Four cases are considered as listed in Table 5. For the first three cases, the thickness D_1 , D_2 or D_3 are increased by one inch separately for each case. For the last case, all thicknesses D_1 , D_2 and D_3 are increased by one inch simultaneously. The design point, reliability index and reliability probability for each case are also listed in Table 5. It can be seen that, among the first approaches, increasing the thickness D_1 of surface material layer by one inch is most effective since the performance function g_w is most sensitive to the D_1 in terms of variance. With new D_i (i=1,2,3), the PDF of $\log W_t$ shift to right hand side as shown in Figure 2(a). For this case, the PDF of $\log W_T$ is kept the same as before since the traffic prediction is kept the same. Consequently, the joint PDF of g_w shifts to the safe zone as illustrated in Figure 2(b); and this joint PDF along the failure plane decreases as shown in Figure 2 (a), resulting in a smaller probability of failure, or larger probability of reliability.

Case	Change	Original	New	DP	β	p _r	p_f
1	D_1	8	9	7.586	1.955	97.5%	2.5%
2	D_2	7	8	7.566	1.650	95.1%	4.9%
3	<i>D</i> ₃	11	12	7.558	1.551	94.0%	6.0%
4	Increase D_i (\models 1,2,3) by 1 in			7.61	2.288	98.9%	1.1%

Table 5. Probabilities of reliability with different pavement layer thicknesses



Figure 2. Probability density functions of traffic data and pavement performance functions (a) and contour view of joint PDF of g_w with failure plane with new D_i (i=1,2,3) (b)

CONCLUSION

The reliability index and design point of geotechnical design problems are related to the statistical moments (i.e., mean and variance) of input variables. Such relationships can be derived from the prospective of the expanding ellipsoid and the joint probability density function of a performance function along its failure surface. They can be expressed in the analytical forms, given that the performance function and input variables have normal probability distributions. The flexible pavement design example indicates that these analytical expressions can be used in practical problems to evaluate different design alternatives in an effective and prompt way. It should be noted that the analytical expressions for reliability index and design point are derived for the performance functions with normal probability distributions and normally distributed input variables. Attentions need to be paid on using these expressions for performance functions with probability distributions other than the normal, since they may produce different results as predicted by Monte Carlo simulations.

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Calibration of Safety Factors for Piping Failure Mechanism in Levees

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Abstract

The Dutch flood defences are periodically tested against statutory safety standards defined in terms of maximum allowable probability of flooding. As such, a set of tools for assessing the safety of flood defences was developed (project WBI-2017), which includes probabilistic as well as semi-probabilistic assessment procedures. To ensure consistency between probabilistic and semi-probabilistic assessments, safety factors had to be calibrated considering the new models and uncertainties involved. Important aspects for this calibration were the derivation of the safety requirements, the definition of design values, and the handling of spatial correlations and multiple failure mechanisms. This paper presents the framework and outcome of such calibration for the piping failure mechanism in levees. Assumptions and choices to achieve the main objectives of the calibration's framework are here discussed, with emphasis placed on the safety format. Lastly, an example of the assessment procedure is also presented.

INTRODUCTION

The Dutch primary flood defences are periodically tested against statutory safety standards. The issue of flood risk management is of particular importance for the Netherlands, since 66% of the country is prone to flooding. Major areas, including Amsterdam and Rotterdam cities, are protected by a system of primary flood defences of a length of almost 3,800 km (MVW 2017; Jonkman and Schweckendiek 2015). Over the past decades significant progress has been made in developing techniques for risk and reliability analysis of flood defence systems in the Netherlands. For example, from 2017, safety standards are defined in terms of maximum allowable probabilities of flooding (Jonkman et al. 2011; Jonkman and Schweckendiek 2015) ranging from 1/100,000 to 1/300 per year for a particular segment. Segments can include stretches of levees and dunes together with hydraulic structures. Each one of these has different possible failure modes. In the case of levees (soil structures, also referred as dikes in the Netherlands), the subject of this paper, the longitudinal length has an important role in the final probability of failure due to soil spatial variability.

In the latest years, a set of instruments for assessing the safety of flood defences was developed, denominated 'Wettelijk beoordelingsinstrumentarium 2017', i.e. 'Research and development of Flood Defence Assessment tools 2017'. This project includes probabilistic as well as semi-probabilistic assessment procedures. The latter rests on a safety factor approach and allows engineers to evaluate the reliability of flood defences without having to perform a probability calculus. To ensure consistency between probabilistic and semi-probabilistic assessments, the safety factors had to be calibrated. Important aspects within this calibration concern the derivation of the reliability requirements, the definition of design values on the basis of influence coefficients, and the handling of spatial correlations (Jongejan and Calle 2013; Jongejan 2016). In summary, this paper presents the framework, on which such calibration is based (schematised in Fig. 1) and focuses on the outcomes for piping failure mechanism in levees. Herein, piping mechanism is considered as a parallel system of the sub-mechanisms uplift, heave and backwards erosion. Assumptions and choices are presented and discussed in order to achieve the main objectives of the framework, such as (a) establishing the reliability requirement accounting for the failure probability budget assigned to a specific failure mechanism, the length effect and the treatment of schematization uncertainties and (b) establishing the safety format in terms of the envisaged characteristic values and safety factors to be applied. This paper extends the state-of-the-art given in current standards and provides an introduction to the reliability of flood defence systems. This forms the basis for a more precise consideration of local conditions and uncertainties, which offers a sound basis of the optimization and cost-effective design of flood defences.



Figure 1. Schematic overview of the calibration procedure, Jongejan and Calle (2013).

FAILURE MECHANISM AND UNCERTAINTIES

The piping mechanism in levees can be split up into three processes: uplift, heave and internal erosion, connected by an AND-Gate. This means that piping failure can only take place if uplift and heave occur in the first place. Here, uplift refers to the uplift of the cover layer causing rupture/crack, heave refers to the movement of the grains inside the crack and finally piping occurs with the backwards internal erosion (Sellmeijer 1988; van Beek 2015). This type of

failure is, in principal, caused by excessive pore pressures. This pore pressures can develop in sand layers due to high (river) water levels. The piezometric head difference over a levee determines the load on the levee, while the resistance of the levee depends on several soil characteristics such as cover layer thickness and weight, permeability of the sand layer and the available seepage length. An overview of relevant parameters for piping and its definitions is given in Fig. 2 and Table 1. The associated uncertainties in the piping mechanism include: the different soil parameters, the spatial variability, the hydraulic loads, the theoretical approaches used to model the failure mechanism's behaviour and also the geometry of the problem (e.g. soil layering and seepage length). Uncertain parameters are here called random variables; these are defined in Table 1 – more in Schweckendiek (2014), van Beek (2015) or Teixeira et al. (2015). Random variables in Table 1 represent the epistemic uncertainty (knowledge), which is directly included in probabilistic analyses. Other type of uncertainty, i.e. aleatory uncertainty, can be taken into account with sub-soil scenarios, each having a certain likelihood. Usually, one sub-soil scenario represents a certain layering schematisation.



Figure 2. Definitions of geometrical properties, phreatic and piezometric levels for piping.

SAFETY STANDARDS AND LENGTH EFFECT

A segment is a part of a flood defence (levees, dunes and/or hydraulic structures), for which a breach would have approximately the same impact/consequence. In the Netherlands, nearly 190 segments are distinguished, which can be over 20 km long (Slomp et al. 2016). In this paper, segments consisting of levees (only) are considered. The safety standard of a segment is defined in terms of maximum allowable probability of flooding. In the national flood safety assessment, a segment is evaluated as 'safe' when the flooding probability of the segment is lower than the maximum allowable probability of flooding set by the standards; otherwise the segment is assessed as 'unsafe'. The safety assessment of a segment is performed per failure mechanism and it is based on a bottom-up approach, in which representative cross sections of levees within the segment are evaluated first. Each failure mechanism may lead to flooding (the fault tree's top event). A detailed explanation of this system behaviour/evaluation is given in TAW (1999), Jonkman and Schweckendiek (2015) or Slomp et al. (2016). The maximum acceptable failure probability for each failure mechanism is defined such that the combined value does not exceed

the safety standard (Jongejan and Calle 2013). The maximum contributions to the probability of flooding per failure mechanism range from 0.02 to 0.3, being 0.04 for macrostability and 0.24 for piping. These are based on statutory prescribed importance of different failure mechanisms to the total failure probability. As an example, if the safety standard of a levee segment is 1/3,000 per year, then the safety requirement for piping failure is $0.24 \times 1/3,000$ per year.

Table 1. Variables and uncertainties taken into account in piping failure mechanism (det = deterministic, norm = normal, log = lognormal, shifted-log (+10) = shifted lognormal with shift 10 μ = mean, σ = standard deviation, CoV = coefficient of variation, t.s. = test set).

Symbol	Unit	Description	Distribution type	Default	Characteristic value			
General variable								
γ _{water}	[kN/m ³]	Volumetric weight of water	det	10	10			
v_{water}	$[m^2/s]$	Kinematic viscosity of water	det	1.33 x 10 ⁻⁶	1.33 x 10 ⁻⁶			
g	$[m/s^2]$	Gravitational constant	det	9.81	9.81			
h	[m+NAP]	Outside water level	Hydra-Ring**	t.s.	Design water level*			
h _{exit}	[m+NAP]	Phreatic level at the exit point	norm	t.s.	5 th -percentile			
D _{cover}	[m]	Total thickness of the cover layer	log	t.s.	5 th -percentile			
Uplift								
m_u	-	Model factor for uplift	log	μ 1.0 ,σ 0.10	1.0			
Ysat,cover	[kN/m ³]	Saturated volumetric weight of the cover layer (cover)	Shifted-log (+10)	t.s.	5 th -percentile			
<i>r</i> _{exit}	-	Damping factor at exit	log	t.s.	5 th -percentile			
	Heave							
<i>i</i> _{c,h}	-	Critical heave gradient	log	μ 0.5 ,σ 0.10	0.3			
	1	Internal eros	ion	ſ	1			
m_p	-	Model factor for piping	log	μ 1.0 ,σ 0.12	1.0			
L	[m]	Seepage length, from entry point to exit point	log	t.s.	5 th -percentile			
d_{70}	[m]	70%-quantile of the grain size distribution of the piping-sensitive layer	log	μ t.s., CoV 0.12	5 th -percentile			
k	[m/s]	Darcy permeability	log	μ t.s., CoV 0.50	95 th -percentile			
D	[m]	Thickness of aquifer	log	t.s.	95 th -percentile			
r _c	-	Head reduction factor	det	0.3	0.3			
η	-	White's drag coefficient	det	0.25	0.25			
Ysub.particles	[kN/m ³]	Submerged volumetric weight of sand particles	det	16.5	16.5			
$d_{70.m}$	[m]	Mean value of the d_{70} in small scale tests	det	2.08 x 10 ⁻⁴	2.08 x 10 ⁻⁴			
$ heta_{sellmeijer.revised}$	[°]	Bedding angle of sand grains for the revised Sellmeijer rule	det	37	37			

*Design water level is defined as the water level with an exceedance probability equal to the maximum allowable probability of flooding of a levee segment.

**Hydra-Ring is the state-of-the-art software, developed by Deltares, for modelling and assessing water level conditions and reliability analysis of levees in the Netherlands (Slomp et al. 2016).

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Levees are long structures, whose performance is strongly influenced by the effects of spatially correlated loads and resistances. The effects of the spatially correlated resistances (i.e. soil properties) are considered via the length-effect, which is defined as the increase of the failure probability with the length of the levee (Kanning 2012). Due to the length-effect, the safety requirement of a levee cross section is more stringent than the safety standard of the levee segment. The general relation between the safety requirement of a levee cross section $P_{T,cross}$ [1/year] and the safety standard of a levee segment P_{norm} [1/year] for piping is defined in Eq. (1), in which the length-effect is described by *a* and *b*. The parameter *a* [-] represents the fraction of the length that is sensitive to piping and *b* [m] is the measure for the intensity of the length-effect within the part of the levee segment that is sensitive to piping (the length of independent, equivalent sections). Furthermore, *T* [year] is the return period and $L_{segment}$ [m] is the total length of the levee segment. According to OI (2015), the recommended value of *b* is equal to 300 m, whereas, the recommended values of *a* are: 0.9 for the upper-river region and 0.4 for the remaining hydraulic regions in the Netherlands.

$$P_{T,cross} = \frac{f \cdot P_{norm}}{\left(1 + \frac{a \cdot L_{segment}}{b}\right)} \quad with \quad P_{norm} = 1/T$$
(1)

CALIBRATION PROCEDURE AND FINDINGS

Below, the calibration procedure is described following the workflow from Fig. 1. Each of the steps involved is explained based on Jongejan and Calle (2013). Note that the procedure is applied per sub-mechanism. The outcomes of the calibration for piping mechanism (Teixeira et al. 2015) are described along with the steps.

Step 1. Establish the piping reliability requirement. In this step, we establish the segment maximum acceptable failure probability concerning piping failure: $P_T = 0.24 \cdot P_{norm} = 0.24 \cdot 1/T$.

Step 2. Establish the safety format. To establish the safety format, a test set needs to be defined first. The test set needs to represent a wide range of inputs (i.e. different load and subsoil conditions) and can concern existing or fictitious cases. Then, the description of the semi-probabilistic rule together with a study on the FORM (First-Order Reliability Method) influence coefficients using the test set, helps with the decision on characteristic values and partial safety factors. So, for each test set member, the corresponding FORM results, consisting of reliability indices (β), design point values (X_d) and influence coefficients (α -values), are used to decide on: characteristic values of random variables (X_k) and the partial safety factors. As an example, characteristic values of random variables with negative α -values¹ are typically defined as 95%-

¹ A random variable with a negative α -value is usually called a load variable. This is because higher values of this variable increase the failure probability. On the other hand, a positive α -value is usually called a strength variable.

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percentiles of these variables. Table 1 presents the characteristic values of all relevant variables. For more information on the choice of the characteristic values see (Teixeira et al. 2015).

Concerning the partial safety factors ($\gamma_X = X_k/X_d$), random variables with high influence coefficients are good candidates for having a partial safety factor different than one. In an ideal situation, partial safety factors are defined for all random variables with high influence coefficients, so that the semi-probabilistic rule corresponds well to the fully probabilistic assessment. However, when not all variables have their dedicated partial safety factor, the assessment is simplified and therefore preferable from the user point of view. Taking this into account, and also other studies (e.g. Lopez de la Cruz et al. 2011), it has been chosen to take all partial safety factors (γ_X) equal to 1.0 in the piping assessment. As such, finding a value of an overall safety factor, the β -dependent safety factor (γ_β), is the subject of the calibration exercise.

Step 3. Applying the calibration criteria. The aim of this step is to establish a functional relation between the β -dependent safety factor (γ_{β}) and the reliability index β (called further the γ - β relation(s)). This involves both probabilistic and semi-probabilistic computations and the application of a calibration criterion. Given a value of the β -dependent safety factor, e.g. [0.5, 1.0, 1.2,...], and the safety format established in step 2 (X_k and γ_X), a design is made. The design variable is the seepage length (for internal erosion) or the thickness of the cover layer (for uplift and heave). Note that, for each design, the condition $\gamma_{\beta}=R_d/S_d$ is met. This means that each designed test set member would just pass the semi-probabilistic assessment. Subsequently, for each designed test set member, (FORM) probabilistic computations are performed entailing reliability indices that correspond to the β -dependent safety factor. Hence, the result is a γ - β scatter (not yet in a functional form). To obtain the γ - β relation, the following is applied: recall that a levee segment fulfils the safety assessment, if the flooding probability of the segment is lower than the maximum allowable probability of flooding (i.e. safety standards). This principle is satisfied, with a sufficient accuracy, when the average of the cross sectional failure probabilities in the segment is smaller than the cross sectional safety requirement ($P_{T,cross}$). The average of cross sectional failure probabilities is therefore used for the calibration of the safety factors. This average value roughly corresponds to the 20%-quantile values of the calculated reliability indices (Jongejan and Calle 2013) – see Fig. 3(a).

In this study, the γ - β relations (per sub-mechanism) were achieved using a test set consisting of almost 3,000 members. This test set originates from the VNK2-study, the National Flood Risk study of 2010 (Jongejan et al. 2011), and represents different sub-soil conditions and water systems in the Netherlands. An individual test set member, located in a certain water system, consists of a realistic levee condition for which we have all the input – as in Table 1. In most studies of this kind, the obtained reliability indices vary greatly per safety factor (Kanning et al. 2017). Furthermore, it is expected that the γ - β relations are clustered (i.e. one relation for test set members having some common feature). As example, the study of Lopez de la Cruz et al. (2011) has shown that the assessment rule behaves differently in different hydraulic load regimes and different cover layer thicknesses. In the present calibration the differentiation was made with respect to safety standard (P_{norm}) – Fig. 3(c). The calibration results are shown in Fig. 3.



Figure 3. Calibrated rules for piping mechanism assessment (Teixeira et al. 2015).

APPLICATION OF THE SEMI-PROBABILISTIC SAFETY ASSESSMENT

The goal of an assessment is herein to compare the target safety ($\beta_{T,cross}$) with the estimated cross section safety (β^*_{cross}) – see Eq.(2). All variables with a '*' are case-specific computed values, while variables with index 'T' are referring to target (based on P_{norm}) values.

$$\beta_{cross}^* \ge \beta_{T,cross} \Leftrightarrow P_{cross}^* \le P_{T,cross} \quad with \quad \beta_{cross}^* = -\Phi^{-1}(P_{cross}^*)$$
(2)

Recall that all three sub-mechanisms have to occur before a levee fails due to piping. Having several sub-soil scenarios, the semi-probabilistic piping assessment is carried out per sub-soil scenario accounting for all sub-mechanisms. Then, the overall result is obtained by combining the results from all sub-soil scenarios. Based on the aforementioned, one should follow the procedure below. To illustrate this, an example is also presented. It is assumed that the levee cross section is situated in a levee segment with the safety standard of 1/T = 1/100,000 per year and there are n = 2 sub-soil scenarios, with probabilities of occurrence of $P(S_1) = 20\%$ and $P(S_2) = 80\%$, respectively. Due to space restrictions, only uplift sub-mechanism is presented in detail. The example's uplift inputs are defined in Table 3.

X	Unit	Distribution type	Parameter values	X_k
h	[m+NAP]	Hydra-Ring	-	h(T) = 3.73
h _{exit}	[m+NAP]	norm	μ -1.0 ,σ 0.10	-1.16
D _{cover}	[m]	log	sce.1: μ 5.5 ,σ 0.27 sce.2: μ 6.9 ,σ 0.52	sce.1: 5.06 sce.2: 6.08
m_u	-	log	μ 1.0 ,σ 0.10	1.0
Ysat,cover	[kN/m ³]	Shifted-log (+10)	μ 16.5 ,σ 0.08	16.37
<i>r</i> _{exit}	-	log	μ 0.35 ,σ 0.0035	0.34

 Table 3. Uplift example: input variables and uncertainties (sce. = scenario)

Piping semi-probabilistic assessment procedure. Considering a segment, the procedure points **a**) to **d**) refer to the estimation of the failure probability of the correspondent levee cross section, whereas point **e**) refers to the assessment itself, where the estimated/occurring '*' and required '*T*' failure probabilities (or reliability indices) are compared – Eq.(2).

For the segment under study:

- a) derive the outside design water level with an exceedance probability equal to the safety standard of that levee segment: e.g.: h(T=100,000) = 3.73 m+NAP (Hydra-Ring),
- **b)** Compute the acceptable probability of failure ($P_{T,cross}$) based on the safety standard and the length-effect parameters. E.g. consider f = 0.24, a = 0.4, b = 300 m, L = 36,500 m and T = 100,000 years see Eq.(1) and (3),
- c) per cross section and per sub-soil scenario (i = 1, ..., n), determine:
 - the characteristic values for all variables (X_k) see Tables 1 and 3,
 - the occurring safety factor for each sub-mechanism (γ^*) see Table 3 and Eq.(4),
 - the conservative estimate of the occurring reliability index and probability of failure ($\beta_{up,i}^*, P_{up,i}^*$), using the γ - β relation(s) and $\gamma_{up,i}^*$ see Fig. 3(c) and Eq.(5),
- **d)** per cross section reach an overall result (P_{cross}^*) by:
 - first combining the sub-mechanisms $(P_{f,i}^*)$ Eq.(6)²,
 - and then by combining the sub-soil scenarios Eq.(7), where P_{cross}^* is a conservative (safe) estimate of the cross sectional probability of failure,
- e) finally, assess the levee cross section as 'safe' or 'unsafe'. The considered levee cross section complies with the safety standard, regarding the piping failure mechanism, if it fulfils Eq. (2). For the analysed cross section P^{*}_{cross} < P_{T,cross} ⇔ 4.2 · 10⁻⁹ < 4.8 · 10⁻⁸. Therefore this cross section is assessed as safe.

$$P_{T,cross} = \frac{0.24 \cdot 1/100,000}{\left(1 + \frac{0.4 \cdot 36,500}{300}\right)} = 4.8 \cdot 10^{-8} \Longrightarrow \beta_{T,cross} = 5.33$$
(3)

² Notice that the minimum of the three failure probabilities (three sub-mechanisms) is equal to the failure probability due to piping mechanism, under the conservative assumption that the three sub-mechanisms are fully correlated.