delays). The task of optimizing the signal timing plan has therefore become a task of determining the state of equilibrium of the corresponding mechanical system.

The determination of the state of equilibrium requires the knowledge of the given structure of the signal plan. The optimum signal timing plan can be found by checking all possible signal plan structures and finding out the optimum structures (stages and stage orders) with respect to the objective function. The possible signal timing plan structures can be listed e.g. according to the procedure by Tully (1976). In this paper, this procedure is not discussed in further detail. The structure of the signal timing plan is considered as predefined.

It can be proven mathematically that the objective function (the potential function U or the delay function W) has exact one minimum, if it is strictly convex in the considered interval (Allsop, 1992). Since the functions which are defined as the potential (delay etc.) during the optimization of the signal timing plan fulfil this condition, it can be assumed that a unique solution for the optimization of the signal timing plan can be found. Also by some non-convex functions the unique optimization can be found (Wu, 1999).

3 Objective Functions and Equilibrium Conditions for Minimization of the Sum of delays

With given traffic volumes and boundary conditions, the objective of the optimization is to obtain the cycle time and the corresponding green times by minimizing the sum of delays over all signal groups. The function for the delay calculation is used as the potential function. The objective function for the minimization of the sum of delays is

$$U_g = \sum_{i=1}^{n} W(q_i.G_i.C) \tag{3}$$

The force function of the signal group F_i (=green time demand B_i), of which the equilibrium is to be found, is

$$F_i = \frac{\partial W(q_i, G_i, C)_i}{\partial G_i} \tag{3b}$$

The stiffness coefficient of the signal group K_i (=M_i) is respectively

$$K_i = \frac{\partial B_i}{\partial G_i} = \frac{\partial^2 W(q_i, G_i, C)_i}{\partial G_i^2} \tag{3c}$$

The parameters in Eq. (3) are

n = number of signal groups
G_i = green time of the signal group i
C = cycle time

$W(q_i, G_i, C)$ $= w(q_i, G_i, C) \cdot q_i$
$=$ delay of signal group i
$w(q_i, G_i, C)$ $=$ average delay per vehicle of signal group i
q_i $=$ traffic volume of signal group i

The boundary conditions for the optimization are
a) $G_i > G_{i,min}$
b) $C_{min} < C < C_{max}$
with
$G_{i,min}$ = predefined minimum green time for signal group i
C_{min} = predefined minimum cycle time
C_{max} = predefined maximum cycle time
and in case of stationary traffic
c) $s_i \cdot G_i > q_i \cdot C$
with s_i = saturation flow of signal group i

If a delay formula defined for temporary over-saturations (e.g. Akcelik, 1980 or Wu, 1990) is used, the boundary condition c) can be omitted.

Furthermore,
d) all boundary conditions which are necessary to ensure safe traffic operations, e.g. intergreens, restricted overlap of green time for permitted left turns, parallel pedestrian crosswalks with permitted right or left turners etc. (for German conditions cf. FGSV, 1992, 2003)
must be hold.

All common delay formulae (Webster, 1958; Miller, 1968; Akcelik, 1980; Kimber and Hollis, 1979; Wu, 1990 etc.) for $W(q_i, G_i, C)$ fulfil the convex condition over the interval $(0,C)$ because

(1) $$\frac{\partial W_i(q_i, G_i, C)}{\partial G_i} < 0$$

(2) $$\frac{\partial^2 W_i(q_i, G_i, C)}{\partial G_i^2} > 0$$

(3) $$\frac{\partial^2 W_i(q_i, G_i, C)}{\partial C^2} > 0 \ .$$

The optimization of the objective function (Eq. (3)) can be carried out for the optimum distribution of green times $G_{i,opt}$ with a fixed cycle time C. In addition, also the cycle time C can be optimized.

4 Determination of the Equilibrium for the Optimization Procedure

According to the principle of virtual work from Dirichlet (cf. Lehmann, 1979)

A mechanical system is in its equilibrium only if the virtual change of the total potential for any virtual displacements is equal to zero, that is

$$\delta U_g = \Sigma \delta U_i = 0$$

For a mechanical system with n degrees of freedom, Dirichlet's theorem corresponds to

$$\frac{\partial U_g(x_i)}{\partial x_i} = 0 \quad i = 1 \text{ to } n \tag{7}$$

with x_i = coordinates of the i-th degree of freedom

Eq. (7) is nothing else but the condition of equilibrium of the conservative forces. If all forces F_i of this mechanical system are proportional to the coordinate x_i - as in a real spring system - a solution for the equation system Eq. (7) can be found for every force F_i and all coordinates. Unfortunately, the virtual forces (green time demand B_i) of a signal timing plan with Eq. (3), (4) and (5) as the potential function (objective) do not fulfil this condition of linearity. Most of the delay formulae are functions of higher orders. Some even contain transcendental functions. An analytical solution of Eq. (7) is therefore very difficult or even impossible to find. Up to now, optimization procedures with the sum of delays as the objective function have been carried out only numerically with an enormous calculation effort.

The state of equilibrium of a mechanical system can also be ascertained iteratively. One of the well-known procedures is the moment distribution method from Cross (cf. Beaufait, 1972) which is based on the stiffness method. This procedure can be explained by Figure 5a.

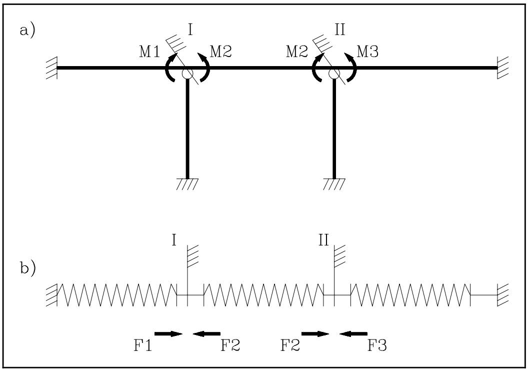

Figure 5. Principle of the moment and force distribution method

Figure 5a shows the structure of a bridge. The bending moments of the beam near the columns are to be determined. The degrees of freedom of this mechanical system are the slope deflections of the beam at both columns. First, these two slope deflections are fastened in their original state by fictitious constraints. Putting weight on the beam causes a surplus of bending moments (*M2-M1, M3-M2*), which is fictitious as well. The bending moments are in a state of imbalance. To determine the state of equilibrium, the constraints are released alternately, i.e.:

(1) Constraint *I* is released first while constraint *II* stays fastened. Constraint *I* turns as an effect of the surplus of the bending moments *M2-M1*. When this surplus is distributed, the bending moments at constraint *I* have achieved the state of equilibrium again. The constraint *I* is fastened again. As a consequence of the turning movement at constraint *I*, constraint *II* gets a new surplus of bending moments.

(2) Then constraint *II* is released while constraint *I* stays fastened. Constraint *II* turns as an effect of the surplus of the bending moments *M3-M2*. When this surplus is distributed, the bending moments at constraint *II* achieve their state of equilibrium. Constraint *II* is fastened again. As a consequence of the turning movement of constraint *II*, constraint *I* gets a new surplus of bending moments.

(3) Steps 1 and 2 are repeated.

The bending moments at the constraints are alternately set into the state of equilibrium. The surplus of the bending moments decreases as the number of iterations increases. The iteration is stopped when the surplus of the bending moments has become small enough for practical application at all constraints. This method is also called the moment-distribution procedure. It can be used for a system with any number of constraints. All constraints are repeatedly released and fastened until all of them have achieved the state of equilibrium.

The state of equilibrium of a mechanical spring system can be determined analogously (cf. Figure 5b). Instead of the surplus of the bending moments M_i, the surplus of the spring forces at the connections F_i is distributed. The connections are first fastened and then released alternately (which causes horizontal displacements). Then they are fastened until the surplus of the spring forces (*F2-F1, F3-F2*) falls below a certain minimum at the connections. This procedure can be called the force-distribution procedure.

The force-distribution procedure does not start with the determination of the state of equilibrium for the total system, but with that of the individual constraints. In a signal timing plan as an analogy to a system of springs, the constraints are clearly defined by the restrictions of the signal timing plan (intergreens, permitted left turners or minimum green times etc.). Thus, a system with n springs (analogue to n signal groups) is simplified as a system with m constraints. In most of cases, m is always smaller than n. The example in Figure 4 has n = 6 signal groups, but only m = 3 constraints if the cycle time is fixed. If the cycle time C is regarded as a variable as well, this example has m = 4 constraints.

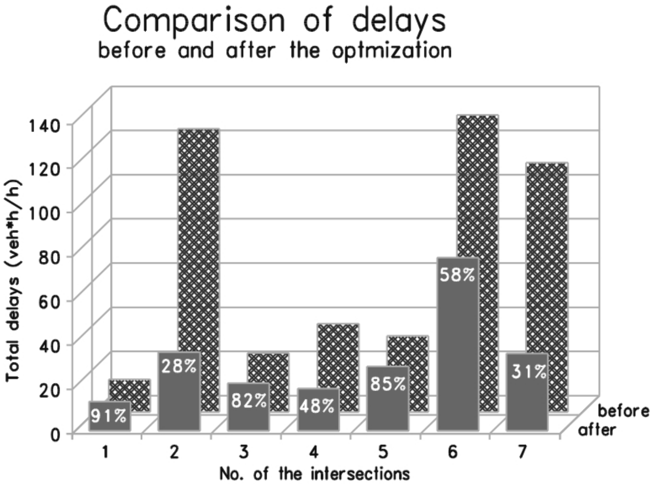

Figure 6. Optimization results for 7 intersections

The new procedure was tested at 7 intersections in Düsseldorf, Germany. The local authorities provided the existing signal timing plans. The existing signal times were used as a basis for the optimization. The result is presented in Figure 6. It shows that the signal timing plans, some of which were several years old and for which the traffic volumes have undoubtedly changed in the meantime, need to be improved. The intersections 2 and 7 were controlled by so-called advanced signals. Here, the green times of some movements were intentionally restricted to limited capacity (capacity < demand) with the purpose of keeping congestion out of the city area. Therefore, the before/after comparison is irrelevant for these two intersections. For the other 5 intersections, improvements could be achieved. The significant improvements at intersections 4 and 6 are results of overloaded left turning traffic streams.

5 Conclusion and outlook

The optimization procedure for a signal timing plan according to the principle of equilibrium has the following advantages compared to the existing procedures:

- Fast computation due to reduction of variables (in place of number of signal groups *n* the number of constraints *m* is used). An optimization procedure for a fixed-time controlled standard intersection normally takes less than 1s.
- Minimum work before the optimization (only the signal timing plan and the corresponding intergreens and the traffic volumes are needed).
- Applicable for any convex objective functions.

- Continuity of the signal timing plan during the optimization (the signal timing plan is not produced totally anew but is only developed in an innovative way).
- Dynamic character (adaptation to changed traffic volumes, minimum green times etc. is possible during optimization).
- The procedure can be influenced manually (new definition of green times, reduction of cycle time etc.).

Regarding the general theoretical background, this procedure can also be easily transferred to the following traffic control strategies:

- Signal timing control with consideration of pedestrian delays
- Signal timing control with special stages for public transport
- Adaptive signal timing control
- Coordinated signal timing control in networks

In the future, the new model will be formulated in details.

References

Akcelik, R. (1980). Time-dependent expressions for delay, stop rate and queue length at traffic signals. Australian Road Research Board, No. 361-1, 10/1980.

Allsop, R.E. (1971). Delay-minimising settings for fixed-time traffic signals at a single road junction. Journal of the Institute for Mathematics and his Applications, 8 (2), 164-185.

Allsop, R.E. (1992). Evolving Application of Mathematical Optimisation in Design and Operation of Individual Signal-Controlled Road Junctions. In: Mathematics in Transport Planning and Control, edited by J.D. Griffith, Clarendon Press, Oxford.

Beaufait, F.W. (1972). Basic Concepts of Structural Analysis. Prentice-Hall, Inc. Englewood Cliffs, New Jersey 07632.

Brilon, W. and Wu, N. (1990). Delays at fixed-time traffic signals under time-dependent traffic conditions. Traffic Engineering and Control. 12/1990.

FGSV (1992, 2003). RiLSA - Guidelines for Traffic Signals - English Version of with minor Modifications. FGSV-Verlag, Köln.

Kimber, R.M. and Hollis, E.M. (1979). Traffic queue and delays at road junctions. TRRL Laboratory Report, LR 909.

Lehmann, Th. (1979). Elemente der Mechanik IV: Schwingungen, Variationsprinzip. Friedr. Vieweg & Sohn, Braunschweig/Wiesbaden 1979.

Miller, A.J. (1968). Signalised intersections - capacity quite. Australian Research Board Bull, No. 4/1968. Reprinted as ARRB Research Report ARR No. 79/1978.

Möller, K. (1987). Signalgruppenorientiertes Modell zur Optimierung von Festzeitprogrammen an Einzelknotenpunkten. Schriftenreihe des Instituts für Verkehrswesen der Universität Karlsruhe, Heft 37.

Rose, R. (1972). Foundations of Mathematical Biology. ACADEMIC PRESS, New York and London.

Silcock, J.P.; Sang, A. (1990). SIGSIGN: a phase-based optimisation program for individual signal-controlled junctions. Traffic Engineering and Control, 5/1990.

Tully, I.M.S.N.Z. (1976). Synthesis of sequences for traffic signal controllers using techniques of the theory of graphs. OUEL Report 1189/77, University of Oxford.

Webster, F.V. (1958), Traffic signal settings. RRL Technical Paper No. 39, HMSO, London.

Wu, N. (1990). Wartezeit und Leistungsfähigkeit von Lichtsignalanlagen unter Berücksichtigung von Instationarität und Teilgebundenheit des Verkehrs (Delay and Capacity at Signalised Intersections under non-stationary and partly-bunched traffic conditions). Schriftenreihe des Lehrstuhls für Verkehrswesen der Ruhr-Universität, Heft 8 (A short English version is given in Brilon, Wu, 1990).

Wu, N. (1999). Optimierung von Signalzeitenplänen nach dem Gleichgewichtsprinzip (Optimization of Signal Timing by Equilibrium Technique). Schriftenreihe des Lehrstuhls für Verkehrswesen, Heft 20, Ruhr-Universität Bochum.

Simulation Research about the Number of Lines in a Passenger Rolling Stock Servicing Yard on ARENA

ZHOU, Lixin[1] and YANG, Xiongbin[2]

[1]Transportation Engineering School of Tongji University, Shanghai 201804, China; PH (021) 68589372; FAX (021) 69599372; email: lxzhou@mail.tongji.edu.cn
[2]Transportation Engineering School of Tongji University, Shanghai 201804, China; PH (021) 69589171; FAX (021) 69589372; email: yxb1983@gmail.com

Abstract: At the design phrase,it is difficult to determine a reasonable number of tracks of passenger rolling stock in a servicing yard. This article focuses on a new idea of simulation technology to find a solution. Using ARENA software, a simulation model for handling passenger train maintenance and overall preparation services is established on the basis of a fixed position working procedure. Several modules are developed and statistical indexes are designed. Finally, an operational evaluation of the design of a 14 track yard is made with a practical case of the passenger rolling stock servicing yard of Shanghai-South Railway Station. Fifteen or more tracks should be equipped with suitable service groups to synchronize all work.

Key words: *passenger rolling stock servicing yard, track, simulation, Arena*

1 Introduction

Passenger rolling stock servicing yards (briefly named "PSY") are important parts of the infrastructure for guaranteeing the safe operation of passenger trains. The main functions of a PSY are as follows:

(1) Technical checking, maintenance, cleaning, etc. for serviceable carriages in advance;

(2) Service preparation for departing passenger trains;

(3) Provision of a temporary stop yard (tracks) for carriages belong to the passenger rolling depot.

The number of tracks designed for the above three tasks in a PSY is the main factor to decide the scale of a PSY. This problem is related to the number of trains in the rolling stock depot, the procedures in the PSY (moving or fixed position working procedure), the location of the PSY relevant to a passenger station and a passenger rolling stock depot, the timetable of passenger trains, etc. The latest "Railway Station and Terminal Design Criterion" (GB-50091-2006) does not give a clear number of tracks in a PSY. Though the "Rail Engineering & Techniques Manual" (No 4.Railway Design and Research Institution, 2004) has listed a table based on train pairs in/out of a PSY (given in Table 1), the reference values do not agree with the actual circumstance. For instance, in the PSY equipped for the Shanghai railway station, the daily in/out train pairs are up to 100, and the existing number of tracks in the PSY is only 16, which is far lower than the recommended value in Table 1.

Table 1. Reference number of tracks in passenger rolling stock yard

Trains in/out(pairs per day)	6～7	8	9～10	11～13	14～15	16～18	19～22	23～26	27～29	30～31	32～34	35～37	38～40	41～43	44～46	47～49	50～52
Number of tracks	5	6	7	8	9	10	11	12	13	14	15	17	17	18	19	20	21

A reasonable design scheme should provide the minimum number of tracks for the technical checks and service tasks of passenger trains without delay. On the other hand, any ineffective investment derived from an excess number of tracks should be avoided. In addition to the technical checking of a train, various operation schedules of passenger trains also have an important impact on the take-up time of the tracks. It is very difficult to decide the scale of a passenger rolling stock depot with a simple calculation formula.

Because PSYs differ in the number of tasks as well as in the content and demand of tasks, it is fruitless to determine the number of tracks in a PSY only according to Table 1. Therefore a new idea with a simulation technique is discussed in the paper to seek for a valid solution to this kind of problem. A case-study of the Shanghai-South railway station is studied in detail in this paper.

2 Model Conceiving

2.1The technical task in PSY

There are various types of rolling stock belonging to a depot and the different carriages may have different task procedures. Generally, they could be divided into three types based on technical procedures: [1]

(1) Type A train

A train does not require a check of running parts of carriages with a ditch and maintenance for air-conditioning package, e.g. type-22 carriages.

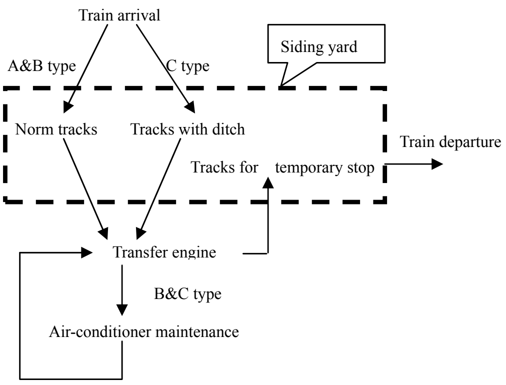

Figure 1. An operations chart of a PSY

1 Here we simply define any carriage groups in PSY as trains.

(2) Type B train

A train requires maintenance for air-conditioning except checking running parts of carriages with a ditch, e.g. type 25K and 25G carriages.

(3) Type C train

Both tasks above are necessary for a train, e.g. type BSP carriages.

For fixed position working procedure, entering PSY trains are hauled into different siding tracks according to their types for checking running parts and equipment, and a check of the appearance of carriages or maintenance of other devices. For example (shown in Figure1）, type A and B trains are first hauled into norm tracks; type C trains are hauled into tracks with ditches which are supplied with the equipment for checking the running parts of carriages. For air-conditioning package maintenance, type B and C trains are then transferred onto an air-conditioner maintenance shed. Finally, all of them are transferred onto a temporary stop yard waiting for departure.

2.2 Modeling Principle

The tasks of a train in/out of a PSY are special technical services. If we refer to the trains as "customers", and the checking groups, tracks and maintenance groups as a "service station", then the service procedure of the train can be abstracted as a typical queuing system. The characteristics of this system are as follows:

(1) The headways of arrival time of trains are asymmetric.

(2) The time used for check and maintenance of each train is not the same.

(3) The time-consuming length to use a track is uncertain.

Under the comprehensive impact of the above three random factors, there likely is a train queuing phenomenon in PSY which could seriously influence the entrance of the arrival trains from a passenger station. Considering the construction cost, it is the core goal of PSY design to decrease or ultimately eliminate train queuing time as far as possible within a limited investment scale.

2.3 Model Design

2.3.1 The Steps of Modeling Based on Arena

Arena is object-orientation simulation software. Aimed at the requirement of process simulation of PSY work, the main steps of modeling are as follows:

(1) Affirm the makeup and number of trains arriving at a PSY per day.

(2) Investigate the service station amount and service procedures according to Figure1.

(3) According to actual investigation and statistics analysis method, define the arrival interval of train flow, the probability distributions of each service station's work time and corresponding confidence interval.

(4) Establish Arena model and set up the service parameters of corresponding service stations. For different procedures (such as type A, B and C trains), the