Aware of Borgman's work Paape [27] measured the forces exerted on piles subjected to irregular waves in the laboratory. Spectral densities of force and wave height calculated from his data were not similar and he failed to find a satisfactory transfer function. Thus one must conclude that the spectra may or may not be similar and that Borgman's assumptions oversimplified matters.

It is unfortunate that the designer cannot rely upon similarity between force and wave height spectral densities. For resonance problems he needs to know the frequency at which the force "energy" peaks. Perhaps Paape's tests and others may be used to find how much the frequency at the peak of the force spectrum may differ from the frequency at the peak of the wave height spectrum.

Because wave forces and heights do not correlate, Bretschneider [28] recommends ranking the heights and maximum forces[6] in order of magnitude and working with cumulative probability plots of wave height and force. His idea makes sense because it avoids any inference that the present wave alone is responsible for the present force. Bretschneider developed the idea into a method for analyzing measurements and predicting magnitudes of design forces for statistically similar wave sequences. An alternative method follows.

Suppose we have force and wave height measurements for a particular body shape in a particular irregular sea. How can one arrive at the force that, say, would be equaled or exceeded five percent of the time if a similar sea of greater magnitude acted upon a similar structure? Note first of all that this is a model-prototype kind of problem and one must be able to identify a similar sea. Criteria will appear presently.

We attribute the force at a given instant to the unbalanced shear and normal stresses exerted by the fluid on the body. They are caused by the instantaneous velocities and accelerations of the fluid particles in a rather large region surrounding the object. The motion pattern in the region, of course, is caused by the waves and the presence of the object, but the only index of velocity available is H/T and of acceleration H/T^2. H may be defined as the height of a crest above the surface depressions before and after it and T as the time interval between the two depressions. Both are often a matter of judgment.

Altho these indices represent the particle motion quite inadequately, the dimensionless acceleration H/gT^2, where g is the acceleration of gravity, is certainly a variable on which the force depends. Using γ as specific weight, we may write for a particular wave

$$\frac{F_{max}}{\gamma D^3} = \frac{F_{max}}{\gamma D^3}\left(\frac{h}{D}, \frac{\gamma DH}{g\mu T}, \frac{H}{D}, \frac{H}{gT^2}, \text{History}\right) \quad (11)$$

The history term is a reminder that a range of values of $F_{max}/\gamma D^3$ may accompany each set of values of the other four variables.

---

[6] He separated drag and inertia forces by picking values at wave peaks and zero crossings respectively.

For a given sea state and a particular structure the variables

$$\frac{\gamma DH}{g\mu T}, \frac{H}{D}, \frac{H}{gT^2}, \text{ as well as } \frac{F_{max}}{\gamma D^3},$$

will have permanent probability distributions. That is, if the gage height record may be taken as a stationary time series, the statistical properties of the force record also will be permanent. Hence we may avoid history and hydrodynamics by writing

$$\left\{\frac{F_{max}}{\gamma D^3}\right\} = \left\{\frac{F_{max}}{\gamma D^3}\left(\frac{h}{D}, \left\{\frac{\gamma DH}{g\mu T}\right\}, \left\{\frac{H}{D}\right\}, \left\{\frac{H}{gT^2}\right\}\right)\right\} \quad \text{.......................... (12)}$$

in which the braces are used (unconventionally) to mean the probability distribution of the quantity enclosed.

Let us compare these variables for two cases, model m and prototype p. If the model is built to the proper length scale ratio and the model sea is similar to the prototype sea,

$$\frac{h}{D}\Big|_m = \frac{h}{D}\Big|_p \quad \text{.. (13)}$$

$$\left\{\frac{H}{D}\right\}_m = \left\{\frac{H}{D}\right\}_p \quad \text{.. (14)}$$

$$\left\{\frac{H}{gT^2}\right\}_m = \left\{\frac{H}{gT^2}\right\}_p \quad \text{.. (15)}$$

$$\left\{\frac{\gamma DH}{g\mu T}\right\}_m \neq \left\{\frac{\gamma DH}{g\mu T}\right\}_p \quad \text{.. (16)}$$

Because of Ineq. (16) $\left\{F_{max}/\gamma D^3\right\}_m$ will not be exactly equal to $\left\{F_{max}/\gamma D^3\right\}_p$. In what follows we shall assume that all the Reynolds numbers are large enough and the structure blunt enough to make the inequality unimportant. Then

$$\left\{\frac{F_{max}}{\gamma D^3}\right\}_p \simeq \left\{\frac{F_{max}}{\gamma D^3}\right\}_m \quad \text{.. (17)}$$

and if the right hand probability distribution is known, the probability distribution of $F_{max}\big|_p$ is obtained simply by multiplying the numbers on the abscissa scale by the constant $\gamma D^3\big|_p$ as indicated in Fig. 4. (In the figure the measurements have been grouped into eight class intervals.) Then the distribution of $F_{max}\big|_p$ may be summed to get the cumulative probability of the prototype force and answer the question of what force will be equaled or exceeded five percent of the time.

Similar seas have been assumed. Now we must face the question of how to identify statistically similar seas. The definition itself can vary, depending on why they are defined. We are interested in making Eq. (17) true when surface tension and viscosity are unimportant.

The first requirement is that Eq. (15) be satisfied, but it says nothing about the sequences of changes in wave height. From a history standpoint these sequences have an important influence on the force distribution. Hence $\left\{\frac{H}{gT^2}\right\}$ is a necessary but insufficient index for similar seas and an equation involving differentials such as

$$\left\{\frac{(H_j-H_i)^2/\overline{H^2}}{(t_j-t_i)^2/\overline{T^2}}\right\}_m = \left\{\frac{(H_j-H_i)^2/\overline{H^2}}{(t_j-t_i)^2/\overline{T^2}}\right\}_p \quad \text{.........................(18)}$$

is indicated. Wave j follows wave i.

Moreover, since Eq. (14) scales the structure to the waves it does not pertain to identifying wave similarity only. Hence for sea similarity an equation such as

$$\left\{H^2/\overline{H^2}\right\}_m = \left\{H^2/\overline{H^2}\right\}_p \quad \text{.......................................(19)}$$

is more appropriate. Thus for the present purpose, similar seas probably exist if Eqs. (15), (18), and (19) are satisfied. Experimental data are required to find if they are adequate or not. For example, altho the orientation of the structure with respect to the dominant wave direction presumably is part of the modeling, no account has been taken of the directions of component waves in the description of the sea.

In addition to dimensional analysis and hopefully some physical insight in the choice of dimensionless variables, the writer has used two rather self-evident requirements if force and wave height records from one situation are to be used to find forces in another:

(a) The wave records at the two locations must have certain similarities.

(b) The structure size must be scaled to the wave size, Eq. (14).

These conditions are no different from the ones required in the case of regular waves, Eq. (10), for which wave similarity is implied by a choice of h/D, H/D, and H/gT^2.

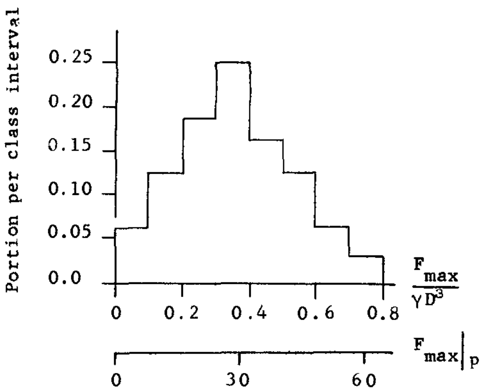

Fig. 4

For regular wave problems engineers use data from tests containing a range of dissimilarities, get a range of coefficients of some sort, and use average coefficient values to calculate approximate forces. Sometimes they calculate extremes to find how much error may be involved.

If similarity requirements are ignored, corresponding errors will occur when statistical quantities, such as probability distributions, are used for analyzing irregular wave and force measurements. To isolate variations due to scale differences, one might well test several sizes of pile, for example, as Wiegel [7] did, but test them simultaneously in the same wave environment.

Bibliography

1. Stokes, G., "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums," Cambridge Trans., Vol. 9, No. 8, 1851 (also Mathematical and Physical Papers, Cambridge Univ. Press, Vol. 3).

2. Yu, Yee-tak, "Virtual Masses and Moments of Inertia of Disks and Cylinders in Various Liquids," Journal of Applied Physics, Vol. 13, Jan. 1942, p. 66.

3. Stelson, T. E., and Mavis, F. T., "Virtual Mass and Acceleration in Fluids," Proceedings, ASCE, Vol. 81, Separate No. 670, April 1955.

4. McNown, J. S., "Drag in Unsteady Flow," Proceedings 9th International Congress of Applied Mechanics, Brussels, 1957, p. 124.

5. Keulegan, G. H., and Carpenter, L. H., "Forces on Cylinders and Plates in an Oscillating Fluid," Journal of Research, National Bureau of Standards, Vol. 60, No. 5, May 1958, p. 423 (also Research Paper 2857).

6. Morison, J. R., et al., "The Forces Exerted by Surface Waves on Piles," Petroleum Transactions, American Institute of Mining, Metallurgical, and Petroleum Engineers, Vol. 189, 1950, TP 2846, p. 149.

7. Wiegel, R. L., et al., "Ocean Wave Forces on Circular Cylindrical Piles," Journal of the Hydraulics Division, ASCE, Vol. 83, No. HY2, April 1957, paper 1199.

8. Harleman, D. R. F., et al., "Experimental and Analytical Studies Of Wave Forces on Offshore Structures, Part I," MIT Hydrodynamics Lab. T. R. No. 19, May 1955.

9. Brater, E. F., et al., "Wave Forces on Submerged Structures," Journal of the Hydraulics Division, ASCE, Vol. 84, No. HY6, Nov. 1958, paper 1833.

10. Grace, R. A., and Casciano, F. M., "Ocean Wave Forces on a Submerged Sphere," Journal of the Waterways and Harbors Division, ASCE, Vol. 95, No. WW3, August 1969, p. 291.

11. Goldstein, S., and Rosenhead, L., "Boundary Layer Growth," Proceedings Cambridge Philosophical Society, Vol. 32, 1936, p. 392.

12. Keim, S. R., "Fluid Resistance to Cylinders in Accelerated Motion," Journal of the Hydraulics Division, ASCE, Vol. 82, No. HY6, Dec. 1956, paper 1113.

13. Laird, A. D. K., et al., "Water Forces on Accelerating Cylinders," Journal of the Waterways and Harbors Division, ASCE, Vol. 85, No. WW1, March 1959, p. 99.

14. Mellsen, S. B., et al., "Real Fluid Effects on an Accelerated Sphere before Boundary-Layer Separation," ASME Paper 66 WA/UNT-6, Annual Meeting Preprint, Nov. 1966.

15. Lai, R. Y. S., and Mockros, L. F., "The Stokes-flow Drag on ... Spheroids ...," Journal of Fluid Mechanics, Vol. 52, Pt. 1, 1972, p. 1

16. Basset, A. B., "On the Motion of a Sphere in a Viscous Liquid," Philosophical Transactions, Royal Society of London, Vol. 179, 1888, p. 43.

17. Hjelmfelt, A. T., and Mockros, L. F., "Stokes Flow Behavior of an Accelerating Sphere," Journal of the Engineering Mechanics Division, ASCE, Vol. 93, No. EM6, Dec. 1967, p. 87.

18. Mockros, L. F., and Lai, R. Y. S., "Validity of Stokes Theory for Accelerating Spheres," Journal of the Engineering Mechanics Division, ASCE, Vol. 95, No. EM3, June 1969, p. 629.

19. Hjelmfelt, A. T., et al., "Dynamic Response of a Restrained Sphere in a Fluid," Journal of the Engineering Mechanics Division, ASCE, Vol. 93, EM1, Feb. 1967, p. 41.

20. Hamilton, W. S., and Lindell, J. E., "Fluid Force Analysis and Accelerating Sphere Tests," Journal of the Hydraulics Division, ASCE, Vol. 97, No. HY6, June 1971, p. 805.

21. Sarpkaya, T., "Separated Flow about Lifting Bodies and Impulsive Flow about Cylinders," Journal of American Institute of Aeronautics and Astronautics, Vol. 4, No. 3, 1966, p. 414.

22. Hjelmfelt, A. T., and Mockros, L. F., "Motion of Discrete Particles in a Turbulent Fluid," Applied Scientific Research, Vol. 16, 1966, p. 149.

23. Lee, T. T., "Research Deficiencies--Wave Force on Piles," Look Lab Hawaii, University of Hawaii, Vol. 1, No. 3, July 1970, p. 38.

24. Odar, F., and Hamilton, W. S., "Forces on a Sphere Accelerating in a Viscous Fluid," Journal of Fluid Mechanics, Vol. 18, Pt. 2, Feb. 1964, p. 302.

25. Garrison, C. J., and Perkinson, B. T., "Wave Forces on a Horizontal Circular Cylinder," Conference Abstracts, Thirteenth International Conference on Coastal Engineering, Vancouver, July 1972, p. 459.

26. Borgman, L. E., "The Spectral Density for Ocean Wave Forces," Coastal Engineering, Santa Barbara Specialty Conference, October 1965, ASCE, 1966, p. 147.

27. Paape, A , "Wave Forces on Piles in Relation to Wave Energy Spectra," Proceedings Eleventh Conference on Coastal Engineering, London, ASCE, Sept. 1968, p. 940.

28. Bretschneider, C. L., "On the Probability Distribution of Wave Force ... Correlation Drag Coefficient ...," Coastal Engineering, Santa Barbara Specialty Conference, Oct. 1965, ASCE, 1966, p. 183.

Notation

A = cross-sectional or wetted area
b = amplitude of a harmonic or periodic motion
C = dimensionless force coefficient
C_A = added mass coefficient--irrotational motion
C_D = velocity coefficient--unsteady motion
C_H = history coefficient
C_M = inertia coefficient
C_V = steady state drag coefficient
D = diameter or other significant length
F = force exerted by fluid
F_{max} = maximum fluid force in repetitive motion or when a wave passes
f = force-variation coefficient
g = acceleration of gravity
H = wave height
h = water depth
i,j = subscripts to indicate one wave and the next
m = mass of fluid displaced by a body; also subscript indicating model
p = subscript indicating prototype
S = displacement during repetitive motion
T = period of a repetitive motion or time interval between waves
t = time at which force is calculated or measured
$\vec{u}$ = velocity of fluid particle
v = speed of body

$\alpha = (2\mu / \rho \sigma D^2)^{\frac{1}{2}}$
γ = specific weight of fluid
μ = viscosity of fluid
ρ = density of fluid
σ = frequency, radians/second
τ = time during history of motion--an integration variable
$\vec{\nabla}$ = space derivative operator

$\{\ \}$ indicate probability distribution of the quantity enclosed

CHAPTER 100

MATHEMATICAL MODELING OF LARGE OBJECTS IN SHALLOW WATER WAVES AND UNIFORM CURRENT

by

Hsiang Wang
University of Delaware, Newark, Delaware

Abstract

A mathematical model is presented which portrays the physical system of a large axially symmetric structure in a flow field of finite water depth, large amplitude wave and strong current. The flow field, which enters as the input, is derived from a velocity potential similar to that of the cnoidal wave of Keulegan and Patterson. The inclusion of a uniform velocity in the derivation of velocity potential results in a cross interference term in addition to the well known Doppler shift effect.

The numerical results are compared with experiments on a bridge pier (Ref. 6) which is partially cylindrical with base diameter equivalent to 100 feet in prototype; close to the surface, where the wave action is greatest it is conical. These results are also compared with theoretical calculations based on linear wave theory and fifth-order wave theory. It is concluded that the results based on the modified cnoidal wave theory come closest to the experimental value.

Introduction

A computer simulation is developed which portrays the physical system of an arbitrarily shaped large structure situated in a flow field where the water depth is finite, the wave is large and the current is not negligible. The structure is large in the sense that its charcteristic length is at least the same order of magnitude as wave length; the wave is large and the water is shallow in that the ratio of wave height to water depth is not infinitesimal. While the presence of a large structure causes wave reflection and diffraction, the existence of a uni-directional current results in the modification of the wave kinematics and, possibly, causes wake formation. As a consequence of large amplitude waves, the convective inertia cannot be neglected. The combination of a large amplitude wave and a large object makes it necessary to compute the wet line around the structure.

The present study is motivated by an earlier experimental work (Wang, 1970). Those laboratory measurements were performed to determine pressures, forces and moments exerted on a large bridge pier. This bridge pier had a cylindrical base and a conical top. When converted from a model into a prototype, such a bridge pier, with a base diameter of 100 feet, would be situated in water 100 feet deep with waves up to 25 feet and current up to 8 knots. The experimental results were later compared with theoretical calculations based on linear wave theory (MacCamy and Fuch, 1954) and on fifth-order wave theory (Clavier, 1967). The comparisons were unfavorable as both theories yielded much too small maximum horizontal forces and moments compared to the experimental values. Since physical situations similar to those described are quite common in engineering, a better predictive technique is, therefore, attempted.

The incoming wave field, in the absence of objects, is first derived from the cnoidal wave of Keulegan and Patterson (1940) incorporated with the effect of a uniform current. The incorporation of a current is not a trivial task as non-linear interaction occurs which results in dispersions of both wave amplitude and wave length.

Since the obstacle is not necessarily in cylindrical shape, the outflows created by the obstacle cannot be expressed in terms of known functions such as Bessel functions of the second kind used by MacCamy and Fuch. A near field wave is sought through Taylor's expansion of wave potential at the obstacle. The outflow potential at the obstacle is then expressed in terms of inflow wave potential and its derivatives normal to the object. The normal derivatives are introduced to fulfill the non-linear free surface condition. Physically, one can reason that the scattering of an incoming wave at a distance should be proportional to the variations of the incoming wave from the distance to the object.

Because of the complicated nature of the problem a computer program is developed using the Burroughs 5500 to facilitate numerical computations of pressure distributions, forces, and moments exerted on the structure.

Incoming Flow

In this section we shall seek a solution for surface waves of finite height superimposed on a uniform current in water of finite depth in the absence of the obstacle. Flow characteristics derived from the wave field will be used as the incoming flow conditions.

General Equations

It will be supposed that the velocity field is irrotational and the fluid is incompressible:

$$\vec{u}' = -\nabla\phi' \tag{1}$$

and

$$\nabla^2\phi' = 0 \tag{2}$$

where $\vec{u}'$ is the velocity vector and ϕ' is the corresponding velocity potential.

A solution of ϕ' will be sought that satisfies the appropriate boundary conditions. Referring to Fig. 1 where the uniform velocity U is oriented into the positive x-direction, we can separate the velocity potential ϕ' into two parts:

$$-\phi' = Ux - \phi \tag{3}$$

where ϕ is the unsteady part of the velocity potential. Then, we have

$$-\phi'_x = U - \phi_x = U + u \tag{4}$$

and

$$-\phi'_y = -\phi_y = v \tag{5}$$

where the subscripts refer to derivatives (as will be used throughout this paper) and where u and v are time-periodic velocities in the x and y directions, respectively.

The boundary conditions to be satisfied are at the surface, i.e., $y = \eta + h$

$$\frac{p}{\rho} = -g(y - h) + \phi_t + U\phi_x - \frac{1}{2}(\phi_x^2 + \phi_y^2) = 0 \tag{6}$$

and

$$\frac{d}{dt}[y - (\eta + d)] = 0 \tag{7}$$

where η is the free surface variation with respect to the calm water and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \nabla\phi\cdot\nabla \tag{8}$$

and at the bottom, $y = 0$

$$\phi_y = 0 \tag{9}$$

For shallow water waves, we adopt for the potential ϕ the power series (Keulegan and Patterson, 1955):

$$\phi = \sum_{n=0}^{\infty} \phi_n y^n, \quad \phi_1 = 0 \tag{10}$$

Substituting this expression for ϕ in Eq. (2), the following series is obtained

$$\phi = \phi_o - \frac{y^2}{2!}\frac{\partial^2 \phi_o}{\partial x^2} + \frac{y^4}{4!}\frac{\partial^4 \phi_o}{x^4} - \frac{y^6}{6!}\frac{\partial^6 \phi_o}{\partial x^6} + \cdots \tag{11}$$

Differentiating with respect to y,

$$\frac{\partial \phi}{\partial y} = -y\frac{\partial^2 \phi}{\partial x^2} + \frac{y^3}{3!}\frac{\partial^4 \phi_o}{\partial x^4} - \frac{y^5}{5!}\frac{\partial^6 \phi_o}{\partial x^6} + \cdots \tag{12}$$

The function ϕ_o is a function of x and t only.

First Order Solutions

If the velocity square terms in Eq. (6) are negligible in comparison with gh and the expansions in Eqs. (11) and (12) are cut short at the first term, wave equations of the first order can be derived. Since (in our case) we are not particularly interested in infinitesimal waves detailed presentations of first order approximation are omitted. It is sufficient to point out that, to the first order, the effect of a uniform stream superimposed on a wave field is the well-known Doppler shift, i.e.,

$$C_1 = U + C_o = U \pm \sqrt{gh} \tag{13}$$

and

$$\omega_1 = \sigma + kU \tag{14}$$

where C is the apparent wave celerity with current
C_o is the wave celerity with no current and is equal to $\sqrt{gh}$ for the shallow water cause
ω_1 is the apparent wave frequency with current
σ is the wave frequency with no current
k is the wave number which remains unchanged with and without the current.

Second Order Solutions

When the ratio of wave height to water depth becomes appreciable, such as the present situation, first order approximation is no longer satisfactory. Second order approximation is sought, therefore. Retaining two terms in Eqs. (11) and (12) and substituting them into Eqs. (2), (6), and (7), the following set of equations are obtained:

$$(g + \frac{UC_o}{h})\eta - \frac{\partial \phi_o}{\partial t} + \frac{g}{2}\left[\frac{\eta^2}{h} + h^2(1 + \frac{U}{C_o})\frac{\partial^2 \eta}{\partial t^2}\right] \doteq 0 \tag{15}$$

and