APPENDIX B

SUMMARY OF REFERENCE EVAPOTRANSPIRATION EQUATIONS USED IN EVALUATION

INTRODUCTION

This appendix contains descriptions of the reference ET methods that were evaluated by the Task Committee at the 82 site-locations. The ET methods included well-known methods, (e.g. ASCE Penman-Monteith, 1982 Kimberly Penman) and hybrids of the ASCE-PM containing modifications to constants or parameterization of components. Definition of calculation procedures are summarized in Table B-1. Additional information for the hybrids of the ASCE-PM is provided in the discussion following Table B-1. Listed in Table B-1 for each parameter of each equation is the equation number, constant value, or procedure used to calculate that parameter. The labels for variations on the ASCE-PM equation are the same as those referred to in Table A-1, Appendix A.

Table B-1. Parameter equation numbers, etc. used in the Reference Equations Evaluated

Parameter	ASCE Penman-Monteith					ASCE Standardized Penman-Monteith	FAO-56 Penman-Monteith	1982 Kimberly Penman	1963 Penman	FAO-24 Penman	CIMIS Penman	1985 Hargreaves
	"ASCE-PM"	"ASCE-PMD"	"ASCE-PMDL"	"ASCE-PMv"	"ASCE-PMDR"							
Reference Types	ET_o, ET_r	ET_o, ET_r	ET_o, ET_r	ET_o, ET_r	ET_o, ET_r	ET_{os}, ET_{rs}	ET_o	ET_r	ET_o	ET_o	ET_o	ET_o
timestep	m, d, h	m, d, h	m, d, h	m, d, h	m, d, h	m, d, h	m, d, h	m, d, (h)[a]	m, d, h	m, d	h	m, d
Δ	5, 36	5, 36	5, 36	5, 36	5, 36	5, 36	5, 36	5	5	5	5	--
γ	B.12	B.12	B.12	B.12	B.12	4	4	B.12	B.12	B.12	B.12	--
λ	B.7	B.7	$\lambda = 2.45$ MJ/kg	B.7	B.7	$\lambda = 2.45$ MJ/kg	$\lambda = 2.45$ MJ/kg	B.7	B.7	B.7	B.7	--
P	B.8	B.8	B.8	B.8	B.8	3	3	B.8	B.8	B.8	B.8	--
α	α=0.23	α=0.23	α=0.23	α=0.23	α=B.25	α=0.23	α=0.23	α=B.25	α=0.23	α=0.23	α=0.23	--
R_n	15-18, 42-46	15-18, 42-46	15-18, 42-46	15-18, 42-46	B.22-B.25	15-18, 42-46	15-18, 42-46	B.22-B.25	15-18, 42-46	15-18	42-46	--
G	30,32, 65-66	30,32, 65-66	30,32, 65-66	30,32, 65-66	30,32, 65-66	30,32, 65-66	30,32, 65-66	B.26 (24-hr), 65-66 (hrly)	30,32, 65-66	30,32	G = 0.	--
R_{so}	19(24-hr), 47 (hrly)	19 (24-hr), 47 (hrly)	19 (24-hr), 47 (hrly)	19 (24-hr), 47(hrly)	19 (24-hr), 47-(hrly)	19 (24-hr), 47 (hrly)	19 (24-hr), 47 (hrly)	19 (24-hr), 47 (hrly)	19 (24-hr), 47 (hrly)	19 (24-hr)	47 (hrly)	--
u_2	Uses u_z	Uses u_z	Uses u_z	Uses u_z	Uses u_z	33, 67	33, 67	33, 67	33, 67	33	67	--
r_s	B.3-B.6	70 and 45 s m^{-1} (24-hr), 50 and 30 s m^{-1} day, 200 s m^{-1}, night (hrly)	70 and 45 s m^{-1} (24-hr), 50 and 30 s m^{-1} day, 200 s m^{-1}, night (hrly)	User defined	70 and 45 s m^{-1} (24-hr), 50 and 30 s m^{-1} day, 200 s m^{-1}, night (hrly)	70 and 45 s m^{-1} (24-hr), 50 and 30 s m^{-1} day, 200 s m^{-1}, night (hrly)	70 s m^{-1} (all time steps)	--	--	--	--	--
r_a	B.2	B.2 for h=0.12m,	B.2 for h=0.12m,	B.2	B.2 for h=0.12m,	B.2 is embedded in	B.2 is embedded	B.18	1.0	1.0	0.29 day 1.14	--

Table B-1. Parameter equation numbers, etc. used in the Reference Equations Evaluated

Parameter	ASCE Penman-Monteith											
	"ASCE-PM"	"ASCE-PMD"	"ASCE-PMDL"	"ASCE-PMv"	"ASCE-PMDR"	ASCE Standardized Penman-Monteith	FAO-56 Penman-Monteith	1982 Kimberly Penman	1963 Penman	FAO-24 Penman	CIMIS Penman	1985 Hargreaves
		h=0.5 m	h=0.5 m		h=0.5 m	Eq. 1 for h=0.12m, h=0.5 m	in Eq. B.15 for h=0.12m				night	
ρ	B.10	B.10	B.10	B.10	B.10	--	--	B.19	0.537	0.862	0.53 day 0.40 night	--
e_s	6, 37	6, 37	6, 37	6, 37	6, 37	6, 37	6, 37	6, 37	6, 37	6	37	--
e_a	order of preference is given in Tables 3 and 4 of the main text											

Numbers in cells refer to equations listed in the main text and appendices.

[a] The Kimberly Penman equations are not intended to be applied hourly, but they were evaluated for hourly timesteps in this study.

"ASCE-PM" is the "full-form" ASCE Penman Monteith using resistance equations by Allen et al. (1989) and ASCE Manual 70 (Jensen et al. 1990) as a function of the height.

"ASCE-PMD" is the "full-form" ASCE Penman Monteith with values for r_s for hourly or shorter timesteps fixed at $r_s = 50$ s m^{-1} for 0.12 m tall grass and $r_s = 30$ s m^{-1} for 0.5 m tall alfalfa during daytime hours and $r_s = 200$ s m^{-1} for both grass and alfalfa during nighttime hours.

"ASCE-PMDL" is the "full-form" ASCE Penman Monteith, identical to ASCE-PMD except that λ is fixed at $\lambda = 2.45$ MJ kg^{-1}.

"ASCE-PMv" is the "full-form" ASCE Penman Monteith with a user supplied resistance.

"ASCE-PMDR" is the "full-form" ASCE Penman Monteith, identical to ASCE-PMD except that net radiation follows Wright (1982).

"ASCE Standardized Penman Monteith" is the standardized form of the ASCE-PM equation (ET_{sz}) specified by equations in the main text.

"FAO 56 Penman Monteith" equation from FAO-56 (Allen et al., 1998).

"1982 Kimberly Penman" is an alfalfa reference from Wright (1982) with wind function expressed as in Jensen et al., (1990).

"1963 Penman" is the Penman equation form and wind function from Penman (1963) but using R_n and G for the standardized method.

"FAO-24 Penman" is the Penman equation from FAO-24 (Doorenbos and Pruitt, 1977) with correction factor = 1.0

"CIMIS Penman" is the hourly Penman equation traditionally used by the California Irrigation Management Information System (Snyder and Pruitt, 1985, Snyder and Pruitt, 1992) using R_n for the standardized method and G=0.

"1985 Hargreaves" is the air temperature based equation of Hargreaves et al., (1985).

The variations on the ASCE Penman-Monteith equation that were evaluated by the task committee are described as follows:

1. ***"ASCE-PM" is the "full-form" ASCE Penman-Monteith*** using resistance equations by Allen et al. (1989) and in ASCE Manual 70 (Jensen et al. 1990). In ASCE-PM, r_s is computed from the leaf area index (LAI), which is a function of the height specified for the reference type (grass or alfalfa). Algorithms for LAI depend on reference type. The value of r_s (and r_a) change with height specified for the reference. The values for r_s for 24-hour timesteps, based on the ASCE LAI algorithms, are $r_s = 70$ s m^{-1} for 0.12 m tall grass and $r_s = 45$ s m^{-1} for 0.5 m tall alfalfa. This equation, when computed using a daily calculation timestep, was the measure against which the other equations were compared. The ASCE-PM method, using resistance parameters as defined in Manual 70 to be functions of vegetation height and computed with a daily timestep, was the method found to perform best against lysimeter measurements in Manual 70.

2. ***"ASCE-PMD" is the "full-form" ASCE Penman-Monteith*** and is the same as (1) except that the values for r_s for hourly or shorter timesteps were fixed at $r_s = 50$ s m^{-1} for 0.12 m tall grass and $r_s = 30$ s m^{-1} for 0.5 m tall alfalfa during daytime hours and $r_s = 200$ s m^{-1} for both 0.12 m tall grass and 0.5 m tall alfalfa during nighttime hours. The purpose of the variation was to evaluate whether use of a lower value for r_s for daytime and higher value for nighttime could improve the prediction for hourly timestep calculations relative to the ASCE-PM computed daily.

3. ***"ASCE-PMDL" is the "full-form" ASCE Penman-Monteith*** and is identical to (2) except that the value for the heat of vaporization was fixed at $\lambda = 2.45$ MJ kg^{-1}.The purpose of the variation was to evaluate whether use of a constant value for λ versus a calculated value impacted calculations significantly.

4. ***"ASCE-PMv" is the "full-form" ASCE Penman-Monteith with a user supplied resistance***. This method is the same as number 1, except that members of the TC had the option of specifying unique values for 24-hour, daytime and nighttime surface resistance, r_s, for each site. The purpose of the variation was to allow the TC members to test data from their region to determine what value for r_s resulted in accurate estimates of ET_{ref} in their region.

5. ***"ASCE-PMDR" is the "full-form" ASCE Penman-Monteith*** and is identical to (2) except that net radiation was calculated following Wright (1982) rather than Eq. 15 – 18 and 42 – 47. The purpose of this variation was to evaluate the degree to which using the Wright (1982) net radiation procedure in place of the standardized procedure impacted the ET_{ref} calculation.

6. ***ASCE Standardized Penman-Monteith equation*** is the standardized form of the ASCE-PM equation (ET_{sz}) specified by equations provided in the main text.

7. ***FAO 56 Penman-Monteith equation.*** The FAO-56 PM method uses essentially identical calculation procedures as the standardized ET_{sz} equation, except for a constant surface resistance (70 s m^{-1}) that is applied to all timesteps and its application is to ET_o, only.

Basic equations and supporting parameter equations for equations other than the standardized equation are listed in the following sections.

ASCE PENMAN-MONTEITH METHOD

The Penman-Monteith form of the combination equation (Monteith 1965, 1981) is:

$$ET_{ref} = \left(\frac{\Delta(R_n - G) + K_{time}\,\rho_a\, c_p \dfrac{(e_s - e_a)}{r_a}}{\Delta + \gamma\left(1 + \dfrac{r_s}{r_a}\right)} \right) / (\lambda \rho_w) \quad \text{(B.1)}$$

where

ET_{ref}	= reference evapotranspiration [mm d^{-1} or mm h^{-1}],
R_n	= net radiation [MJ m^{-2} d^{-1} or MJ m^{-2} h^{-1}],
G	= soil heat flux [MJ m^{-2} d^{-1} or MJ m^{-2} h^{-1}],
$(e_s - e_a)$	= vapor pressure deficit of the air [kPa],
e_s	= saturation vapor pressure of the air [kPa],
e_a	= actual vapor pressure of the air [kPa],
ρ_a	= mean air density at constant pressure [kg m^{-3}],
c_p	= specific heat of the air [MJ kg^{-1} $^oC^{-1}$],
Δ	= slope of the saturation vapor pressure temperature relationship [kPa $^oC^{-1}$],
γ	= psychrometric constant [kPa $^oC^{-1}$],
r_s	= (bulk) surface resistance [s m^{-1}],
r_a	= aerodynamic resistance [s m^{-1}],
λ	= latent heat of vaporization, [MJ kg^{-1}],
ρ_w	= density of water, [Mg m^{-3}] (taken as 1.0 Mg m^{-3}),
K_{time}	= units conversion, equal to 86,400 s d^{-1} for ET in mm d^{-1} and equal to 3600 s h^{-1} for ET in mm h^{-1}.

The aerodynamic resistance, applied for neutral stability conditions, is:

$$r_a = \frac{\ln\left[\dfrac{z_w - d}{z_{om}}\right] \ln\left[\dfrac{z_h - d}{z_{oh}}\right]}{k^2\, u_z} \quad \text{(B.2)}$$

where

r_a = aerodynamic resistance [s m^{-1}],
z_w = height of wind measurements [m],
z_h = height of humidity and or air temperature measurements [m],
d = zero plane displacement height [m], = 0.67 h
z_{om} = roughness length governing momentum transfer [m], = 0.123 h
z_{oh} = roughness length for transfer of heat and vapor [m], = 0.0123 h
k = von Karman's constant, 0.41 [-],
u_z = wind speed at height z [m s^{-1}]
h = mean height of the vegetation [m].

Bulk surface resistance is:

$$r_s = \frac{r_l}{LAI_{active}} \tag{B.3}$$

where

r_s = (bulk) surface resistance [s m^{-1}],
r_l = effective stomatal resistance of a well-illuminated leaf [s m^{-1}],
LAI_{active} = active (sunlit) leaf area index [m^2 (leaf area) m^{-2} (soil surface)]

For ASCE calculations for dense vegetation, LAI_{active} is calculated as:

$$LAI_{active} = 0.5\,LAI \tag{B.4}$$

where

LAI = leaf area index [m^2 of leaf per m^2 of soil surface = dimensionless]

For clipped grass:

$$LAI = 24\ h \tag{B.5}$$

For alfalfa:

$$LAI = 5.5 + 1.5\ln(h) \tag{B.6}$$

where

h = vegetation height [m]

In the "full-form" ASCE Penman-Monteith method, the following "full-form" ancillary equations are used. Many of these have been simplified for use with the ET_{sz} form of the Penman-Monteith equation and are listed in the main text.

Latent Heat of Vaporization (λ)[1]

$$\lambda = 2.501 - (2.361 \times 10^{-3})\, T_{mean} \quad \text{(B.7)}$$

where:

λ = latent heat of vaporization [MJ kg^{-1}]
T_{mean} = mean air temperature for the time interval [°C]

The value of the latent heat varies only slightly over normal temperature ranges. For ET_{sz}, a single value is taken: λ = 2.45 MJ kg^{-1}. The inverse of λ is presented as 0.408.

Atmospheric Pressure (P)[2]

Mean atmospheric pressure for a location is predicted from site elevation using a lapse-based integration of the universal gas law:

$$P = P_o \left(\frac{T_{Ko} - \alpha_1 (z - z_o)}{T_{Ko}} \right)^{\frac{g}{\alpha_1 R}} \quad \text{(B.8)}$$

where:

P = atmospheric pressure at elevation z [kPa]
P_o = atmospheric pressure at reference level (sea level = 101.3) [kPa]
z = weather site elevation [m]
z_o = elevation at reference level (i.e., sea level = 0) [m]
g = gravitational acceleration = 9.807 [m s^{-2}]
R = specific gas constant = 287 [J kg^{-1} K^{-1}]
α_1 = constant lapse rate of moist air = 0.0065 [K m^{-1}]
T_{Ko} = reference temperature [K] at pressure P_o and elevation z_o.

List (1984) defined P_o = 101.3 kPa, z_o = 0 m and T_{Ko} = 288 K for the U.S. and International Standard Atmospheres,. However, Smith et al., (1991) recommended

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[1] Reference: Harrison (1963)

[2] Reference: List (1984), Burman *et al.* (1987)

using a reference temperature of T_{mean} = 20 °C to represent mean daytime conditions during growing seasons, so that:

$$T_{Ko} = 293\ K \qquad (B.9)$$

Using T_{Ko} = 293 K from equation (B.9), equation (B.8) becomes equation 3 of the main text. The difference in prediction of P using T_{Ko} = 288 and T_{Ko} = 293 K is less than 0.7% for elevations less than 3000 m.

Air Density (ρ_a)[3]

$$\rho_a = \frac{1000\,P}{T_{Kv}\,R} = 3.486\,\frac{P}{T_{Kv}} \qquad (B.10)$$

where:

ρ_a = air density [kg m^{-3}]
R = specific gas constant = 287 [J kg^{-1} K^{-1}]
T_{Kv} = mean virtual temperature for period [K]

$$T_{Kv} = T_K \left(1 - 0.378\,\frac{e_a}{P}\right)^{-1} \qquad (B.11)$$

where:

T_K = mean absolute temperature [K] : T_K = 273.16 + T_{mean} [°C]
e_a = actual vapor pressure [kPa]

In derivation of the ET_{sz} equation, equation (B.11) was reduced to $T_{Kv} \approx 1.01\,(T_{mean} + 273)$ that holds for most conditions. T_{mean} is set equal to mean daily temperature for 24-hour calculation time steps.

Psychrometric Constant (γ)[4]

The pyschrometric constant, γ, is used in the numerator and denominator of the standardized Penman-Monteith equation:

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[3] Reference: Smith *et al.* (1991)

[4] Reference: Brunt (1952)

$$\gamma = \frac{c_p P}{\varepsilon \lambda} \tag{B.12}$$

where:

γ	=	psychrometric constant [kPa °C^{-1}]
c_p	=	specific heat of moist air = 1.013 x 10^{-3} [MJ kg^{-1} °C^{-1}]
P	=	atmospheric pressure [kPa]
ε	=	ratio of the molecular weight of water vapor/dry air ("epsilon") (ε = 0.622 for standard, dry air)
λ	=	latent heat of vaporization [MJ kg^{-1}] (λ = 2.45 MJ kg^{-1} for standardized calculations)

The simplification of λ = 2.45 MJ kg^{-1} in equation B.12 and reduction results in Eq. 4 for the ET_{sz} equation. This simplification causes less than 2% error in γ over the range of $0 < T_{mean} < 40$ °C and less than 1% error over the range of $11 < T_{mean} < 31$ °C. This translates into errors in ET_{os} and ET_{rs} that are generally less than 0.2%.

Soil Heat Flux Density (G) for hourly periods[5]

The full equation for hourly G, on which equations 66 and 67 for ET_{sz} are based, is:

$$G_{hr} = K_G \exp(-0.5\,LAI)\, R_n \tag{B.13}$$

where

K_G	=	0.4 during daytime (defined as when $R_n > 0$)
K_G	=	2.0 during nighttime (defined as when $R_n \le 0$)
LAI	=	leaf area index [dimensionless]

Units for G_{hr} and R_n are the same.

[5] Reference: Choudhury et al. (1987), Choudhury (1989)