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Prediction of the Slope Stability of Municipal Solid Waste Landfills Using the Reliability Analysis

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Abstract: Landfill stability is one of the most important concerns in the landfill design and operation. Several slope failures in the past decade highlight the need for the detailed analysis of landfill stability. These failures are attributed to reasons such as heterogeneous composition, improper evaluation of shear strength parameters and the effects of mechanical creep and biodegradation on strength response as well as non-consideration of material variability issues. MSW properties vary due to heterogeneous nature, overburden pressure, and degradation. This heterogeneity and uncertainty in MSW characteristics makes the analysis and design of landfills complicated. In the present study, stability as well as deformation of a typical lined landfill is analyzed using the Mohr Coulomb model for MSW, implemented in FLAC2D. The objective is to demonstrate the influence of variability in strength and stiffness parameters on stability of MSW landfill slopes. To capture the effect of variability, response surface method (RSM) is used in conjunction with Mohr-Coulomb model to develop multi-linear relationships among all the design variables. The influence of variation of unit weight with depth on stability analysis of landfill systems has been particularly studied and discussed. Results show that reliability index decreases with increase in variability of parameters and consideration of unit weight variation with depth provides more reliable estimates of factor of safety and deformations.

INTRODUCTION

Global stability of the sanitary landfill is one of the dominant problems in the landfill design from the geotechnical point of view. In order to ensure the stability of landfill, the design considerations should include the stability of waste, bottom liner and drainage layers, final cover system, as well as the stability of foundation soil. The stability of landfill is affected by the wide range of parameters. Probabilistic stability analysis, as a tool for estimation of safety level in geotechnical problems, became in the last few years a quite popular method. Moreover, in the case of landfills where uncertainties of waste strength parameters are very high, the use of probabilistic methods is fully justified. The objective of the present study is to demonstrate the influence of variability in model input parameters on stability of MSW landfill slopes. To capture the effect of variability, response surface method (RSM) is used in conjunction with Mohr-Coulomb model to develop multi-linear relationships among all the design variables. The following sections present the methodology, analysis results and implications.

NUMERICAL MODELLING

The landfill is modelled as a two dimensional plain strain model using FLAC2D 5.0. The slope geometry has been incorporated from Reddy et al. (1996). A schematic of the landfill composite liner configuration analysed in this study is shown in Fig. 1. The top of the side slope was assumed to be at a height of 30.5 m above the base grade. A base length of 122 m was selected for the MSW. The MSW was assumed to be placed in horizontal layers. The side slope of the landfill was taken as 1V:2H and face slope was assumed to be 1V:3H.

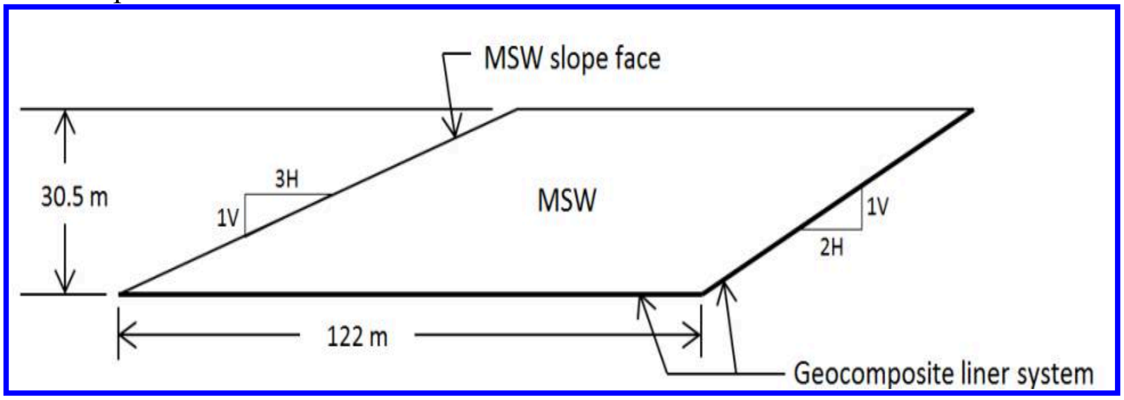

FIG. 1. Schematic diagram of the landfill

Mesh generation and boundary conditions

The mesh was generated using FLAC 2D version 5.0 (Fig. 2). It was assumed that the MSW was placed in eight lifts to reach the final height of 30.5 m. There are two prominent interfaces which are at the base and side slope of the given lined landfill. The interface parameters are taken from Reddy et.al (1996) as enlisted in Table 1. The geomembrane liner was assumed to be placed on a rigid foundation material which is considered to be fixed in both X and Y direction. The top and face slope are free to move.

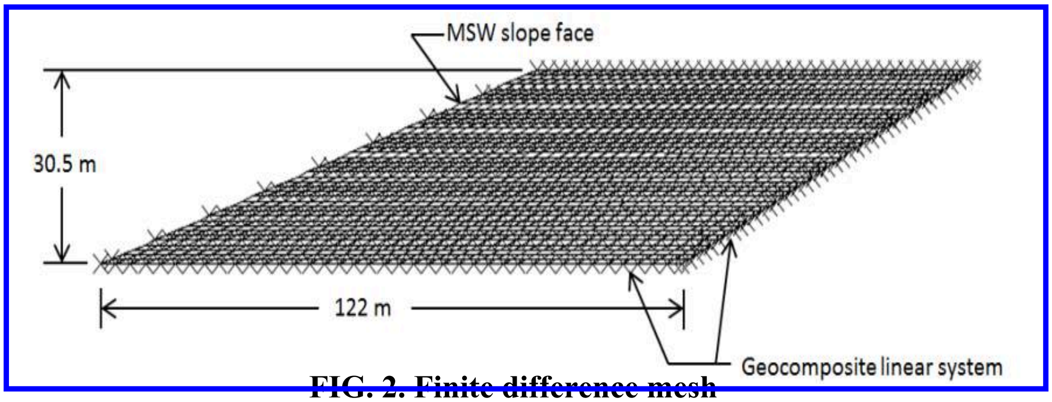

FIG. 2. Finite difference mesh

TABLE 1. Summary of interface model parameters (Reddy et.al 1996)

Parameter	Smooth HDPE geomembrane-nonwoven geotextile
Tangent shear stiffness, K_s	1490
Normal stiffness, K_n	10000
Cohesion, c_l	1.4 kPa
Friction angle, δ_l	11°

Material model

The Mohr-Coulomb model in FLAC 2D is the most conventional model to represent shear failure in soils. This model involves parameters such as unit weight (ρ), bulk modulus (K), shear modulus (G), cohesion (c), and friction angle (ϕ).

The parameters and their variation used in the analysis are presented in Table 2. The variability considered in these parameters is having a wide range and have been considered from published literature (Table 3) in order to generate response surface equations and carry out reliability analysis.

TABLE 2. Model input parameters and their variation

Parameter	Minimum	Maximum	Mean	COV (%)	Distribution
Unit Weight (kN/m^3)	7	20	13.5	48.15	Lognormal
Bulk Modulus (MPa)	2.69	107.53	55.11	95.12	Lognormal
Shear Modulus (MPa)	2.1	82.03	42.07	95.01	Lognormal
Cohesion (kPa)	0	15	7.5	100	Lognormal
Friction Angle (degrees)	20	30	25	20	Normal

Slope Stability

The Mohr-coulomb model which is an elasto-plastic model computes the factor of safety using strength reduction method in the finite difference program FLAC 2D. This method follows the load advancement number of steps. The incremental multiplier is used to specify the increment of the strength reduction of the first calculation step. The strength reductions are reduced successively in each step until all the steps have been performed. The final step should result in a fully developed failure mechanism. Here, the static analysis of slope is carried out under gravity loading and drained conditions.

VARIABILITY OF MSW PARAMETERS

For the engineering design of landfill, design parameters and their variability play vital role in design and decision making. Literature review indicates that the influence of all these parameters and their variations have significant effects on prediction of MSW slope stability analysis. It should be pointed out that the values shown in Table 3 are not the ''material'' properties, but the ''matrix'' properties, representing the elastic properties of the set of constituents.

TABLE 3. Variability of MSW shear strength and stiffness parameters.

Parameters	Range	Reference
Cohesion (c'), kPa		
1. Based on CD tests	0-27.5 43 50-75 16-19	Gabr and Valero (1995) Kavazanjian et al. (1999) Machado et al. (2002) Reddy et al. (2009)
2. Based on DST	10-23 5 2.5-4 31-64	Landva and Clark (1986) Houston at al. (1995) Mahler and Netto (2003) Reddy et al. (2009b)
Friction angle (ϕ'), degrees		
1. Based on CD tests	20.5-39 31 21-28 27-29	Gabr and Valero (1995) Kavazanjian et al. (1999) Machado et al. (2002) Reddy et al. (2009)
2. Based on DST	24-42 33-35 21-36 26-30	Landva and Clark (1986) Houston et al. (1995) Mahler and Netto (2003) Reddy et al. (2009b)
Elastic modulus (E), MPa 1. Soil-like material 2. Degradable and compressible 3. Reinforcing and tensile elements	10-20 0.5-0.7 1.5-3.0	Singh et al. (2007)
Poisson's ratio (ν) 1. Soil-like material 2. Degradable and compressible 3. Reinforcing and tensile elements	0.25-0.33 0.05-0.15 0.28-0.32	Singh et al. (2007)

The effect of variability is examined in terms of its influence on reliability index or probability of failure.

RESPONSE SURFACE METHODOLOGY (RSM)

RSM is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes (Myers and Montgomery, 2002). Once the important factors have been identified in an experiment, the next step is to determine the settings for these factors that result in the optimum value of the response. Methodologies that help the experimenter reach the goal of optimum response are referred to as response surface methods. These methods are exclusively used to examine the "surface," or the relationship between the response and the factors affecting the response. In this study, RSM is performed using Central Composite Design, which allows the estimation of all the regression parameters required to fit a second order model to a given response. After the designed experiment is performed, linear regression is used to obtain results.

Mohr-Coulomb model has been used to determine the responses – factor of safety (f.o.s) of the slope, maximum horizontal and vertical deformations. The correlation coefficients among the different input parameters and the responses have been calculated using the Pearson’s product-moment method and are provided in Table 4.

TABLE 4. Correlation coefficients among input parameters and responses

Parameter	Unit weight	Bulk modulus	Shear modulus	Cohesion	Friction angle	FOS	Maximum horizontal deforma-tion	Maximum vertical deforma-tion
Unit weight	1	-0.003	-0.003	-0.003	0	-0.235	0.406	0.351
Bulk modulus	-0.003	1	-0.003	-0.003	0	-0.032	-0.372	-0.665
Shear modulus	-0.003	-0.003	1	-0.003	0	0.018	-0.460	-0.035
Cohesion	-0.003	-0.003	-0.003	1	0	0.658	-0.048	-0.081
Friction angle	0	0	0	0	1	0.662	-0.028	-0.074
FOS	-0.235	-0.032	0.018	0.658	0.662	1	-0.119	-0.147
Maximum horizontal deforma-tion	0.406	-0.372	-0.460	-0.048	-0.028	-0.119	1	0.774
Maximum vertical deforma-tion	0.351	-0.665	-0.035	-0.081	-0.074	-0.147	0.774	1

The central composite design summary for the general values of COV is provided in Table 5 as follows. The best possible model with high correlation coefficient (R^2) has been adopted from the design, ensuring the adequacy of the model which effectively defines the relationship between the parameters and the responses.

TABLE 5. Central composite design summary

Response	Name	Minimum	Maximum	Mean	Model
fos	Factor of safety	1.01	2.85	1.79	Quadratic
$\delta_{hor,\ max}$	Maximum Horizontal Displacement (mm)	4	700	149.7	2 factor interaction
$\delta_{ver,\ max}$	Maximum Vertical Displacement (mm)	12.5	2000	459.2	2 factor interaction

RELIABILITY ANALYSIS

Reliability evaluation has been done using First Order Reliability Method (FORM). FORM is an analytical approximation in which the reliability index is interpreted as the minimum distance from the origin to the limit state surface in standardized normal space and the most likely failure point (design point) is searched using mathematical programming methods. To quantify the probability of the system failure, a reliability index, introduced by Hasofer and Lind (1974) has been used. An efficient reliability evaluation has been done using spreadsheets as introduced by Low and Tang (2004) and later modified in 2007. The index (β'_{HL}) can be calculated by minimising the quadratic form (in this case an ellipsoid) subjected to the constraint that the ellipsoid just touches the surface of the failure region. The matrix formulation of the Hasofer–Lind index (β') as modified in Low and Tang (2007) is:

$$\beta' = \min_{\underline{x} \in F} \sqrt{[\underline{n}]^T \underline{R}^{-1} [n]^T} \tag{1}$$

where, $\underline{n}$ is a column of vector n_i, and R is the correlation matrix. For each trial n_i, the value of the original basic random variable x_i is computed automatically:

$$x_i = F^{-1}[\varphi(ni)] \tag{2}$$

The above inverse distribution functions are either closed forms, or computed via a refined newton method. The “probability of failure”, from reliability index should be regarded as the probability that the performance function will yield unacceptable values for the analytical and statistical models adopted.

RESULTS

Response Surface Equations

RSM equations representing factor of safety for different percentage of COV have been developed to study the variability of input parameters on the response and shown in Table 6.

TABLE 6. Response Surface Equations representing factor of safety

COV (%)	Response Surface Equations	R^2
General values	$fos = 1.76 - 0.14\rho - 0.02K + 0.012G + 0.38c + 0.34\phi - 0.008\rho K + 0.006\rho G - 0.14\rho c$ $- 0.0053\rho\phi - 0.014KG - 0.0053Kc - 0.0016K\phi + 0.006Gc + 0.0022G\phi$ $+ 0.018c\phi + 0.035\rho^2 + 0.016K^2 + 0.0019G^2 - 0.052c^2 + 0.016\phi^2$	0.999
20	$fos = 1.81 - 0.064\rho - 0.001K - 0.001G + 0.07c + 0.34\phi - 0.001\rho K + 0.001\rho G$ $- 0.008\rho c - 0.004\rho\phi + 0.001KG + 1.38e-16Kc + 0.001K\phi + 1.54e-16Gc$ $- 0.001G\phi - 0.01c\phi + 0.021\rho^2 - 0.0024K^2 - 0.003G^2 - 0.00421c^2 + 0.008\phi^2$	0.998
10	$fos = 1.80 - 0.033\rho + 0.001K - 0.001G + 0.034c + 0.17\phi + 0.001\rho K - 0.001\rho G$ $- 0.003\rho c + 2.94e-16\rho\phi + 3.07e-16KG + 2.86e-16Kc + 0.001K\phi$ $- 2.86e-16Gc - 0.001G\phi - 0.001c\phi + 0.004\rho^2 + 0.001K^2$ $+ 0.001G^2 - 0.0001c^2 + 0.034\phi^2$	0.977

Similar RSM equations have been developed for vertical and horizontal deformations for the MSW slope. From the above equations, it is clearly observed that unit weight, cohesion and friction angle are the major influencing parameters on the stability of MSW slope, since the coefficients of the variables in the equations vary considerably with different percentages of COV. Computed values of coefficients of regression (R^2) give description of adequacy of fitted model. For a good model, values of R^2 should be close to 1. The calculated values are in the range of 0.98 in Table 6 indicating the adequacy of the regression equations.

Influence of variation of unit weight

Unit weight is one of the most influencing parameters in the stability analysis of landfill systems. The unit weight of MSW varies depending on the initial composition, compaction effort, decomposition, settlement and moisture content. The unit weight profile of MSW changes with depth of the landfill due to degradation and overburden stress of the above layers. In this study, an attempt has been made to observe the effect of unit weight variation with depth on the stability analysis of the slope. As proposed by Zekkos et al. (2006), eq. 3 has been used for the 30.5 height landfill in the FLAC model and stability analysis has been carried out, the values of other four parameters (c, φ, K, G) remaining the same.

$$\gamma = \gamma_i + \frac{z}{\alpha + \beta z} \tag{3}$$

where, γ_i= near-surface in-place unit weight (kN/m^3), z = depth (m) at which the MSW unit weight is to be estimated, α (m^4/kN) and β (m^3/kN) = modeling parameters. For the analysis, α and β values have been considered to be 3 and 0.2, respectively, which corresponds to a typical compaction effort and soil amount. It is observed that the factor of safety increases if the unit weight variation with depth is implemented in the analysis. Fig. 3 clearly explains the observation, with the other parameters assuming their minimum, mean and maximum values.

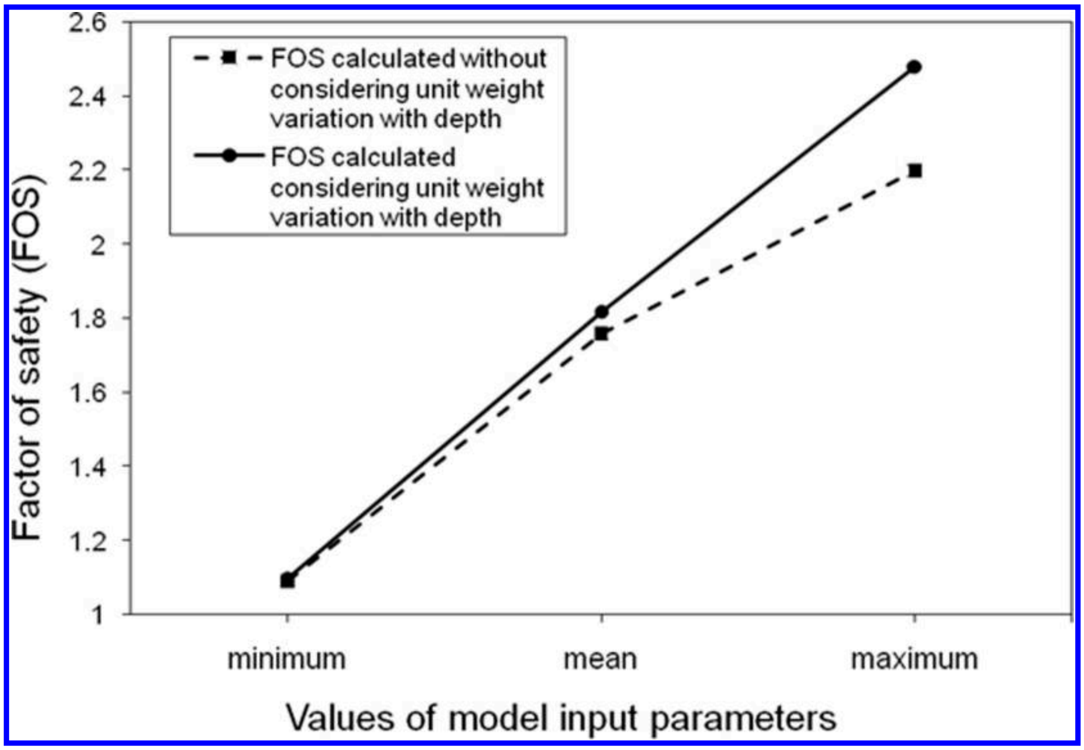

FIG. 3. Influence of variation of unit weight with depth on factor of safety of MSW slope

Reliability Analysis

One of the most useful figures which can be obtained with probabilistic stability analysis is the reliability index of analysed structure. The reliability index value expresses the distance of the mean margin of safety M (M is defined as the difference between the resistance and the load) from its critical value. It is necessary to know the variability associated with the factor of safety, horizontal and vertical displacement, knowing that the input parameters are random variables. Under these conditions, it is useful to evaluate the probability of obtaining a certain value less than the expected values.

Reliabilty index (β') is calculated using AFOSM method, where the RSM equations have been used as performance functions subjected to constraint values. Constraint values for deformations are evaluated corresponding to factor of safety of 1. *β'* is calculated against factor of safety greater than or equal to 1 and deformations less than the limiting values as provided in Table 7(a), whereas Table 7(b) represents the effect of variation of unit weight (γ) with depth on reliability index. The reliability index values increases after considering the variation.

TABLE 7(a). Reliability index values

COV of model input parameters	RESPONSES					
	Factor of safety		Maximum Vertical deformation		Maximum Horizontal deformation	
	Limiting values	β'	Limiting values corresponding to FOS=1	β'	Limiting values corresponding to FOS=1	β'
General values of COV	1	0.28	636.3 mm	13.16	187.6 mm	14.78
20 %	1	8.38	72.4 mm	7.07	19.7 mm	0.77
10 %	1	14.14	65.8 mm	10	21.8 mm	14.14

TABLE 7(b) Influence of unit weight variation with depth on reliability index (β)

	RESPONSES					
	Factor of safety		Maximum Vertical deformation		Maximum Horizontal deformation	
	Limiting values	β'	Limiting values corresponding to FOS=1	β'	Limiting values corresponding to FOS=1	β'
Consideration of unit weight variation with depth	1	0.58	434.7 mm	15.45	155.4 mm	15.77
No consideration of unit weight variation with depth	1	0.28	636.3 mm	13.16	187.6 mm	14.78

CONCLUSIONS

Central composite design has proved effective in generating response surface equations using five parameters (γ, c, φ, K, G), where interaction terms and quadratic model suitably describes the behavior of MSW. Stability analysis of landfill slope showed that reliability index decreases (from 14.14 to 8.38) with increase in variability of parameters (from 10 % to 20 % coefficient of variation in parameters). Consideration of unit weight variation with depth gives higher values of factor of safety and more reliable estimates of factor of safety and deformations.