

## Stability Analysis on Highway Slopes in Rainy Region

Wenhui Tan<sup>1</sup>, Shijie Qu<sup>2</sup> and Danqing Gao<sup>3</sup>

<sup>1</sup>Associate Professor, Department of Civil Engineering, University of Science and Technology Beijing, 30 Xueyuan Road, Haidian District, Beijing100083, P.R. China; wenhui.t@163.com

<sup>2</sup>Professor, Department of Civil Engineering, University of Science and Technology Beijing, 30 Xueyuan Road, Haidian District, Beijing100083, P.R. China; qushijie1229@yahoo.com.cn

<sup>3</sup>Master Candidate, Department of Civil Engineering, University of Science and Technology Beijing, 30 Xueyuan Road, Haidian District, Beijing100083, P.R. China; gaodanqing@gmail.com

**ABSTRACT:** Rainfall infiltration is an important factor affecting slope stability. To investigate the influence of rainfall on highway slope stability, a typical cut slope of highway in rainy region has been adopted for the study. Based on saturated - unsaturated seepage theory and coupled solid-liquid analysis, the features of seepage, pore-water pressure, stress and displacement of the highway slope are analyzed under two different rainfall intensity and duration conditions. At the same time, modified Mohr-Coulomb strength criteria and 2-D limit equilibrium method are used to calculate the safety factors of the slope. The results showed that the range of unsaturated zone decreased, while the transient saturated zone enlarged, the pore-water pressure, displacements and the negative shear stress on the surface of the slope rose with increasing rainfall intensity and duration. The safety factors of the slope also decreased with rising of the rainfall intensity and duration and were all lower than 1.0. To mitigate, anchor-shotcrete reinforcement with adhesive bolt suspended net was applied to the slope. The results from field monitoring showed that the reinforced slope is stable even after continuous heavy rain. Therefore, anchor-shotcrete reinforcement is a useful method for strengthening slopes in rainy regions.

### INTRODUCTION

The causes of landslides are attributed to a number of factors, such as geologic features, topography, vegetation, weather, and groundwater/seepage conditions. A large number of cases show that rainfall infiltration is a principal factor impacting slope stability, since almost 90% of landslides were induced by rainfall infiltration in China (Huang et al., 2002). Hydrostatic pressure and hydrodynamic pressure in a rock mass slope are generated by seepage flow of rainwater, as these pressures undermine the stress balance of slopes, leading to the resisting force of the slopes being reduced and the sliding force increased. At the same time, rainfall infiltration also makes the suction of the slopes and the safety factor of slopes decrease, which result in landslides (Lam et al., 1987).

Yiwan highway is located in Henan Province. The rock mass of the highway slopes are mainly strong weathered granite, strong weathered metamorphic quartz schist and clastic tuff. The strata are water-bearing or permeable. The permeability is very high especially in the shallow unloaded weathered stratum because most of the joints are splitting. The groundwater level is high; the distribution of rainfall is uneven and mainly concentrates in June, July and August. The average yearly precipitation is up to 1000 ~ 1200mm (Duan, 2007). Water is infiltrated in the unloaded zone of slopes more easily during the rainy season, and the hydrostatic pressure and hydrodynamic pressure will lead to serious adverse effects on the stability of slopes. If slopes are excavated according to the original design, the safety of construction and long-term stability of the slopes can not be guaranteed. Therefore, it is necessary to analyze the stability of the cut slope suffering from rainfall infiltration and to select an appropriate reinforcement method for keeping slopes safe at all stages.

Some researchers (Gasmo et al., 2000; Cho et al., 2001; Tsaparas et al., 2002; Gerscovich et al., 2006; Zhou et al., 2008; Rong et al., 2005) have studied the problem of slope stability under the condition of rainfall infiltration. In order to provide useful advice to the design of drainage and reinforcement of slopes in rainy region, the features of seepage, pore-water pressure, stress and displacement of a typical slope of Yiwan highway are analyzed using finite element method in two scenarios with different rainfall intensity and duration, based on saturated-unsaturated seepage theory of rock-soil medium and coupled solid-liquid analysis method. At the same time, modified Mohr-Coulomb strength criteria and the 2-D limit equilibrium method are used to evaluate the stability of the slope under all the conditions. To improve the stability of the slope, a program of anchor-shotcrete reinforcement with adhesive bolt suspended net is adopted.

## MATHEMATICAL MODEL OF RAINFALL INFILTRATION IN SLOPES

### Model of Infiltration Field

An equivalent continuum model is used to analyze infiltration field in intensive crack rock mass. According to Darcy's law and the law of conservation of mass, the basic mathematical model of the seepage slope (Mao et al., 1999) is given by

$$\frac{\partial}{\partial x_i} \left[ k_{ij} k_r(h) \frac{\partial h}{\partial x_j} + k_{i3} k_r(h) \right] + S = [C(h) + \beta S_s] \frac{\partial h}{\partial t} \quad (1)$$

As boundary condition is defined as

$$\left\{ \begin{array}{l} h(x, y, z, t_0) = h_0(x, y, z, t_0) \\ h(x, y, z, t) \Big|_{\Gamma_1} = h_1(x, y, z, t) \\ - \left[ k_{ij} k_r(h) \frac{\partial h}{\partial x_j} + k_{i3} k_r(h) \right] n_i \Big|_{\Gamma_2} = q_n \\ - \left[ k_{ij} k_r(h) \frac{\partial h}{\partial x_j} + k_{i3} k_r(h) \right] n_i \Big|_{\Gamma_3} \geq 0, h \Big|_{\Gamma_3} = 0 \\ - \left[ k_{ij} k_r(h) \frac{\partial h}{\partial x_j} + k_{i3} k_r(h) \right] n_i \Big|_{\Gamma_4} = i(t) \end{array} \right.$$

where,  $h$  is pressure head;  $k_r(h)$  is the relative permeability coefficient, in the saturated zone,  $k_r(h) = 1$ , in the unsaturated zone,  $k_r(h)$  has range of  $[0, 1]$ ;  $k_{ij}$  is the saturated permeability tensor;  $S_s$  is a unit storage coefficient, for the non-saturated body,  $S_s = 0$ , for the saturated body,  $S_s$  is a constant;  $C(h)$  is the degree of water content, on the positive pressure zone,  $C(h) = 0$ , on the negative pressure zone,  $C(h) = \partial\theta/\partial h$ ;  $\theta$  is volumetric water content,  $\theta = nS_w$ ,  $S_w$  is the media saturation;  $n$  is the medium of saturated water content;  $\beta$  is to determine the saturation and non-saturation parameters, in the unsaturated zone,  $\beta = 0$ , in the saturated zone,  $\beta = 1$ ;  $t$  is time variable;  $S$  is the source and sink items.  $k_{ij}$  is the saturated permeability tensor,  $k_{i3}$  is the saturated hydraulic conductivity when  $j=3$ ;  $q_n$  is the normal flux;  $n_i$  is the normal direction cosine of the outside unit boundary;  $t_0$  is the initial time;  $i(t)$  is infiltration rate;  $\Gamma_1$  is head boundary;  $\Gamma_2$  is flow boundary;  $\Gamma_3$  is the saturation of spilling boundary;  $\Gamma_4$  is border of infiltration.

The differential equation (1) can be solved by Galerkin weighted residual method considering the boundary conditions shown above.

### Coupled Solid-liquid Model

Rainfall infiltration is a complicated process in which seepage flow and stress interact with each other. The deformation characteristic, stress state and failure mechanism of slopes resulting from rainfall can be modeled by a coupled solid-liquid analysis. The equilibrium equations used in the coupled analysis are formulated on the basis of the principle of virtual work, that is, the total internal virtual work is equal to the external virtual work, so the equations of coupled analysis can be formulated using incremental displacement and incremental pore-water pressure (Krahn(a), 2004), the coupled equations for finite element analysis can be written as

$$\begin{cases} [K]\{\Delta\delta\} + [L_d]\{\Delta u_w\} = \{\Delta F\} \\ \beta[L_f]\{\Delta\delta\} - \left(\frac{\Delta t}{\gamma_w}[K_f] + \omega[M_N]\right)\{\Delta u_w\} = \Delta t \left( \{Q\}_{t+\Delta t} + \frac{1}{\gamma_w}[K_f]\{u_w\}_t \right) \end{cases} \quad (2)$$

where,  $[K]$  is stiffness matrix,  $[K] = \sum [B]^T [D] [B]$ ;  $[B]$  is the gradient matrix (also called response matrix);  $[D]$  is the draining of the constitutive matrix;  $[L_d]$  is the coupling matrix,  $[L_d] = \sum [B]^T [D] \{m_H\} \langle N \rangle$ , in the saturation,  $\{m_H\} = \left\langle \frac{1}{H} \quad \frac{1}{H} \quad \frac{1}{H} \quad 0 \right\rangle$ ,  $[L_d] = [B]^T \{m\} \langle N \rangle$ ,  $\{m\}^T = \langle 1, 1, 1, 0 \rangle$ ;  $\Delta\delta$  is the displacement increment vector;  $\Delta u_w$  is the pore pressure increment vector;  $[K_f]$  is the element stiffness matrix,  $\{m\}^T = \langle 1, 1, 1, 0 \rangle$ ,  $[K_w]$  is the permeability coefficient matrix;  $[M_N]$  for the mass matrix,  $[M_N] = \sum \langle N \rangle^T \langle N \rangle$ ,  $\langle N \rangle$  is linearity shape function vector;  $[L_f]$  for the fluid coupling matrix,  $[L_f] = \sum \langle N \rangle^T \{m\}^T [B]$ ;  $\{Q\}$  is the flow of the boundary nodes;  $\beta = \frac{E}{H(1-2\nu)} = \frac{3K_B}{H}$ ,  $K_B$  for the bulk modulus;  $\omega = \frac{1}{R} - \frac{3\beta}{H}$ ,  $R$  is with the change of the matrix suction, the modulus of changing volumetric water content, in the full saturation,  $\beta = 1$ ,  $[L_f] = [L_d]^T$ .

The flow coefficient, pore pressure, stress and strain at every calculation step can be obtained by iterative calculation of formula (2).

## Calculation Model of Slope Stability

In a saturated – unsaturated slope, the water pressure above the water level is negative, while the negative water pressure will impact on the suction and the stability of slopes. In order to consider the effect of suction on the resistant shear strength and the stability safety factor of slopes, the Mohr-Coulomb failure criteria is modified as follows (Fredlund, 1993):

$$\tau_f = C' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (3)$$

$$\tau = \frac{(\sigma_y - \sigma_x)}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha \quad (4)$$

Therefore, according to the limit equilibrium theory, the safety factor of saturated - unsaturated slopes can be calculated according to the following formula (Cho et al., 2001) :

$$F_s = \frac{\int_{\Gamma} \tau_f d\Gamma}{\int_{\Gamma} \tau d\Gamma}, \quad (5)$$

where  $\Gamma$  is the length of the slip surface.

Using the slice method, the slope safety factor can be calculated by the following formula according to the balance of forces (Krahn(b), 2004):

$$F_f = \frac{\sum \left( C' \beta \cos \alpha + \left[ N - u_w \beta \frac{\tan \phi^b}{\tan \phi'} - u_a \beta \left( 1 - \frac{\tan \phi^b}{\tan \phi'} \right) \right] \tan \phi' \cos \alpha \right)}{\sum N \sin \alpha + \sum kW - \sum D \cos \omega \pm \sum A} \quad (6)$$

where,  $C'$  is the effective cohesion;  $\phi'$  is the internal friction angle of the net normal stress's state variable ( $\sigma_n - u_a$ );  $u_w$  is the pore water pressure of the sliding surface;  $\phi^b$  is the internal friction angle of suction-related, in the saturated zone,  $\phi^b$  equals to  $\phi'$ ;  $u_a$  is the pore air pressure of the sliding surface;  $\beta$  is the slice base length;  $\alpha$  is the angle between bottom slice and horizontal tangent;  $N$  is the total normal force from the bottom slice;  $kW$  is the earthquake load from the slice-shaped heart level;  $D$  is the external line load,  $\omega$  is an angle between the line load and horizontal direction;  $A$  is the external water pressure.

## CASE ANALYSIS

### Model and Parameters for Calculation

The subject investigated is in the first sublevel of Yiwan highway. The height of the cut slope is approximately 27 meters. According to geometry and rock properties of the slope, a typical model is established as shown in FIG. 1. The continuum model is selected for the macroscopic seepage analysis because of intense fissures and no large fault in the slope. In the seepage analysis, the head of water at the bottom of slope is defined as the height of the bottom. The total head of groundwater is 48 m.

In order to study the effect of rainfall on the slope stability, two scenarios are evaluated with different intensities and durations: Scenario 1, maintain the duration of

rainfall as two days, but the rainfall intensities are varied at 30mm/day, 60mm/day, 120mm/day respectively; Scenario 2, maintain rainfall intensities as in Scenario 1, but the durations of rainfall are changed to 1 day, 2 day, 3 day and 4 day differently. Characters of seepage flow, stress, displacement and the stability of the slope are analyzed in the two scenarios. Mechanical parameters and permeability coefficients of rock mass in the slope are shown in Table 1.

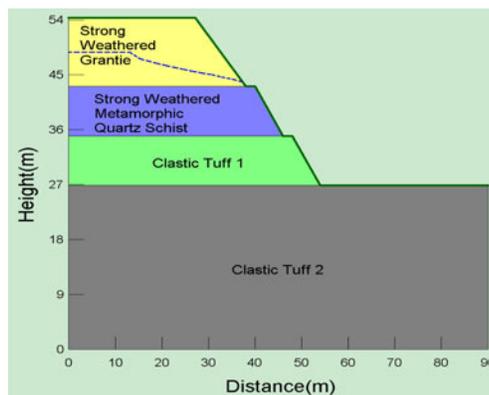


FIG. 1. Calculation model of a slope

Table 1. Parameters of Rock Mass

Rock type	Strong weathered granite	Strong weathered metamorphic quartz schist	Clastic tuff 1	Clastic tuff 2
Density of Bulk $\gamma$ ( $\text{kN}\cdot\text{m}^{-3}$ )	24.01	25.1	21.17	21.17
Young's Modulus $E$ (GPa)	3.59	7.63	1.01	1.01
Poisson Ratio $\nu$	0.246	0.202	0.301	0.301
Tensile Strength $\sigma_t$ (MPa)	0.0125	2.716	6.83	6.83
Cohesive Strength $C$ (kPa)	19.62	139.05	17.85	27.85
Friction Angle $\Phi$ ( $^\circ$ )	32.51	41.57	37.81	37.81
Permeability Coefficient $k$ ( $\text{m}\cdot\text{s}^{-1}$ )	$1.73\times 10^{-6}$	$1.16\times 10^{-7}$	$1.06\times 10^{-7}$	$9.03\times 10^{-8}$

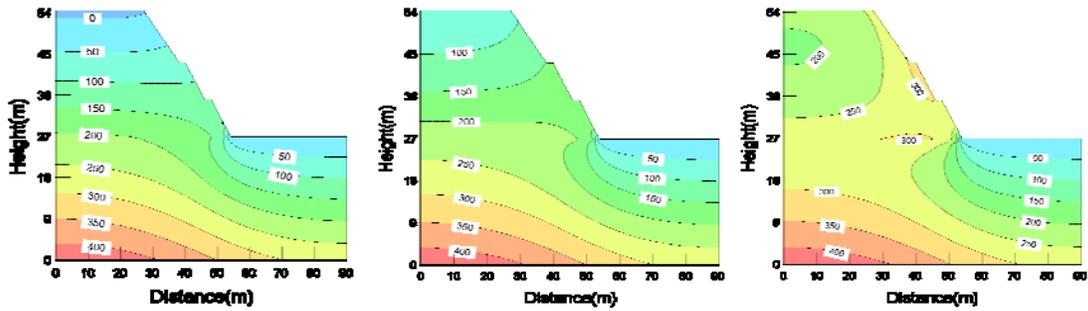
### Numerical Simulation of Slope under Rainfall Infiltration Condition

#### ( 1 ) Numerical simulation using Finite Element Method

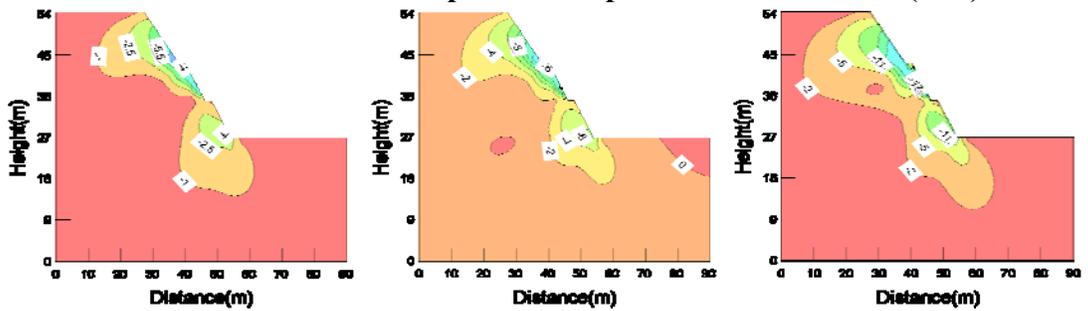
In numerical simulation, rainfall is distributed along the ground surface. Seepage field analysis and solid-liquid coupled analysis are accomplished to develop the characters of seepage, pore-water pressure, shear stress and displacements in the two projects. FIG. 2 and FIG. 3 are parts of the results of Scenario 1. FIG. 4 and FIG.5 illustrates the changes of the maximum negative shear stress on surface of the slope and the maximum x-displacement of the slope in Scenario 2.

In Scenario 1, the total head and pore water pressures (FIG.2) at the top and on the surface of the slope increased significantly with the rise of the rainfall intensity. The maximum pore water pressure on the slope surface reached 0.3MPa. The reason is that a

transient saturated zone is formed in the former unsaturated zone at the top of the slope by the rainfall infiltration and the range of the transient saturated zone enlarged with increasing of the rainfall intensity. As shown in FIG.3, the negative shear stress on the surface of the slope and the deformations of the slope increased. The maximum x-displacements are 0.152m, 0.215m and 0.336m respectively. The sliding region extends gradually to the toe of the slope and leads to an ultimate slide.



(a) 30 mm/d (b) 60 mm/d (c) 120 mm/d  
 FIG. 2. Distribution of pore-water pressure in Scenario 1(kPa)



(a) 30 mm/d (b) 60 mm/d (c) 120 mm/d  
 FIG. 3. Distribution of shear stress in Scenario 1 (MPa)

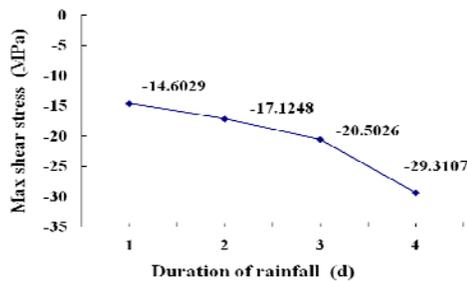


FIG. 4. Change of the Max shear stress in Scenario 2

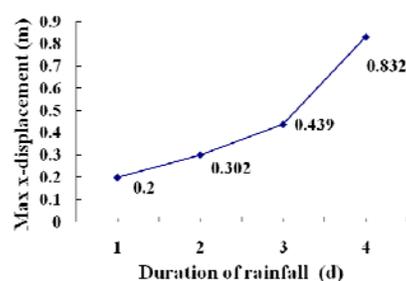


FIG. 5. Change of the Max x-displacement in Scenario 2

In Scenario 2, the maximum negative shear stress (FIG.4) on the surface of the slope and deformation (FIG.5) of the slope increases with the rainfall duration increment. After one day's rainfall, a tensile stress zone appeared at the top of the slope. As the duration of the rainfall increases, the sliding area extends gradually to the toe of the slope. A penetrating slip surface is formed after 4 days' rainfall.

## ( 2 ) Slope stability analysis using limit equilibrium method

Modified Mohr-Coulomb criterion and 2-D limit equilibrium method are adopted to evaluate slope stability. When there is no water in the slope, the calculated safety factor is 1.656. But when there is groundwater in the slope, the safety factor is reduced to 1.112. The safety factors in Scenario 1 and Scenario 2 are listed in Table 2.

**Table 2. Safety Factors in Scenario 1 and Scenario 2**

<b>Program</b>	<b>Rainfall Intensity</b>	<b>Rainfall Duration</b>	<b>Safety Factor</b>
<b>Scenario 1</b>	30 mm/d	2d	0.952
	60 mm/d	2d	0.772
	90mm/d	2d	0.620
	120 mm/d	2d	0.609
<b>Scenario 2</b>	90mm/d	1d	0.719
	90mm/d	2d	0.620
	90mm/d	3d	0.615
	90mm/d	4d	0.601

From Table 2, the safety factors declined as the rainfall intensity and duration increased. In all situations, the safety factor is less than 1.0, that is, the slope is unsafe when raining. Hence, reinforcement is needed to strengthen the slope.

**Stability Analysis on the Reinforced Slope**

A program of anchor-shotcrete reinforcement with adhesive bolt suspended net is adopted to improve the stability safety factor of the slope. Three rows of anchors are laid out at every bench of the slope with a spacing of 2 m, the length of anchor is 9 m, the adhesive bolt suspended net on the surface of the slope is formed with wire iron of  $\Phi 8$  and C20 cement grout. After 4 days of heavy rain, the reinforced slope is verified as stable by field monitoring. After being reinforced by anchor-shotcrete, the rainfall infiltration capacity of the slope becomes lower, so the rainfall intensity can be discounted by 10% in the calculation. As a result, the safety factor of the reinforced slope calculated by the limit equilibrium method is 1.151. Therefore, the technique of anchor-shotcrete reinforcement with an adhesive bolt suspended net is useful for strengthening slopes in rainy regions.

**CONCLUSIONS**

The influence of rainfall intensity and duration of rainfall on highway slope stability is studied with coupled solid - liquid analysis and limit equilibrium methods in this paper and the results show that:

(1) As the duration of rainfall keeps constant, while rainfall intensity changed, the pore water pressures at the top and on the surface of the slope increased significantly with the rise of the rainfall intensity. The maximum negative shear stresses increased at the top and the toe of the slope. Also, the maximum horizontal displacement continued to enlarge with the rising of the rainfall intensity, gradually developing a potential slip surface.

(2) As the rainfall intensity and rainfall duration increased, the safety factors became lower. The safety factors were all less than 1.0 at all the situations of rainfall, that is, the slope was unsafe when raining.

(3) Changes of the water table were small when raining because the rock mass of the slope is heavily weathered and jointed. When the height of the water table reaches to the strong weathered stratum, the water is easily to drained away due to the characters of large porosity and high hydraulic conductivity of the rock mass. Therefore, the level of groundwater fluctuates very small. However, when the drainage is poor for continuous rainfall, the pore-water pressure of the slope increases, a transient saturated zone forms above the groundwater level, which affects slope stability.

(4) Continuous rainfall on the slope will lead to a landslide. However, the reinforced slope is stable, even after continuous heavy rains. In conclusion, anchor-shotcrete reinforcement with adhesive bolt suspended net is useful for strengthening slopes in rainy regions.

## REFERENCES

- Cho, S. E. and Lee, S. R. (2001). "Instability of unsaturated soil slopes due to infiltration . " *Computers and Geotechnics* , No.28:185 - 208.
- Duan, W.Q. (2007). "Monitoring and analysis on reinforced freeway slope. " Master thesis, University of Science and Technology Beijing. (in Chinese)
- Fredlund, D. G. and Rahard, J. O. (1993). "Soil mechanics for unsaturated soils. " New York: John Wiley & Sons, INC.
- Gasmo, J. M., Rahardjo, H. and Leong, E. C. (2000). "Infiltration effects on stability of a residual soil slope . " *Computers and Geotechnics*, No.26: 145 - 165.
- Gerscovich, D. M. S. , Vargas, E. A. and Campos, T. M. P. de. (2006). "On the evaluation of unsaturated flow in a natural slope in Rio de Janeiro, Brazil. " *Engineering Geology*, No.88 : 23–40.
- Huang, R.Q. and Qi, G.Q.(2002). "Matric suction in unsaturated seepage on slope stability. " *Journal of Engineering Geology*, Vol. 10 (4): 343-347. (in Chinese)
- Krahn, J. (a). "Stress and deformation modeling with SIGMA/W: an Engineering Methodology (1st edit). " Geo-Slope International Ltd., Alberta, Canadian. 2004
- Krahn, J. (b). "Stability modeling with SLOPE/W: an Engineering Methodology.(1st edit). " Geo-Slope International Ltd., Alberta, Canadian. 2004
- Lam, L. , Fredlund, D. G. and Barbour, S. L. (1987)."Transient seepage model for saturated -unsaturated soil systems: a geotechnical engineering approach." *Canadian Geotechnical Journal* , No.24 : 565 ~ 580.
- Mao, C.X., Yin, X.B. and Li, Z.Y.(1999). "Numerical Methods of seepage and application procedures. " Nanjing: Press of Hehai University, 1999. (in Chinese)
- Rong, G., Zhang, W. and Zhou, C.B. (2005). "Numerical analysis of saturated-unsaturated seepage problem of rock slope under rainfall infiltration. " *Rock and soil mechanics*. Vol. 26 (10) :1545 - 1550. (in Chinese)
- Tsaparas, I., Rahardjo, H., Toll,D.G., et al.(2002). "Controlling parameters for rainfall-induced landslides . " *Computers and Geotechnics*, No.29: 1 - 27.
- Zhou, J.W. , Xu, W.Y. , Deng, J.Y. and Liu, X.N.(2008) . "Stability analysis of slope under the condition of rainfall infiltration. " *Journal of hydraulic engineering*. Vol. 39(9): 1066 -1073 (in Chinese)

## Upper Bound Stability Analysis for Soil Slope with Non-associated Flow Rule

Zhi-gang Zhao<sup>1</sup>, Lian-heng Zhao<sup>2</sup>, Fu Huang<sup>3</sup>, Yang Feng<sup>4</sup>,

<sup>1</sup> Civil engineer, China Railway Seven Engineering Group Co. , Ltd , Zhengzhou, Henan Province, People's Republic of China, Email: zzg71@163.com

<sup>2</sup> Ph.D, Lecturer, School of Civil and Architectural Engineering, Central South University, Changsha, Hunan Province, People's Republic of China, Email: zlh8076@163.com

<sup>3</sup> Doctoral candidate, School of Civil and Architectural Engineering, Central South University, Changsha, Hunan Province, People's Republic of China, Email: hfzndx2002@yahoo.com.cn

<sup>4</sup> Ph.D, Lecturer, School of Civil and Architectural Engineering, Central South University, Changsha, Hunan Province, People's Republic of China, Email: yf5754@126.com

**ABSTRACT:** The non-associated flow rule was introduced by means of modification shear strength parameters and a kinematically admissible velocity field was considered for slope stability analysis. The effectiveness and validity of the analysis method in this paper was illuminated through a case study. Comparative analysis has shown that the proposed method gives better calculation results. The results show that the consideration of parameters of non-associated flow rule plays a remarkable role on the slope stability. Therefore, the presumption that geomaterials subject to associated flow rule takes disadvantageous influence on evaluating the real stability characteristics of slope.

## INTRODUCTION

Soil slope stability analysis plays a considerably vital part in the field of geotechnical and civil engineering, and it has generated a lot of investigation. Many efforts have been carried out to analyze slope stability on the basis of associated flow rule. However, in the case of dense granular materials, a key factor in constitutive behavior is described as the characteristic of dilatancy angle, which is different from friction angle (Radenkovic (1961), Collins (1969), Mróz and Drescher (1969), Chen (1975, 90), De Borst R, Vermeer P A.(1984), Lade P V, (1992), Drescher and Detournay (1993)). According to plasticity theory of geomaterials, dilatancy means the geomaterials follow the non-associated flow rule, which influences the safety factor of the slope (Yang et al (2007), Yang and Huang (2009)). Therefore, a number of researchers have employed some quite complex process and non-associated flow rule to calculate the safety factor of the slope (Wang et al (2001a, 2001b), Wang et al (2002), Kumar (2004), Yang et al (2007), Yang and Huang (2009)), and some conclusions have been drawn.

The main objective of the present paper is to get the upper bound solution of safety factor ( $F_s$ ) under the assumption of non-associated flow rule by utilizing a short-cut method, in which the upper bound limit analysis and strength reduction technique have been used. The influence of non-associated flow rule on slope safety factor ( $F_s$ ) and latent slide surface were examined by using the sequential quadratic programming algorithm. Finally, the paper compacts some conclusions with those from previous research.

## NON-ASSOCIATED FLOW RULE

The material subjected to non-associated flow rule when the dilation angle is not equal to the friction angle. The included angle between velocity vector and discontinue line is dilation angle, therefore, the dilation coefficient can be expressed as follows:

$$\eta = \psi / \varphi \quad (1)$$

where  $\psi$  is dilation angle. In theory, variation range of dilation coefficient  $\eta$  is  $0 \leq \eta \leq 1$ .  $\eta=1$  means the material subjected to associated flow rule. For the coaxial non-associated material, if the soil satisfied Mohr-Coulomb failure criterion, the energy calculation can be achieved by the following equations (Yang et al (2007), Yang and Huang (2009))

$$c^* = c \frac{\cos \psi \cos \varphi}{1 - \sin \varphi \sin \psi} \quad (2)$$

$$\tan \varphi^* = \tan \varphi \frac{\cos \psi \cos \varphi}{1 - \sin \varphi \sin \psi} \quad (3)$$

in which  $c^*$  and  $\varphi^*$  are modified parameters of Mohr-Coulomb failure criterion respectively. As a result, the limit analysis theorem can be used in the non-associated flow soil material also. According to equations (2) and (3), non-associated flow rule is introduced by using modified friction angle and cohesion in this paper.

## DISCRETIZATION MODE AND CALCULATION FOR SAFETY FACTOR OF SLOPE

### Strength reduction technique

Strength reduction technique was proposed by Bishop in 1955. The shear strength parameters ( $c^*$ ,  $\varphi^*$ ) are divided by slope safety factor ( $F_s$ ), which is analytically defined as Equation (1), and makes the slope reach a critical state.

$$c_f = c^* / F_s, \quad \varphi_f = \arctan(\tan \varphi^* / F_s) \quad (4)$$