (Lowry, 1964) has been widely used for this purpose. It was developed to simulate location patterns of residential and service activities. A maximum likelihood approach was employed to calibrate the Lowry model(Putman and Ducca, 1978).

The maximum likelihood approach is a large-scale nonlinear optimization problem with a large number of variable and uncertain parameters. Recently, in order to get a better solution, the artificial intelligence methods, such as genetic algorithm (GA) (Wong, 1998), parallelized GA(Wong, 2001), neural networks (Rodrigue, 1997), etc, were proposed in many papers, and their promising performance was approved. But analyzing these methods, problem solutions are not very optimistic due to objections coming from these algorithms, e.g. local optimization solution, slow constringency speed etc. According to the disadvantages, this paper cites the theory of biological immune system (Huang, 1999), and constructs an immune genetic algorithm (IGA) to calibrate the Lowry model.

The remainder of the article is organized as follows. Section 2 describes the structure of the maximum likelihood function to calibrate the Lowry model. IGA for the calibration process is given in section 3. A case study is presented in section 4, where modeling results are given and interpreted. Finally, section 5 concludes the article with a brief summary and implications for further research.

2 The Lowry Model

The Lowry model uses three categories of activities: basic employment, service employment, and the household sector or residential population. Two spatial interaction functions form the basis of the Lowry model; the first distributes zonal employment to residences, and the second distributes service employment required by the residential population dependent on these employees. Beginning from an exogenously supplied distribution of basic employment, these interaction functions are used iteratively until no further increments to population are generated and a static equilibrium is achieved.

The parameters used to predict the population and employment distributions can be specified with the vector format as follows: $\mathbf{B}=(\beta_i, i=1,2,\ldots,N_P)$, $\mathbf{W}_P=(w_{Pik}, i=1,2,\ldots,N_P, k=1,2,\ldots,N_Z)$ and $\mathbf{W}_E=(w_{Ejl}, j=1,2,\ldots,N_E, l=1,2,\ldots,N_Z)$; and the results from the Lowry model as $\mathbf{P}=(P_{ik}, i=1,2,\ldots,N_P, k=1,2,\ldots,N_Z)$ and $\mathbf{E}=(E_{jl}, j=1,2,\ldots,N_E, l=1,2,\ldots,N_Z)$.
where

$$P_{ik}=\alpha\sum_{j=1}^{N_E}\sum_{l=1}^{N_Z}E_{jl}\frac{w_{Pik}\exp(-\beta_i c_{lk})}{\sum_{i=1}^{N_P}\sum_{k=1}^{N_Z}w_{Pik}\exp(-\beta_i c_{lk})}$$

$$E_{jl}=\frac{1}{\alpha}\sum_{i=1}^{N_P}\sum_{k=1}^{N_Z}P_{ik}\frac{w_{Ejl}\exp(-\beta_i c_{kl})}{\sum_{j=1}^{N_E}\sum_{l=1}^{N_Z}w_{Ejl}\exp(-\beta_i c_{kl})}$$

P_{ik} and w_{Pik} are, respectively, the number of and housing potential for household of category i in zone k; E_{jl} and w_{Ejl} are, respectively, the number of and employment potential for employment of category j in zone l; N_P, N_E and N_Z are, respectively, the number of household categories, employment categories and spatial zones in the study area; c_{lk} is the travel cost from zone l to zone k; β_i is the travel impedance coefficient of household category i; α is the regional household to employment ratio. The household and employment potential are defined as measures of the relative zonal attractiveness to residents and employees, respectively, in the study area. These potentials state only the aggregate effects and it is believed that these attractions are contributed by the level of development or the land-use intensities of a zone. The above-mentioned procedures can be summarized by

$$(\mathrm{P},\mathrm{E})=\Gamma(\mathrm{B},\mathrm{W}_P,\mathrm{W}_E) \tag{1}$$

Let $\overline{P}_{ik}$ and $\overline{E}_{jk}$ be respectively the observed population for population category *i* and observed employment for employment category *j*, at zone *k*. Assuming that the population or employment in a zone for a particular category is normally distributed around the results of the Lowry model. The probability of obtaining the observed values can be calculated as

$$P_r\left\{\overline{P_{ik}} = P_{ik}\right\} = \frac{1}{\sigma_{Pik}\sqrt{2\pi}}\exp(-(\overline{P_{ik}} - P_{ik})^2 / 2\sigma_{Pik}^2)$$

$$P_r\left\{\overline{E_{jk}} = E_{jk}\right\} = \frac{1}{\sigma_{Ejk}\sqrt{2\pi}}\exp(-(\overline{E_{jk}} - E_{jk})^2 / 2\sigma_{Ejk}^2)$$

Where σ_{Pik}^2 and σ_{Ejk}^2 are respectively the variances of the observed population for population category *i* and observed employment for employment category *j*, at zone *k*. Further assuming that the occurrence of the observed values are independently distributed, the likelihood function *L* is

$$L = \prod_{i=1}^{N_P}\prod_{k=1}^{N_Z}\frac{1}{\sigma_{Pik}\sqrt{2\pi}}\exp(-(\overline{P_{ik}} - P_{ik})^2 / 2\sigma_{Pik}^2)\prod_{j=1}^{N_E}\prod_{k=1}^{N_Z}\frac{1}{\sigma_{Ejk}\sqrt{2\pi}}\exp(-(\overline{E_{jk}} - E_{jk})^2 / 2\sigma_{Ejk}^2)$$

The problem is solved by finding a set of parameters (**B, W**$_P$, **W**$_E$) such that the household and employment distribution obtained from the Lowry model as specified in equation (1) maximizes the objective function ln*L*, i.e.

$$\underset{\mathrm{B,W_P,W_E}}{Maximize}\ln L = -\left[\sum_{i=1}^{N_P}\sum_{k=1}^{N_Z}\frac{(\overline{P_{ik}} - P_{ik})^2}{\sigma_{Pik}^2} + \sum_{j=1}^{N_E}\sum_{k=1}^{N_Z}\frac{(\overline{E_{jk}} - E_{jk})^2}{\sigma_{Ejk}^2}\right] \tag{2}$$

Subject to

$$(\mathrm{P,E}) = \Gamma(\mathrm{B,W}_P,\mathrm{W}_E)$$

3 Application IGA to Calibrate the Lowry Model

Immune genetic algorithm (IGA) is put forward by adding the theory immune system to genetic algorithm in order to calibrate the Lowry model. IGA regards evolutional individuals as antibodies and objective function as antigens. With its ability of self-regulate， the antibody population achieves a good regulation of dynamic balance between individual diversity and population convergence after encountering foreign invading (Ma, 2006). Main processes using IGA to calibrate the Lowry model are followed as:

Step 1 *Generation the initial antibodies*

Gene coding adopts the decimal coding rather than binary coding. This method avoids the process of frequent coding and recoding, increases the speed and accuracy of calculation, and has advantage to solve the large-scale optimization problems. The control variables including the travel impedance coefficient **B**, the population potential **W**$_\mathrm{P}$ and the employment potential **W**$_\mathrm{E}$ are coded into antibodies. **B** adopts the real number code, while **W**$_\mathrm{P}$ and **W**$_\mathrm{E}$ adopt the integer code. The initial antibodies population of control variables is generated randomly from the set of uniformly distributed control variables ranging over their upper and lower limits.

Step 2 *Calculation the affinities*

The affinity A_{bgw} between antibodies and antigens is calculated as follows:

$$A_{bgw} = \mu\left[f(v)\right].$$

Where $f(v)$ is objective function, $\mu(x)$ is the monotony function of x. Here, the negative of the reciprocal of objective function is used to express affinity:

$$A_{bgw} = -\frac{1}{\ln L}.$$

Affinity between antibodies can reflect their analogical extent. In other words, a greater affinity value indicates a greater similarity between antibodies. The affinity between two antibodies w and v can be expressed by followed expression:

$$B_{w,v} = 1/(1+H_{w,v}).$$

Where $H_{w,v}$ is Euclidean space between antibody w and v, and it can be calculated as follows:

$$H_{w,v} = \left\{\sum\left[(B_{iw}-B_{iv})(B_{iw}-B_{iv})^T + (W_{Piw}-W_{Piv})(W_{Piw}-W_{Piv})^T + (W_{Eiw}-W_{Eiv})(W_{Eiw}-W_{Eiv})^T\right]\right\}^{\frac{1}{2}}$$

Where $B_{iw}, B_{iv}, W_{Piw}, W_{Piv}, W_{Eiw}, W_{Eiv}$ are values of the *ith* item of antibody w and v.

Step 3 *Calculation the density of antibodies*

The density of antibody w, c_w can be defined as $c_w = \left(\sum_{v=1}^{N} B_{w,v}\right)\Big/N$. Where N is the number of antibodies. According to their corresponding densities, antibodies are ranked in ascending order, and the antibodies with great density are eliminated by the abandoning rate. The new ones by random generation are substituted for the eliminating antibodies. Suppressing the high density antibodies can greatly keep the diversity of population, and avoid trapping into the local optimal solution.

Step 4 *Selection calculation*

The selection operation is executed with the rank method, while adopting the rank method, the fitness of antibody is only decided by its order in the population rather than by its actual value of the objective function. The fitness of antibody w is composed of two sections: the affinity A_{bgw} and the density c_w, that is,

$$p = \alpha A_{bgw} + (1-\alpha)c_w.$$

Where α is a proportional factor, $0<\alpha<1$. The selecting operation can retain the antibodies with the small affinity to antigen by activating and suppressing based on the densities of antibodies, which can ensure the diversity of population and improve the convergence near the optimal solution.

Step 5 *Crossover and mutation*

The antibodies with smaller fitness have higher probability to be selected by the rank method, and to carry out the crossover and mutation operation. The crossover operator adopts the middle recombination suitable for the real variables, after crossover, the antibodies possess a good distribution. The mutation operation adopts the even real number mutation.

Step 6 *Judging the conditions of end*

If the number of end generation of evolution is arrived and the average density of antibodies is stable, then the program is ended, else goes back to step 2.

4 Numerical Analysis of Example

Hu Zhou city, in Zhe Jiang province of China, is used as a case to demonstrate the performance of the IGA. The travel characteristics survey (TCS) was carried out by Southeast University to provide information for transportation planning in 2004. It consists mainly of a large scale home interview survey to collect information on household, personal and trip characteristics. Some land use data are measured from the outline zoning plans for the present study.

Using MATLAB6.5 toolbox, two kinds of algorithm are performed. The first is simple genetic algorithm (SGA) with only the GA operators and the second is IGA adding the immune operators. The city is divided into 32 zones. The number of unknowns is 65. In the IGA, the size of population is 100 and the mutation rate is set as 1/500,000. The crossover rate is 0.8 and the maximum number of generation is taken as 1000. The fitness function proportional factor α is set as 0.98. Fig.1 shows the evolving curves of the SGA vs. IGA. It is apparent that the proposed method is superior to the SGA.

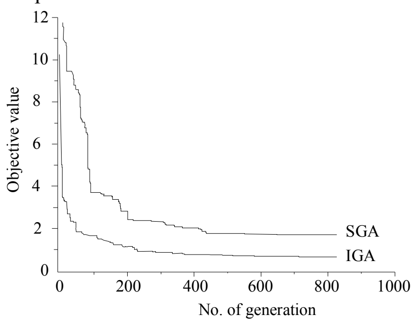

Fig. 1. Evolving curves of SGA vs. IGA

By minimizing the discrepancy between the observed and modeled distributions of population and employment, a set of zonal population potential and employment potential are obtained. They are set as dependent variables and a set of ten types of land use data are set as independent variables in the stepwise regression analysis. The coefficient of determination, R^2, is employed as a statistical measure for the assessment. Table 1 summarizes some performance index. It was observed that very high R^2 were obtained in IGA. The resulting potentials are reliable and able to replicate the base year conditions. It is shows that the Lowry model can be applied to forecast future distributions of population and employment based on different land use zoning policies.

Table 1 calibration results of the Lowry model

	Population R^2	Employment R^2	Travel impedance coef. β
SGA	0.778	0.753	5.75×10^{-4}
IGA	0.929	0.899	9.03×10^{-4}

5 Conclusions

A novel algorithm, immune genetic algorithm (IGA), is proposed to calibrate the Lowry model, which can be applied to forecast future population and employment distribution. IGA, compared with SGA, possesses a better global convergence and a quicker calculation speed. The results indicate that the calibrated Lowry model is acceptable for forecasting the future population and employment distribution due to different land-use planning and policy schemes, as well as changes in transportation systems.

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